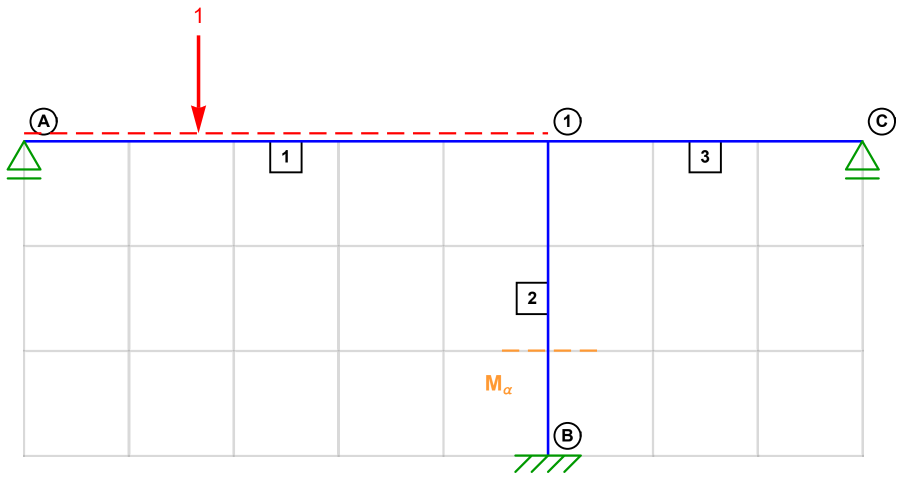
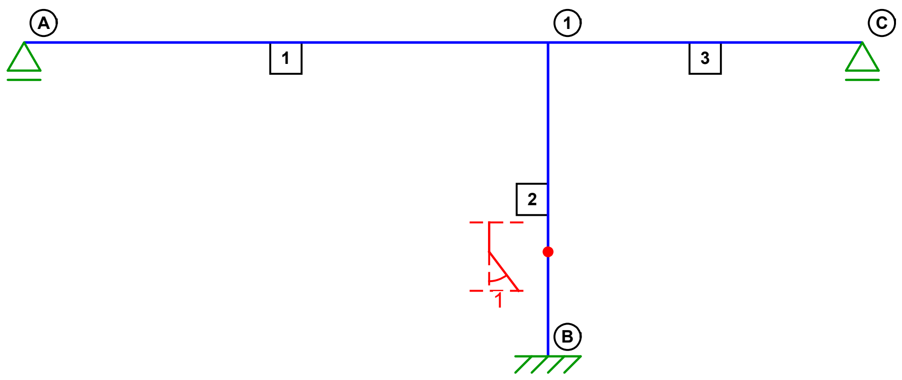
Określenie zadania linii wpływu (wymiar oczka siatki – 1):



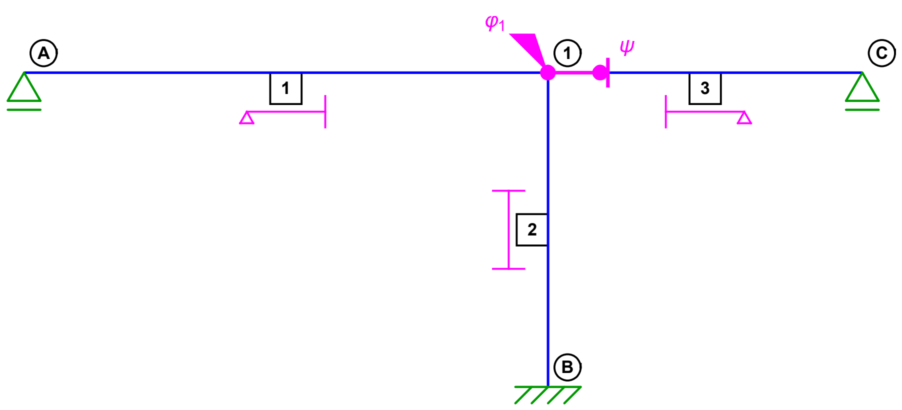
Zadanie statyki konstrukcji wg. twierdzenia Bettiego:



