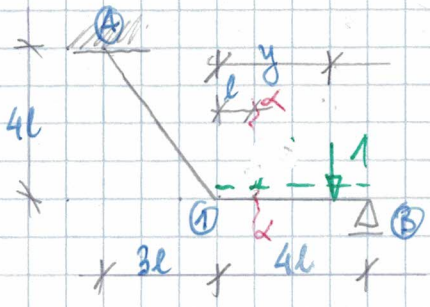
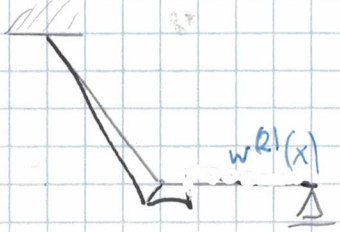
Kolokwium 1.2. MK2 r.a. 2023/24

Układ nr 1 Układ nr 2

EJ = const. $EA \to \infty$





2 tw. Bettiego

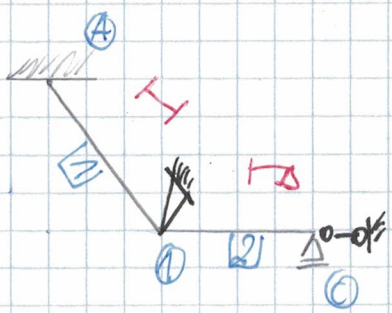
$L_{(1)(2)} = M_\alpha(-1) + 1 \cdot w^{(2)}(y)$

$L_{(2)(1)} = 0$

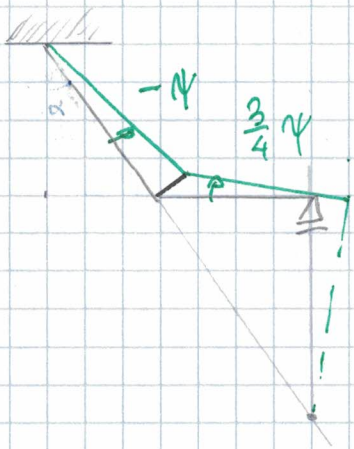
$L_{(1)(2)} = L_{(2)(1)}$

$M_\alpha(y) = w^{(2)}(y)$

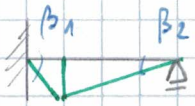
Funkcja ugięcia $w^{(2)}(x)$ - odc. A - B



$q = \begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix}$



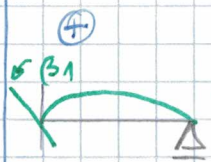
Pręt nr 2



$\beta_1 + \beta_2 = 1$

$\beta_1 \cdot 4 = \beta_2 \cdot 3 \cdot 4$

$\beta_2 = \frac{1}{4}$ $\beta_1 = \frac{3}{4}$



Równania równowagi MP

$\Phi_1^{(1)} + \Phi_1^{(2)} = 0$ (1)

$\left(\Phi_A^{(2)} + \Phi_1^{(1)}\right)(-\bar{\psi}) + \Phi_1^{(2)}\left(\frac{3}{4}\bar{\psi}\right) = 0$ (2)

Moment wyjściowy

$\Phi_1^{02} = \frac{3EJ}{4l}\left[-\frac{3}{4}\right] = -\frac{9}{16}\frac{EJ}{l}$

→ WT

$\Phi_A^{(1)} = \frac{2EJ}{5l}\left[\varphi_1 + 3\psi\right] = \frac{EJ}{l}\left[\frac{2}{5}\varphi_1 + \frac{6}{5}\psi\right]$ $\Phi_1^{(1)} = \frac{EJ}{l}\left[\frac{4}{5}\varphi_1 + \frac{6}{5}\psi\right]$

(1)

$\Phi_1^{(2)} = \frac{3EJ}{4l}\left[\varphi_1 - \frac{3}{4}\psi\right] + \Phi_1^{02} = \frac{EJ}{l}\left[\frac{3}{4}\varphi_1 - \frac{9}{16}\psi\right] - \frac{9}{16}\frac{EJ}{l}$ $\left(-\frac{3}{4}\right)$

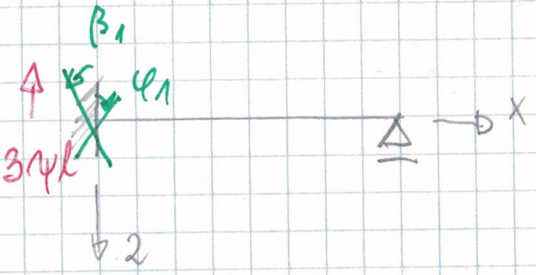
$$\frac{EJ}{l}\left[\begin{array}{c|c} \frac{4}{5}+\frac{3}{4} & \frac{6}{5}-\frac{9}{16} \\ \hline \frac{6}{5}-\frac{9}{16} & \frac{12}{5}+\frac{27}{64} \end{array}\right]\left[\begin{array}{c} \varphi_1 \\ \hline \psi \end{array}\right] = \frac{EJ}{l}\left[\begin{array}{c} \frac{9}{16} \\ \hline -\frac{27}{64} \end{array}\right]$$

$\varphi_1 = 0{,}468$

$\psi = -0{,}255$

Funkcja ugięcia $\tilde{w}(x)$ – odc. ① – ②

$$\tilde{w}(x) = Ax^3 + Bx^2 + Cx + D$$
$$\tilde{w}'(x) = 3Ax^2 + 2Bx + C$$
$$\tilde{w}''(x) = 6Ax + 2B$$



$\tilde{w}(0) = -3\varphi l = -0{,}766\,l$ $\qquad \tilde{w}(4l) = 0$

$\varphi(0) = w'(0) = \varphi_1 - \beta_1 = -0{,}282$ $\qquad M(4l) = 0 \Rightarrow w''(4l) = 0$

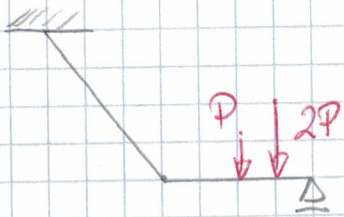
$$A = \frac{0{,}0028}{l^2} \qquad B = \frac{0{,}0339}{l} \qquad C = -0{,}282 \qquad D = 0{,}766l$$

$$\tilde{w}(x) = 0{,}0028\frac{x^3}{l^2} + 0{,}0339\frac{x^2}{l} - 0{,}282x + 0{,}766l$$

$$w_{00}(x) = \begin{cases} \frac{3}{4}x & , x < l \\ \frac{1}{4}(4l - x) & , x \geq l \end{cases}$$

$$M_\alpha(y) = w_{00}(y) + \tilde{w}(y)$$

Wartość momentu od układu sił P



$$M_\alpha^? = P \cdot M_\alpha(2l) + 2P\,M_\alpha(3l) =$$
$$= 1{,}613\,Pl$$