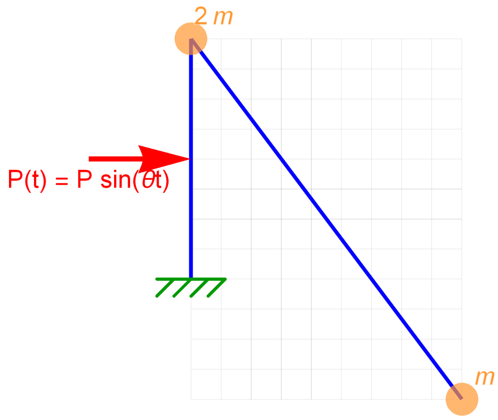
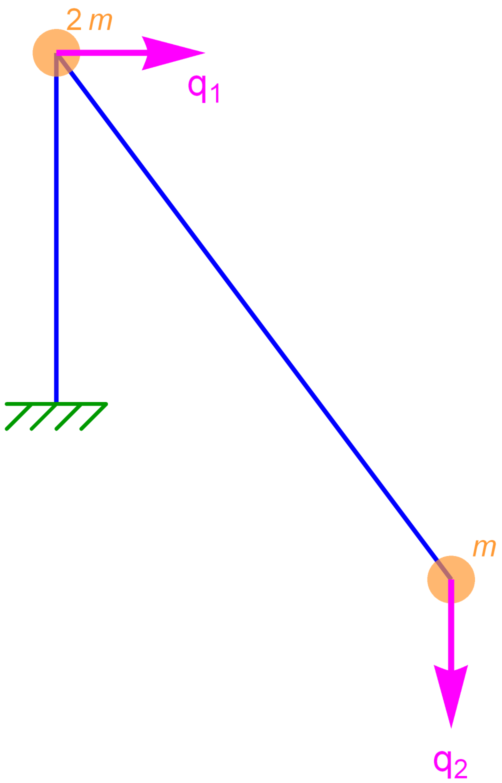
Geometria oraz obciążenia konstrukcji (wymiar oczka siatki - l, $\theta=\sqrt{\frac{EJ}{l^3 m}}$):

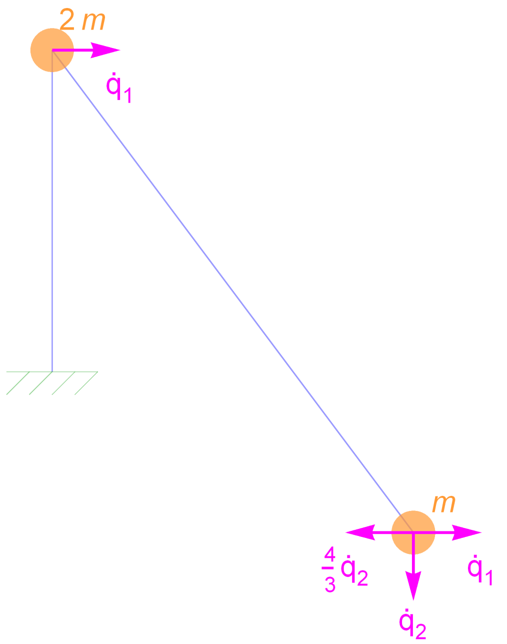


Zadanie statyki konstrukcji jest statycznie wyznaczalne.

Wspolrzedne Lagrange'a:



Plan predkosci:



Energia kinetyczna jako forma kwadratowa wektora $\dot{\mathbf{q}}$:

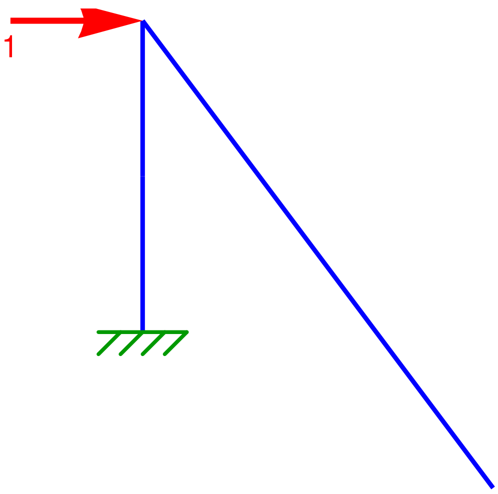
$$2\,E_k\,(\,\dot{\mathbf{q}}\,) = 2\,m\,\dot{q}_1^{\,2} + m\,[\,(\,\dot{q}_1 - \frac{4}{3}\,\dot{q}_2\,)^2 + \dot{q}_2^{\,2}\,] = 3\,m\,\dot{q}_1^{\,2} - \frac{4}{3}\,m\,\dot{q}_1\,\dot{q}_2 - \frac{4}{3}\,m\,\dot{q}_2\,\dot{q}_1 + \frac{25}{9}\,m\,\dot{q}_2^{\,2} = \dot{\mathbf{q}}^T\,\mathbf{M}\,\dot{\mathbf{q}}$$

Macierz mas:

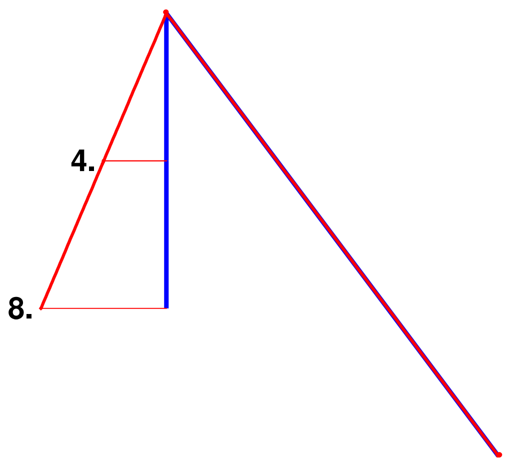
$$\mathbf{M} = m\begin{pmatrix} 3 & -\frac{4}{3} \\ -\frac{4}{3} & \frac{25}{9} \end{pmatrix}$$

Wykresy momentow zginajacych od jednostkowych sil bezwladnosci:

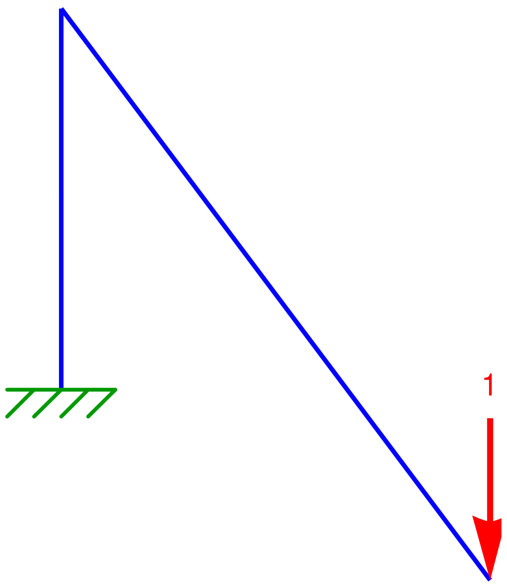
- od q_1:



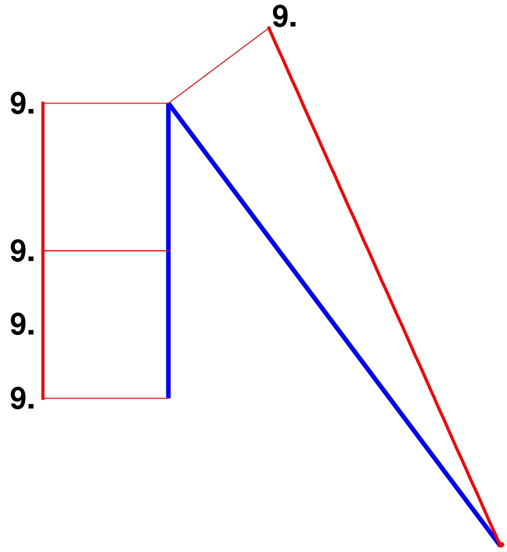
M_1[0. +1. 1]:



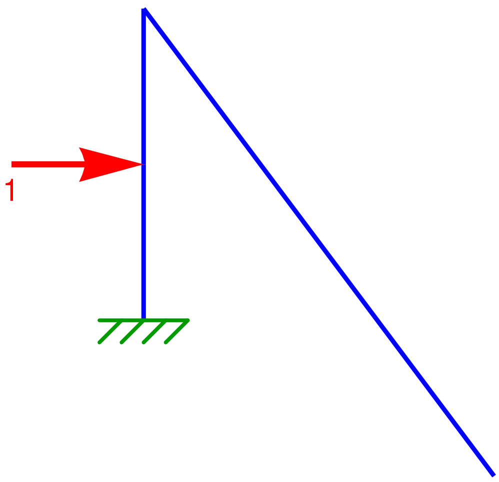
– od q_2:



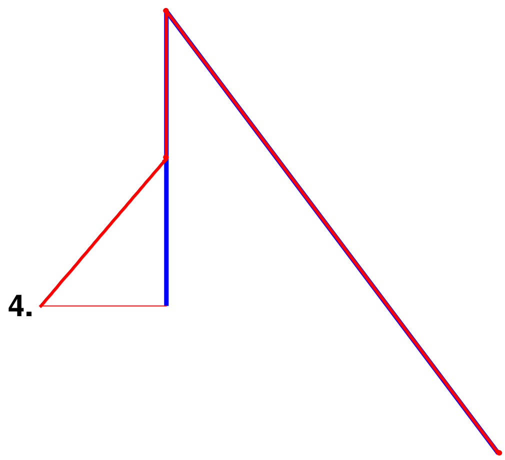
M_2[0. + 1. l]:



Wykresy momentow zginajacych od jednostkowego obciazenia wymuszajacego:



M_0[0. + 1. l]:



Macierz podatnosci:

$d_{11} = \frac{1}{EJ}[(\frac{1}{2}\cdot 8.1.\,l\cdot 4.\,l)(\frac{2}{3}\cdot 8.1.\,l+\frac{1}{3}\cdot 4.1.\,l)+(\frac{1}{2}\cdot 4.1.\,l\cdot 4.\,l)(\frac{1}{3}\cdot 8.1.\,l+\frac{2}{3}\cdot 4.1.\,l)]_1 + \frac{1}{EJ}[(\frac{1}{2}\cdot 4.1.\,l\cdot$

$d_{12} = d_{21} = \frac{1}{EJ}[(\frac{1}{2}\cdot 8.1.\,l\cdot 4.\,l)(9.1.\,l)+(\frac{1}{2}\cdot 4.1.\,l\cdot 4.\,l)(9.1.\,l)]_1 + \frac{1}{EJ}[(\frac{1}{2}\cdot 4.1.\,l\cdot 4.\,l)(9.1.\,l)]_2 =$

$d_{22} = \frac{1}{EJ}[(9.1.\,l\cdot 4.\,l)(9.1.\,l)]_1 + \frac{1}{EJ}[(9.1.\,l\cdot 4.\,l)(9.1.\,l)]_2 + \frac{1}{EJ}[(\frac{1}{2}\cdot 9.1.\,l\cdot 15\,l)(\frac{2}{3}\cdot 9.1.\,l)]_3 = 1$

$$\mathbf{D} = 0. + \frac{1.\,l^3}{EJ}\begin{pmatrix} 170.667 & 288. \\ 288. & 1053. \end{pmatrix}$$

Przemieszczenia od jednostkowego obciazenia wymuszajacego:

$d_{10} = \frac{1}{EJ}[(\frac{1}{2}\cdot 4.1.\,l\cdot 4.\,l)(\frac{2}{3}\cdot 8.1.\,l+\frac{1}{3}\cdot 4.1.\,l)]_1 = 53.33330. + \frac{1.\,l^3}{EJ}$