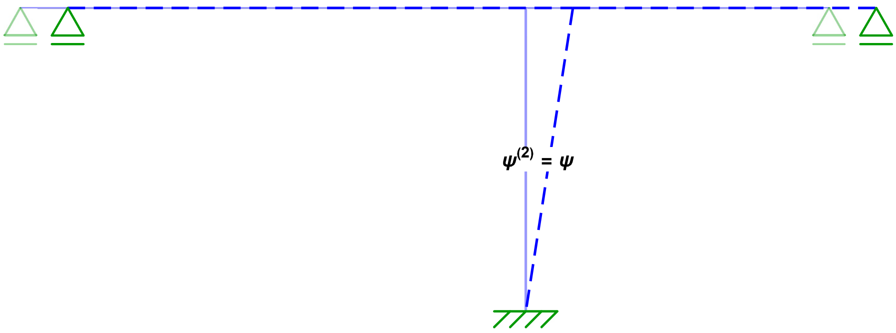
Wektor niewiadomych:

$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \psi \end{pmatrix}$$

Układ geometrycznie wyznaczalny:



Plan przemieszczeń:



$\psi^{(1)} = 0$

$\psi^{(2)} = \psi$

$\psi^{(3)} = 0$

Momenty wyjściowe:

$\Phi_B^{02} = \frac{2}{3}\frac{EJ}{1}$

Wzory transformacyjne:

$\Phi_1^1 = \frac{EJ}{1}\left[\frac{3}{5}\varphi_1\right]$

$\Phi_1^2 = \frac{EJ}{1}\left[\frac{4}{3}\varphi_1 - 2\psi\right]$

$\Phi_B^2 = \frac{EJ}{1}\left[\frac{2}{3}\varphi_1 - 2\psi\right] + \frac{2}{3}\frac{EJ}{1}$

$\Phi_1^3 = \frac{EJ}{1}\left[\varphi_1\right]$

Równania równowagi:

$\Phi_1^1 + \Phi_1^2 + \Phi_1^3 = 0$

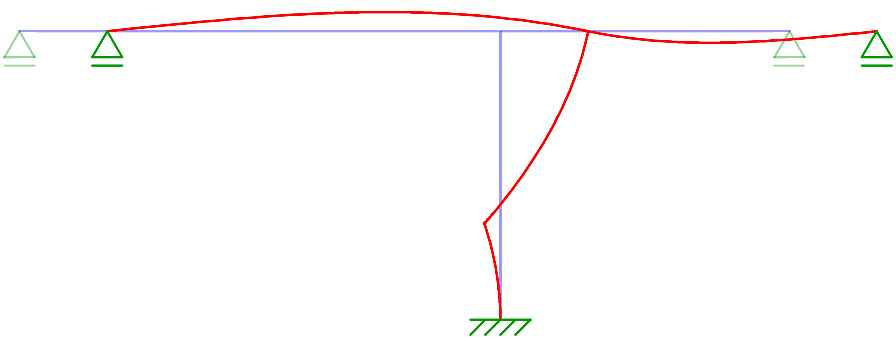
$(\Phi_1^2 + \Phi_B^2)\ \psi = 0$

$$\frac{EJ}{1}\begin{pmatrix}\frac{44}{15} & -2 \\ -2 & 4\end{pmatrix}\begin{pmatrix}\varphi_1 \\ \psi\end{pmatrix} = \frac{EJ}{1}\begin{pmatrix}0 \\ \frac{2}{3}\end{pmatrix}$$

Rozwiązanie metody przemieszczeń:

$$\mathbf{q} = \begin{pmatrix}\varphi_1 \\ \psi\end{pmatrix} = \begin{pmatrix}0.172 \\ 0.253\end{pmatrix}$$

