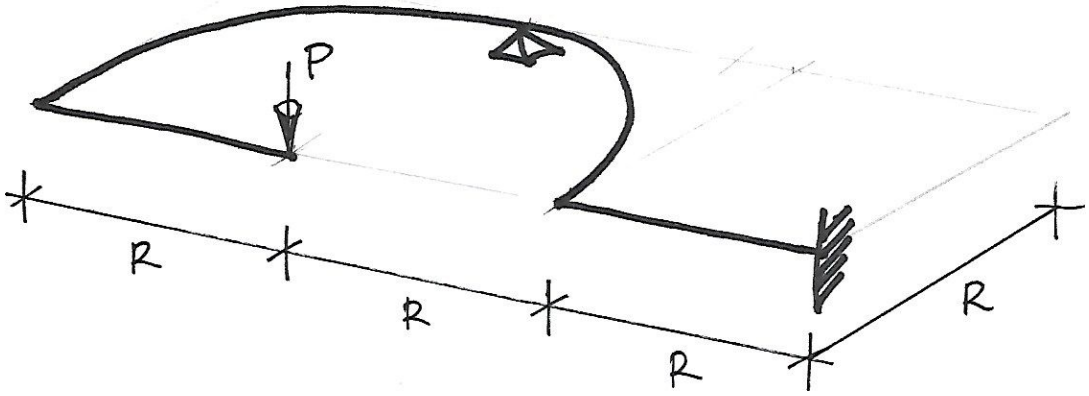
Mechanics of Structures, Civil Engineering Structures

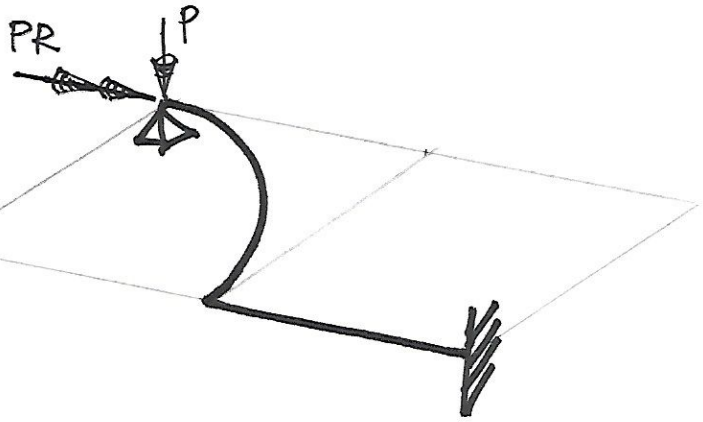
Test 1.1 27 March 2018

Find reactions in a rigid-joint grillage.