$d_{20} = \frac{1}{EJ}[(\frac{1}{2}\cdot 4.1.\, l \cdot 4.\, l)(9.1.\, l)]_1 = 72.0. + \frac{1.\, l^3}{EJ}$

ZADANIE DRGAN HARMONICZNYCH

- poszukiwanie funkcji przemieszczen postaci:

$$\mathbf{q}(t) = \mathbf{a}\sin(\theta\, t) = \mathbf{a}\sin\left(\left(\frac{EJ}{l^3\, m}\right)^{0.500} t\right)$$

- rownania ruchu:

$$(\,\mathbf{I} - \theta^2\, \mathbf{D}\, \mathbf{M}\,)\, \mathbf{a} = \mathbf{d_0}\, P$$

$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 1.0000 \begin{pmatrix} 170.667 & 288. \\ 288. & 1053. \end{pmatrix} \begin{pmatrix} 3.000 & -1.333 \\ -1.333 & 2.778 \end{pmatrix}\right) \mathbf{a} = 0. + \frac{1.\, l^3\, P}{EJ} \begin{pmatrix} 53.3333 \\ 72. \end{pmatrix}$$

$$\begin{pmatrix} -127. & -572.444 \\ 540. & -2540. \end{pmatrix} \mathbf{a} = 0. + \frac{1.\, l^3\, P}{EJ} \begin{pmatrix} 53.3333 \\ 72. \end{pmatrix}$$

Amplituda wektora przemieszczeń:

$$\mathbf{a} = \frac{1.\, l^3\, P}{EJ} \begin{pmatrix} -0.149202 \\ -0.0600665 \end{pmatrix}$$

Wektor uogolnionych sil bezwladnosci:

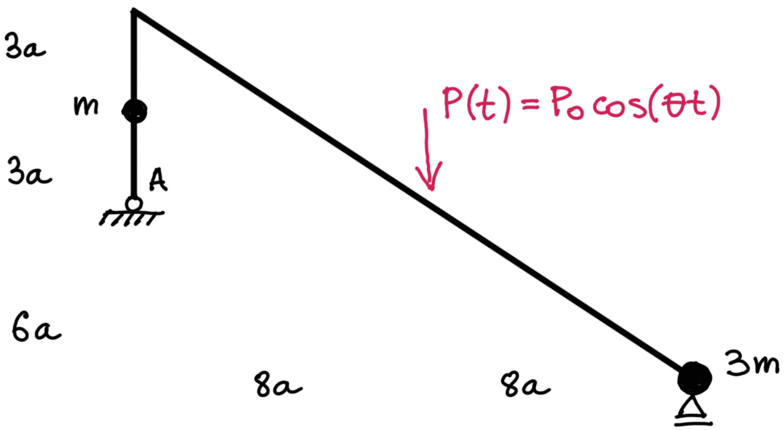
$$\hat{\mathbf{B}} = \theta^2\, \mathbf{M}\, \mathbf{a} = \frac{EJ}{l^3\, m}\, m \begin{pmatrix} 3.000 & -1.333 \\ -1.333 & 2.778 \end{pmatrix} \frac{1.\, l^3\, P}{EJ} \begin{pmatrix} -0.149202 \\ -0.0600665 \end{pmatrix} = 1.\, P \begin{pmatrix} -0.367516 \\ 0.0320841 \end{pmatrix}$$

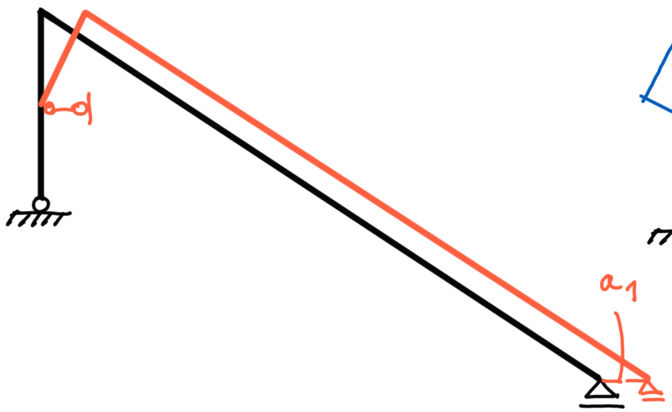
Wykres amplitudy momentow zginajacych:

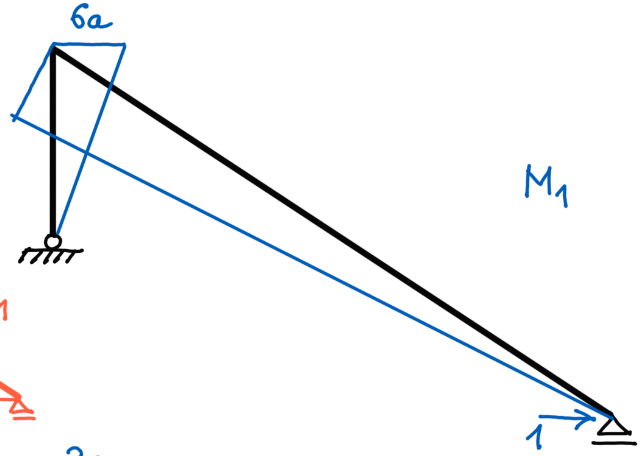
M[0. + 1. l P]:

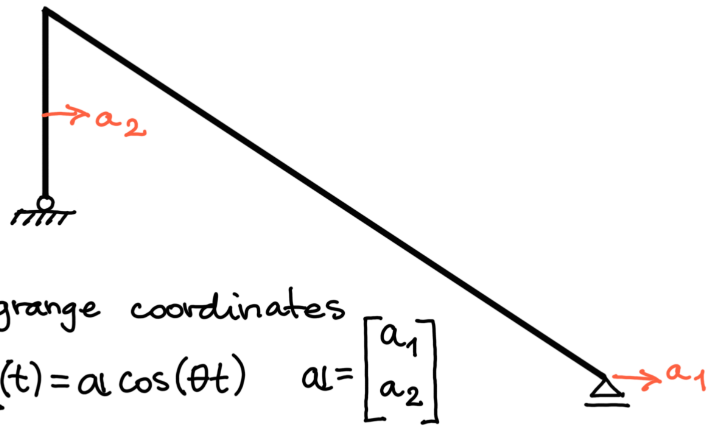
Test 2.2   2025/26 (MoS2 course) $EA=\infty$, EJ − const., $\mu=0$, $\theta=\sqrt{\frac{EJ}{ma^3}}$

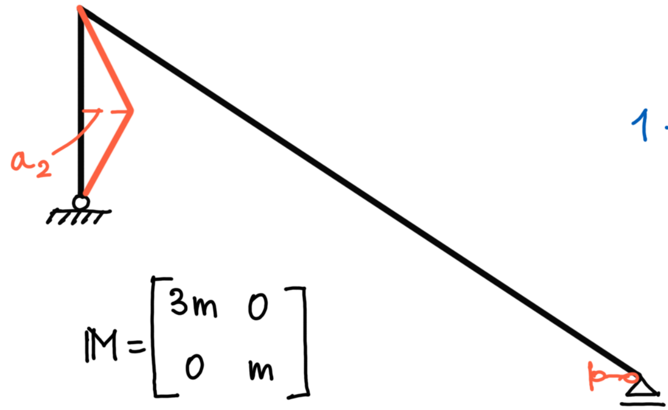
Compute the amplitude of the horizontal component of reaction at A.

