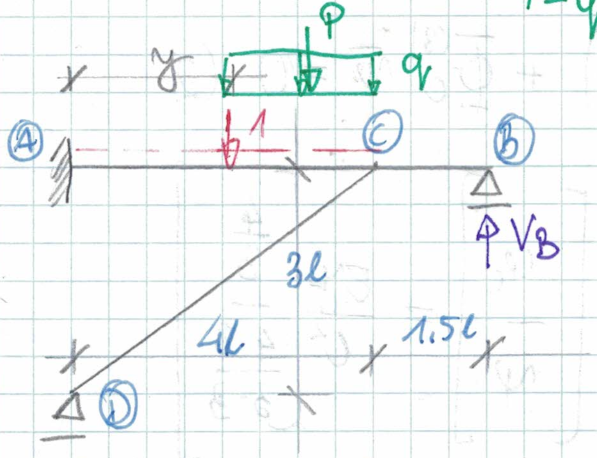
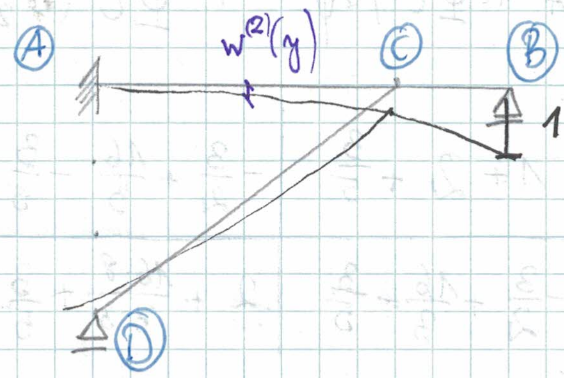
Kolokwium 1.1. r.a. 2023/24

układ nr 1 $P = ql$ układ nr 2





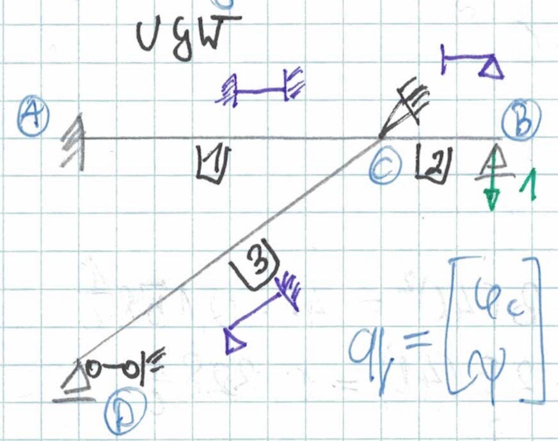
$EJ = const.$ $EA \to \infty$

2 tw. Bettiego $L_{12} = 1 \cdot w^{(2)}(y) + V_B(-1)$ $L_{21} = 0$

$V_B(y) = w^{(2)}(y)$

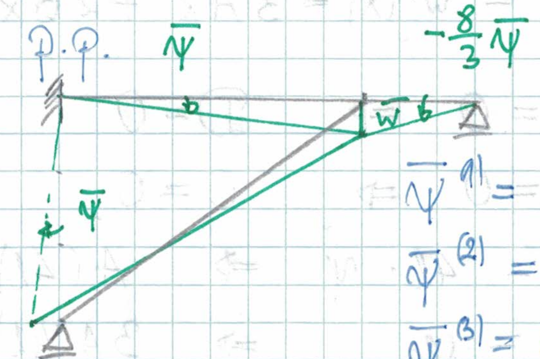
Deformacja układu nr 2 - MP

UGW



$q = \begin{bmatrix} \varphi_C \\ \psi \end{bmatrix}$

p.p.