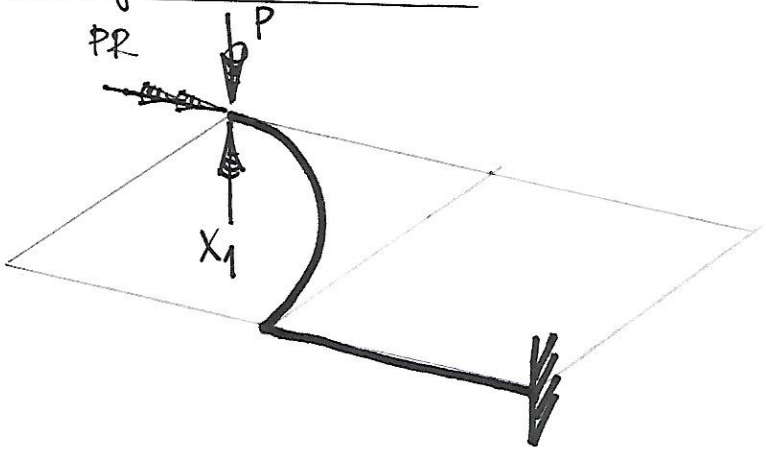
$EJ = GJ_s$



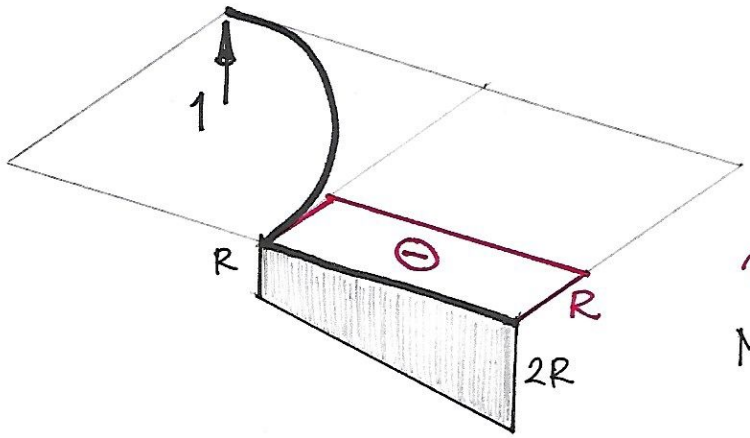
Note that the original problem reduces to the following:



Primary structure:



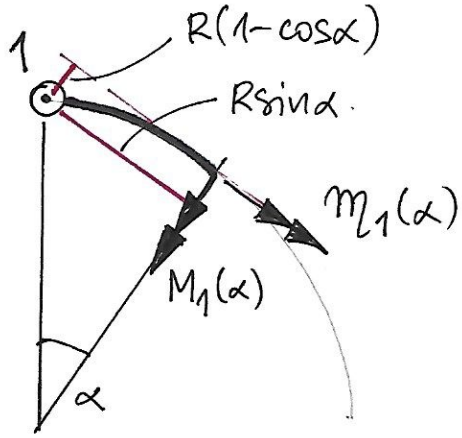
$X_1=1$ Load case



m_1

M_1

Bending and twisting moment diagrams for the straight part of the grillage.



Equilibrium of the curved part of the grillage.

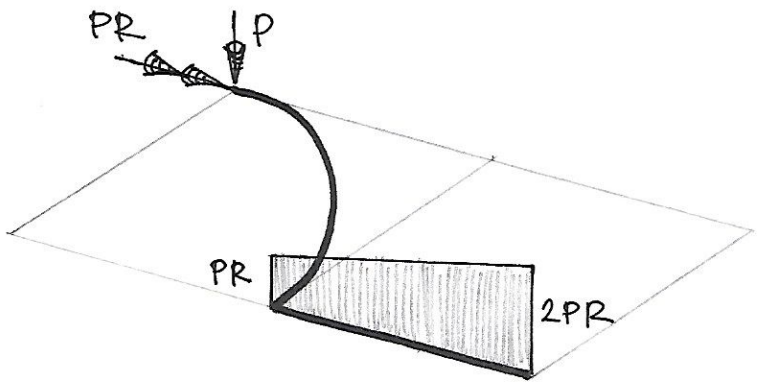
From the equilibrium equations for the curved part:

$$M_1(\alpha) - 1 \cdot R\sin\alpha = 0$$

$$m_1(\alpha) - 1 \cdot R(1-\cos\alpha) = 0$$

$$\alpha \in \left(0, \frac{\pi}{2}\right)$$

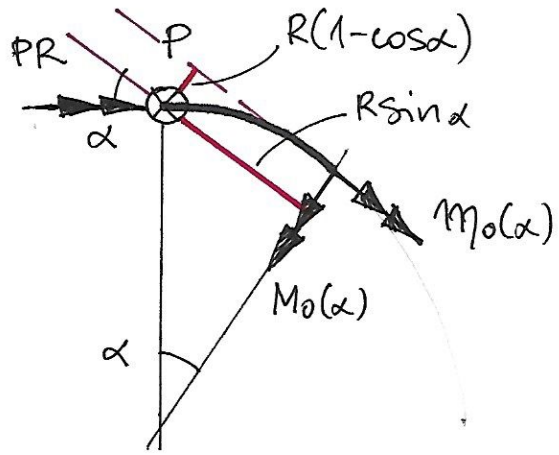
"0" load case



M_0

$m_0 = 0$

Diagrams for the straight part of the grillage.



