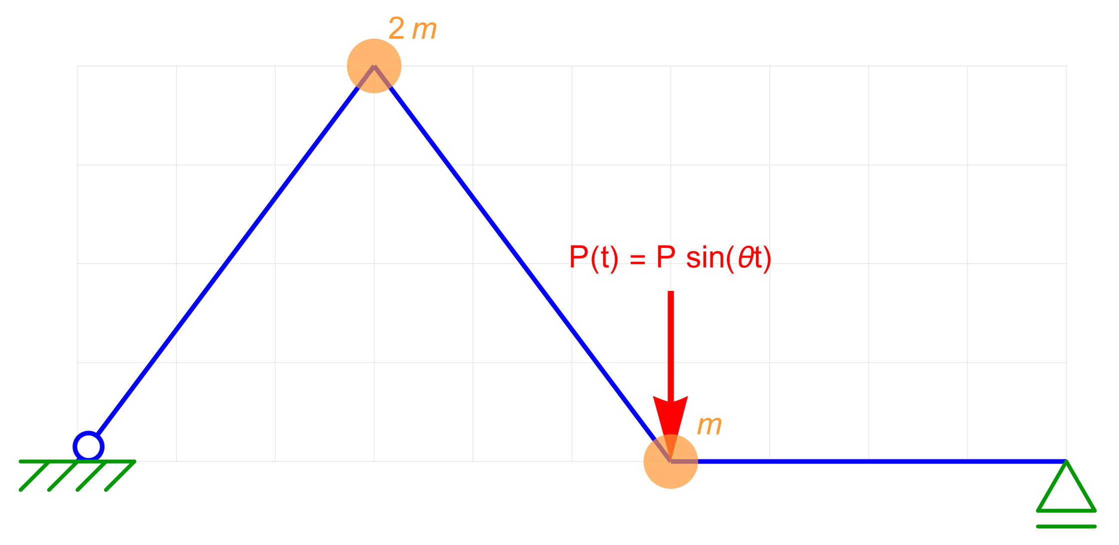
Kol 2.2 R.A. 2022/23

Obliczyć amplitudę reakcji pionowej w podporze przesuwnej.

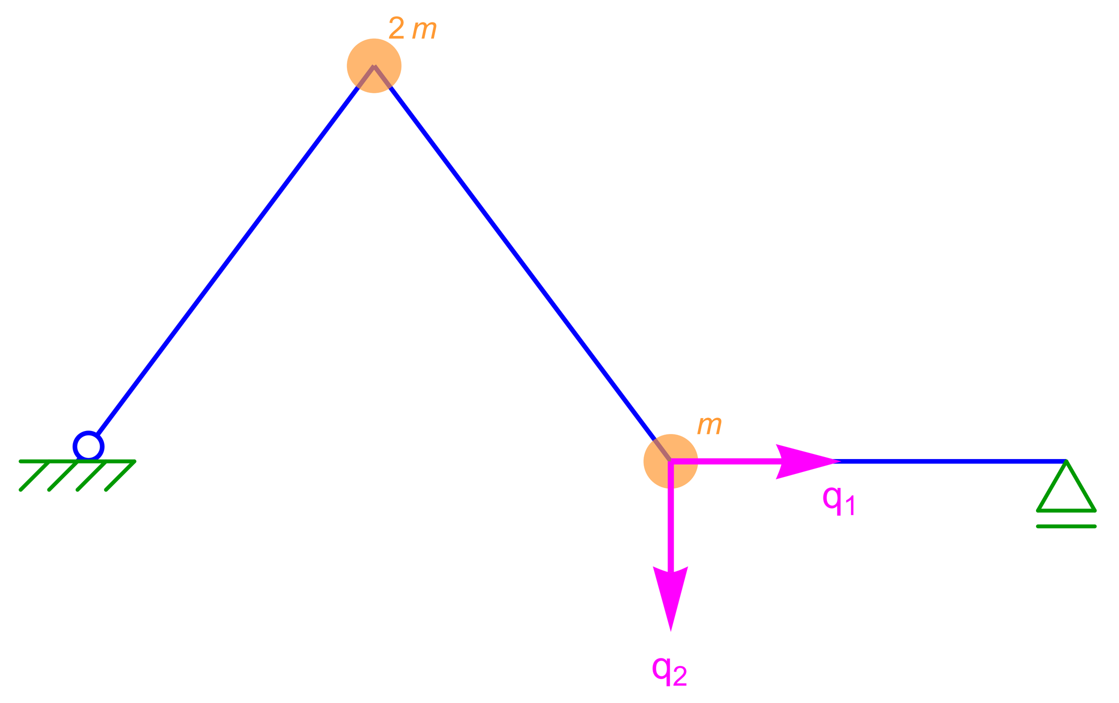
(Compute the amplitude of the vertical reaction in the roller support).