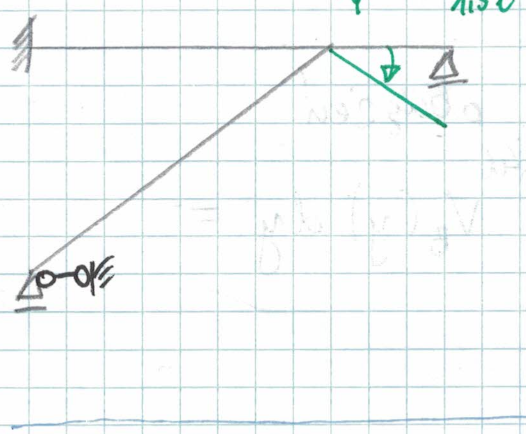
$\bar\psi^{(1)} = \bar\psi$

$\bar\psi^{(2)} = -\frac{8}{3}\bar\psi$

$\bar\psi^{(3)} = \bar\psi$

zerowy p.p.

$\psi^{02} = \frac{1}{1,5l}$



WT 1]

$$\Phi_A^{(1)} = \frac{2EJ}{4l}[\varphi_C - 3\psi] = \frac{EJ}{l}\left[\frac{1}{2}\varphi_C - \frac{3}{2}\psi\right]$$

$$\Phi_C^{(1)} = \frac{EJ}{l}\left[\varphi_C - \frac{3}{2}\psi\right] \quad (-1)$$

2]

$$\Phi_C^{(2)} = \frac{3EJ}{1,5l}\left[\varphi_C + \frac{8}{3}\psi\right] + \frac{3EJ}{1,5l}\left[-\frac{1}{1,5l}\right] =$$

$$= \frac{EJ}{l}\left[2\varphi_C + \frac{16}{3}\psi\right] + \frac{EJ}{l}\left[-\frac{4}{3l}\right] \quad \left(\frac{8}{3}\right)$$

3]

$$\Phi_C^{(3)} = \frac{3EJ}{5l}[\varphi_C - \psi] = \frac{EJ}{l}\left[\frac{3}{5}\varphi_C - \frac{3}{5}\psi\right] \quad (-1)$$

r. r. MP

$$\Phi_c^{(1)} + \Phi_c^{(2)} + \Phi_c^{(3)} = 0 \quad (1)$$

$$\left(\Phi_A^{(1)} + \Phi_c^{(1)}\right)\bar{\psi} + \Phi_c^{(2)}\left(-\frac{8}{3}\bar{\psi}\right) + \Phi_c^{(3)}\bar{\psi} = 0 \quad (2)$$

$$\frac{EJ}{l}\left[\begin{array}{c|c} 1 + 2 + \frac{3}{5} & -\frac{3}{2} + \frac{16}{3} - \frac{3}{5} \\ \hline -\frac{3}{2} + \frac{16}{3} - \frac{3}{5} & 3 + \frac{16 \cdot 8}{3 \cdot 3} + \frac{3}{5} \end{array}\right]\left[\begin{array}{c} \varphi_c \\ \hline \psi \end{array}\right] = \frac{EJ}{l^2}\left[\begin{array}{c} \frac{4}{3} \\ \hline \frac{4 \cdot 8}{3 \cdot 3} \end{array}\right]$$

$$\varphi_c = 0{,}228\,\frac{1}{l} \qquad \psi = 0{,}158\,\frac{1}{l}$$

Linia ugięcia (A) – (C)

$$w(x) = Ax^3 + Bx^2 + Cx + D$$

$$\varphi(x) = w'(x) = 3Ax^2 + 2Bx + C$$

$$w(0) = 0 \Rightarrow D = 0$$

$$\varphi(0) = 0 \Rightarrow C = 0$$

$$w(4l) = 4l \cdot \psi \Rightarrow A(4l)^3 + B(4l)^2 = 4l \cdot 0{,}158\,\frac{1}{l}$$

$$\varphi(4l) = \varphi_c \Rightarrow 3A(4l)^2 + 2B(4l) = 0{,}228\,\frac{1}{l}$$

$$w(x) = -0{,}0055\,\frac{x^3}{l^3} + 0{,}0615\,\frac{x^2}{l^2}$$

$$V_B(y) = w(y)$$

Wartość reakcji od zadanych obciążeń

$$V_B = q\,l \cdot V_B(3l) + q \cdot \int_{2l}^{4l} V_B(y)\,dy =$$

$$= 1{,}223\,ql$$