Loads and internal forces acting on the curved part of the grillage.

From the equilibrium equations for the curved part:

$$M_0(\alpha) + P \cdot R\sin\alpha - PR \cdot \sin\alpha = 0$$

$$m_0(\alpha) + P \cdot R(1-\cos\alpha) + PR \cdot \cos\alpha = 0$$

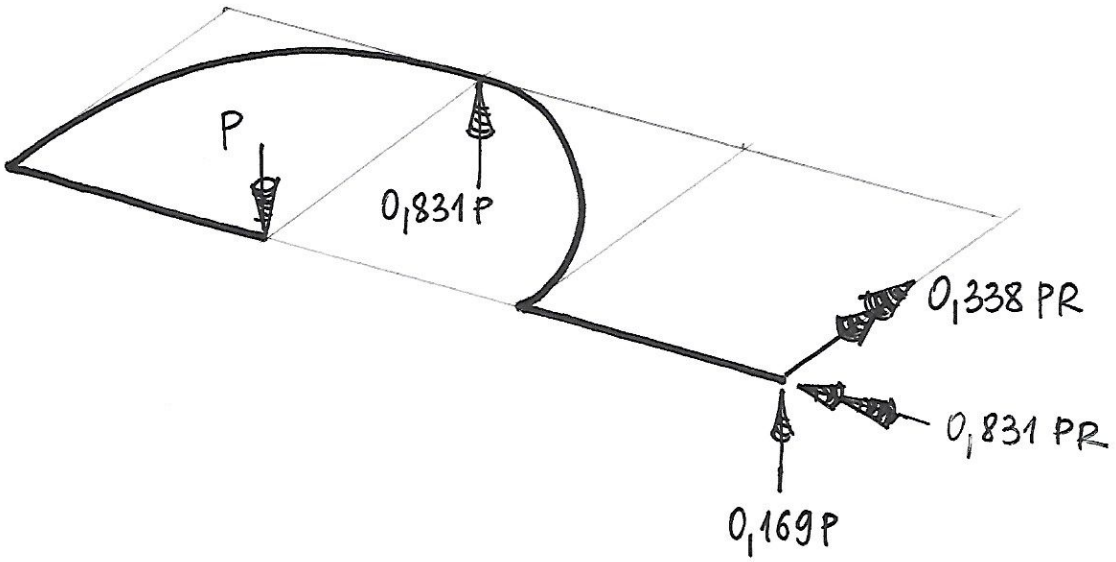
Calculations :

$$\delta_{11} = \left(\frac{13}{3} + \frac{\pi}{2}\right)\frac{R^3}{EJ} \approx 5{,}904\,\frac{R^3}{EJ}$$

$$\delta_{10} = -\left(\frac{10}{3} + \frac{\pi}{2}\right)\frac{PR^3}{EJ} \approx -4{,}904\,\frac{PR^3}{EJ}$$

$$X_1 = 0{,}831\,P$$

Reactions :



Note :

$$\delta_{11} X_1 + \delta_{10} = 0$$

$$\delta_{11} = \frac{1}{EJ}\left\{\left[M_1^2 + m_1^2\right]_{\substack{\text{straight}\\ \text{parts}}} + \int_0^{\frac{\pi}{2}} \left[M_1^2 + m_1^2\right] R\,d\alpha\right\}$$

$$\delta_{10} = \frac{1}{EJ}\left\{\left[M_1 M_0 + m_1 m_0\right]_{\substack{\text{straight}\\ \text{parts}}} + \int_0^{\frac{\pi}{2}} \left[M_1 M_0 + m_1 m_0\right] R\,d\alpha\right\}$$