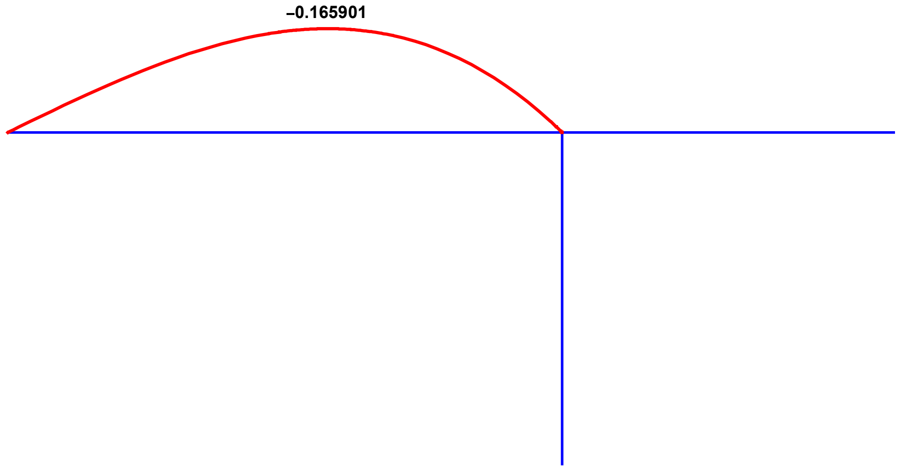
Deformacja konstrukcji:



Funkcja linii wpływu na poczególnych prętach:

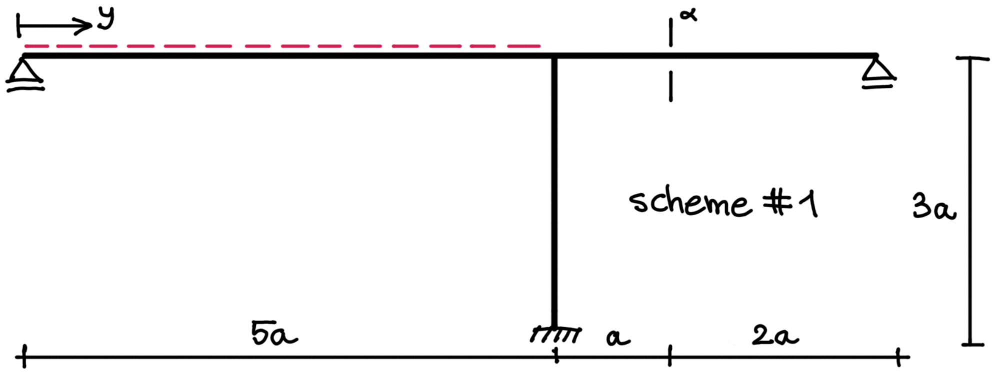
$Lw^{(1)}(\eta) = -0.431\,l\,\eta + 0.431\,l\,\eta^{3}$

Linia wpływu[l]:

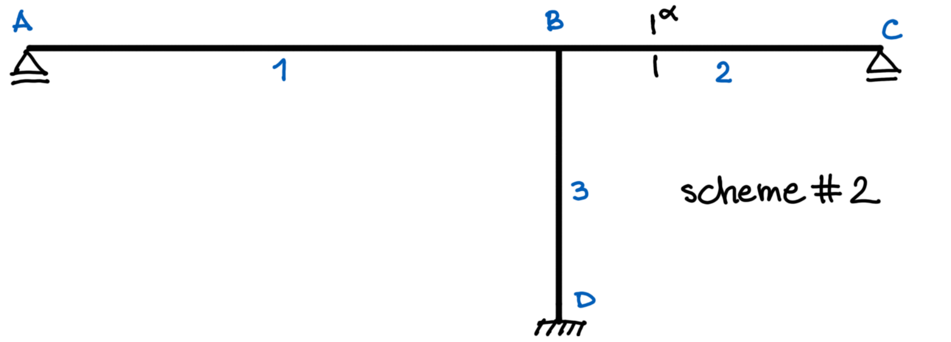


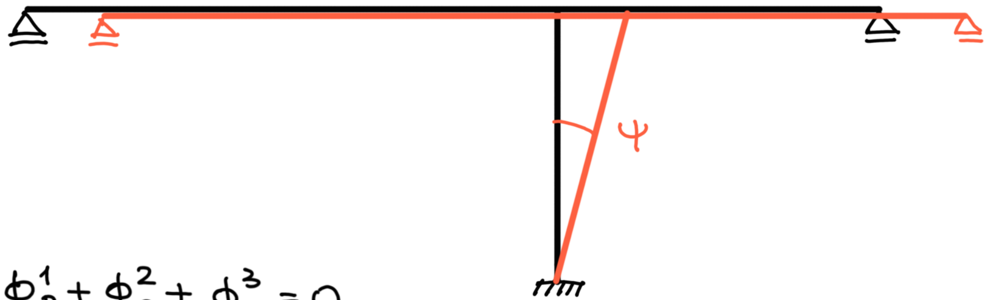
Test 1.3 2025/26 (MoS 2 course)