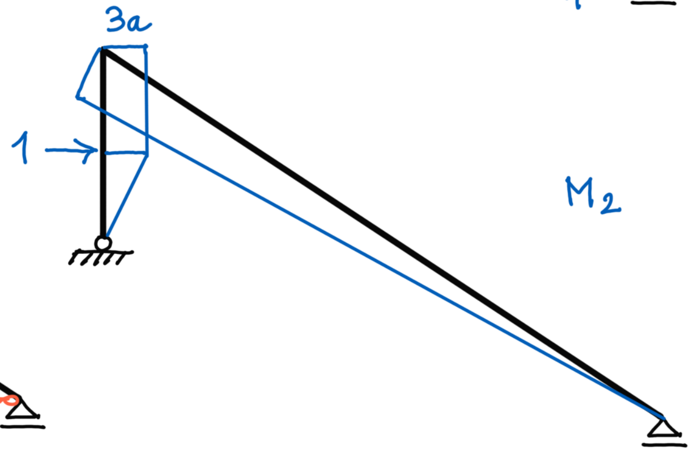












Lagrange coordinates

$\mathbb{q}(t)=\mathbb{a}\cos(\theta t)$ $\mathbb{a}=\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$\mathbb{M}=\begin{bmatrix} 3m & 0 \\ 0 & m \end{bmatrix}$

$d_{11}=M_1 \square M_1 = 312\frac{a^3}{EJ}$

$d_{12}=d_{21}=M_1 \square M_2 = \frac{339}{2}\frac{a^3}{EJ}$

$d_{22}=M_2 \square M_2 = 96\frac{a^3}{EJ}$

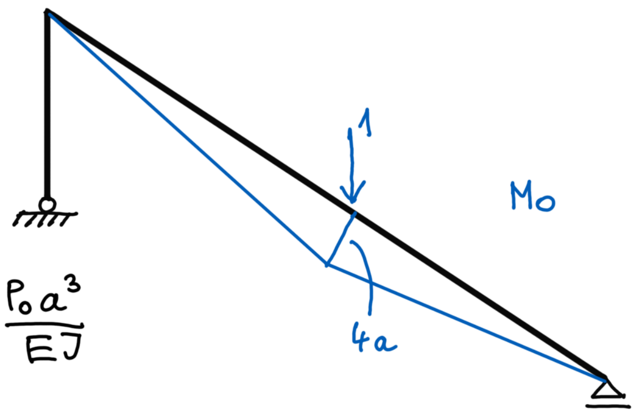
$d_{10}=M_1 \square M_0 = 120\frac{a^3}{EJ}$

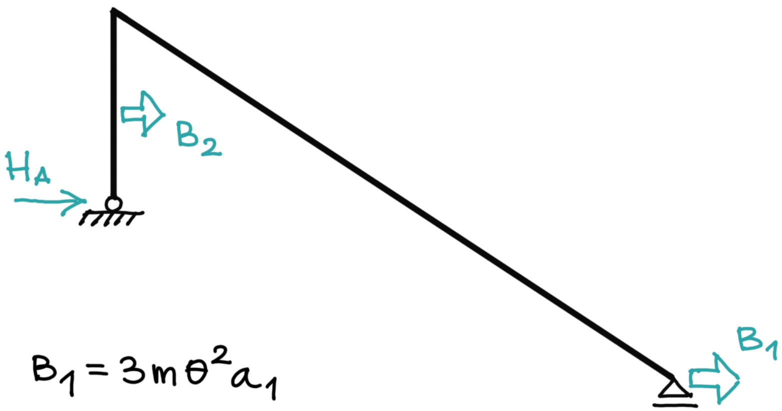
$d_{20}=M_2 \square M_0 = 60\frac{a^3}{EJ}$

$\mathbb{D}=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$ $\mathbb{d}_0=\begin{bmatrix} d_{10} \\ d_{20} \end{bmatrix}$

$\mathbb{I}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(\mathbb{I}-\theta^2\mathbb{D}\mathbb{M})\mathbb{a}=\mathbb{d}_0 P_0 \rightarrow \mathbb{a}=\begin{bmatrix} -0{,}467 \\ 1{,}868 \end{bmatrix}\frac{P_0 a^3}{EJ}$





$$B_1 = 3m\theta^2 a_1$$

$$= -1{,}401\ P_0$$

$$B_2 = m\theta^2 a_2$$

$$= 1{,}868\ P_0$$

$$H_A + B_1 + B_2 = 0$$

$$H_A = -0{,}467\ P_0$$