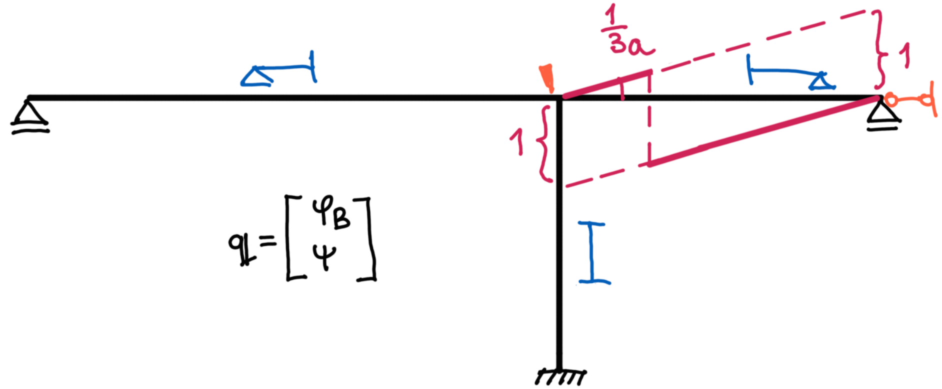
Compute $T_\alpha(y)$, $y \in (0, 5a)$. $EA = \infty$, EJ – const.



By Betti's Theorem, $T_\alpha(y) = \tilde{w}(y)$, where $\tilde{w}(y)$ stands for deflection of element 1 due to geometric imperfection imposed at α in scheme #2.







$$q = \begin{bmatrix} \varphi_B \\ \psi \end{bmatrix}$$

$$\begin{cases} \Phi^1_B + \Phi^2_B + \Phi^3_B = 0 \\ (\Phi^3_B + \Phi^3_D)\cdot\psi = 0 \quad \Big| \cdot \left(-\frac{1}{\psi}\right) \end{cases}$$

$$\frac{EJ}{a}\left\{\left[\frac{3}{5} + 1 + \frac{4}{3}\right]\varphi_B + [-2]\psi\right\} + \frac{EJ}{a^2}\cdot\frac{1}{3} = 0$$

$$\frac{EJ}{a}\left\{\left[-\frac{4}{3} - \frac{2}{3}\right]\varphi_B + [2+2]\psi\right\} = 0$$

$$\Rightarrow q = \begin{bmatrix} -\frac{5}{29} \\ -\frac{5}{58} \end{bmatrix}\cdot\frac{1}{a}$$

$\mathring{\varphi}_B = \frac{1}{3a}$ compensates for the violated boundary condition at B in element 2.

$$\mathring{\Phi}^2_B = \frac{EJ}{3a}\left[3\cdot\frac{1}{3a}\right] = \frac{EJ}{3a^2}$$

$\tilde{w}_0(y) = 0 \Rightarrow \tilde{w}(y) = \tilde{w}_q(y) = C_0 + C_1 y + C_2 y^2 + C_3 y^3$ with boundary conditions: $\tilde{w}(0) = 0$, $\tilde{M}(0) = 0$, $\tilde{w}(5a) = 0$, $\tilde{\varphi}(5a) = \varphi_B$

Then, $C_0 = 0$, $C_1 = \frac{5}{58a} = 0{,}086\cdot\frac{1}{a}$, $C_2 = 0$, $C_3 = -\frac{1}{290}\cdot\frac{1}{a^3} = -0{,}003\,\frac{1}{a^3}$ and

$$T_\alpha(y) = \tilde{w}(y) = \frac{25}{58}\left(\frac{y}{5a}\right) - \frac{25}{58}\left(\frac{y}{5a}\right)^3 = 0{,}431\left(\frac{y}{5a}\right) - 0{,}431\left(\frac{y}{5a}\right)^3$$

G. Dzierzanowski