Geometria oraz obciążenia konstrukcji (wymiar oczka siatki – l, $\theta=\sqrt{\frac{EJ}{l^3 m}}$):

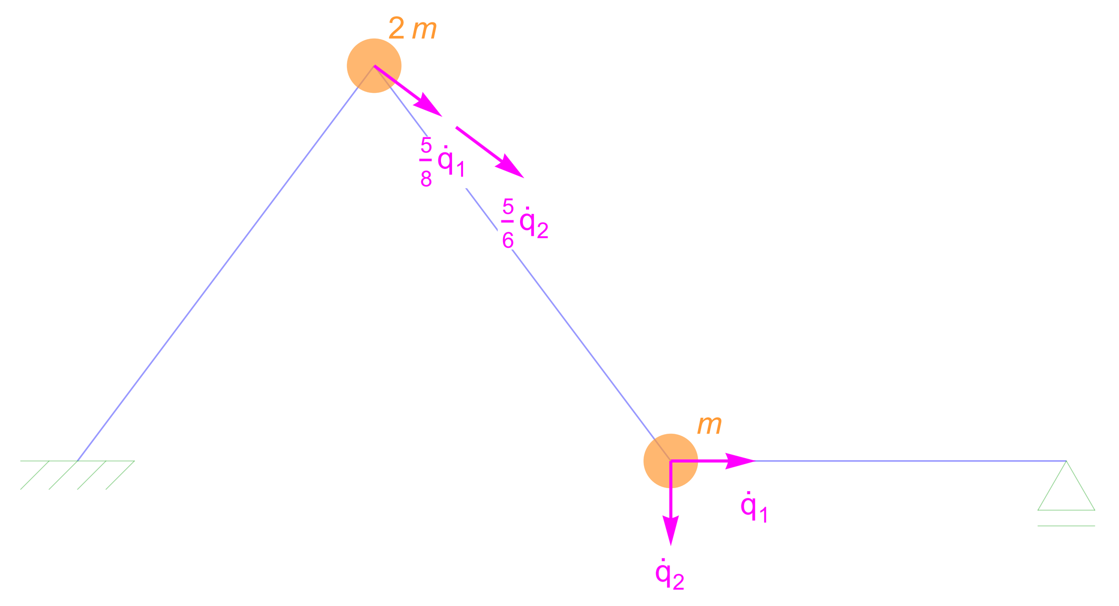


Zadanie statyki konstrukcji jest statycznie wyznaczalne.

Współrzędne Lagrange'a:



Plan predkosci:



Energia kinetyczna jako forma kwadratowa wektora $\dot{\mathbf{q}}$:

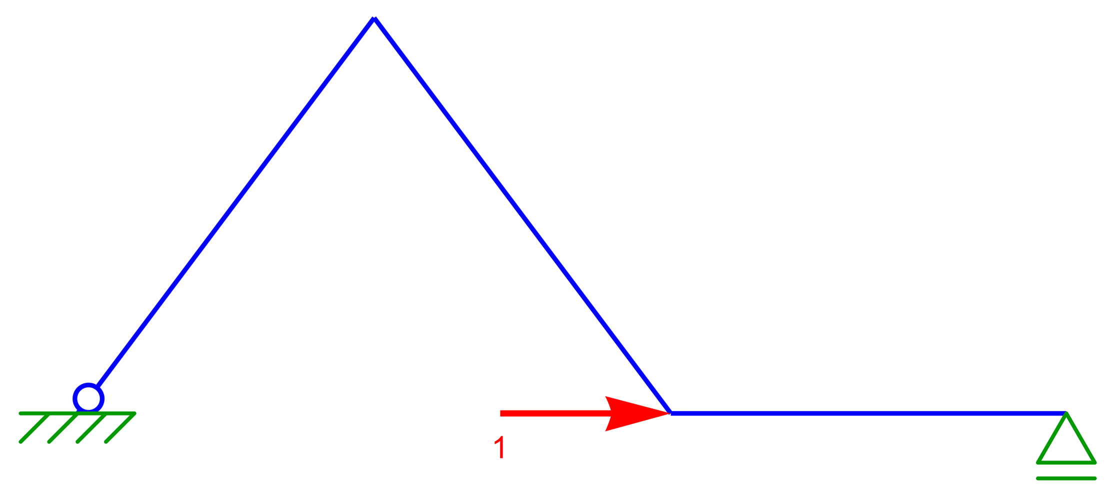
$$2\,E_k\,(\,\dot{\mathbf{q}}\,) = 2\,m\,(\,\tfrac{5}{8}\,\dot{q}_1 + \tfrac{5}{6}\,\dot{q}_2\,)^2 + m\,[\,\dot{q}_2{}^2 + \dot{q}_1{}^2\,] = \tfrac{57}{32}\,m\,\dot{q}_1{}^2 + \tfrac{25}{24}\,m\,\dot{q}_1\,\dot{q}_2 + \tfrac{25}{24}\,m\,\dot{q}_2\,\dot{q}_1 + \tfrac{43}{18}\,m\,\dot{q}_2{}^2 = \dot{\mathbf{q}}^T\,\mathbf{M}\,\dot{\mathbf{q}}$$

Macierz mas:

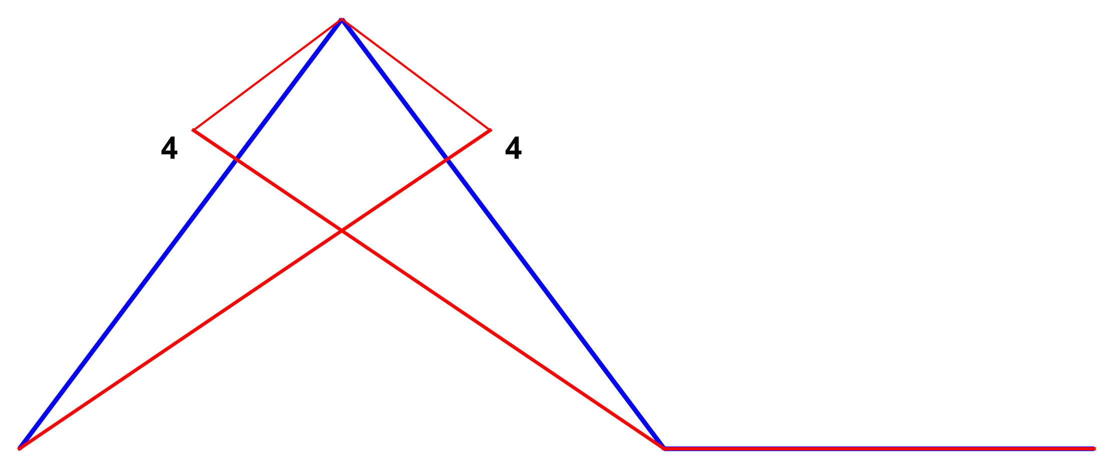
$$\mathbf{M} = m\begin{pmatrix} \frac{57}{32} & \frac{25}{24} \\ \frac{25}{24} & \frac{43}{18} \end{pmatrix}$$

Wykresy momentów zginających od jednostkowych sił bezwładności:

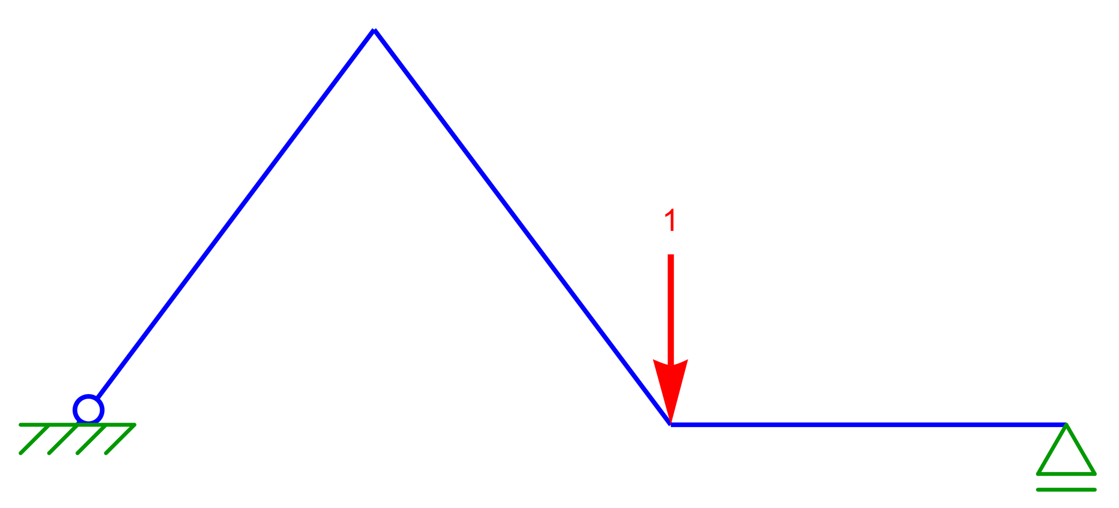
– od q_1:



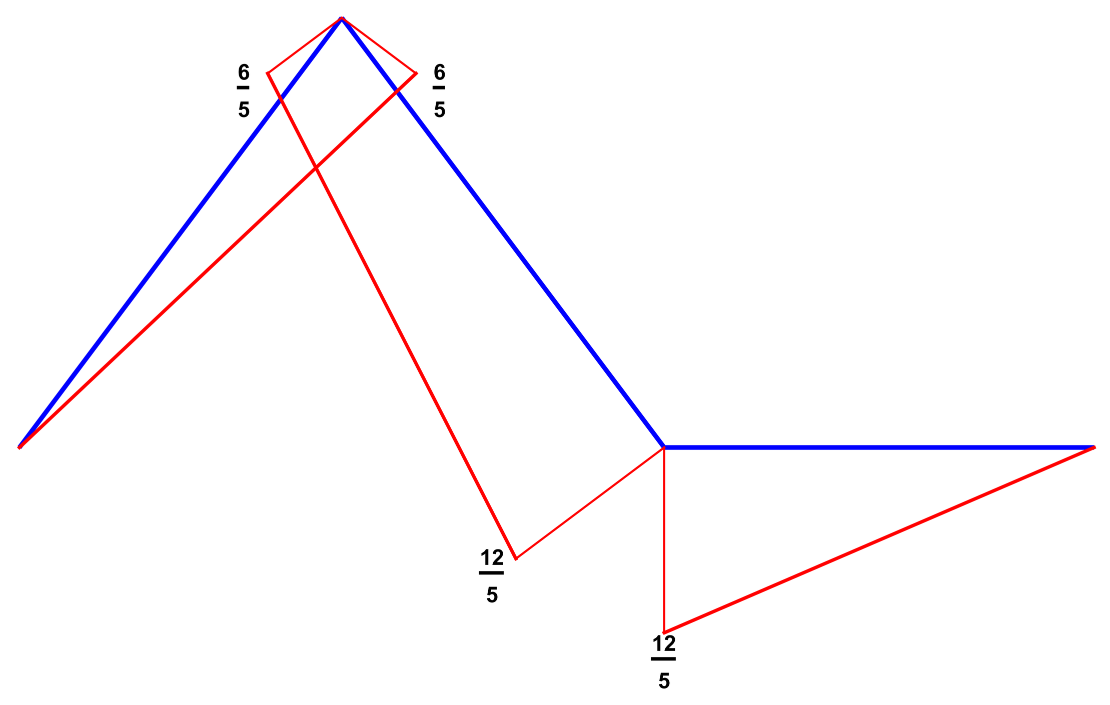
$M_1\,[l]$:



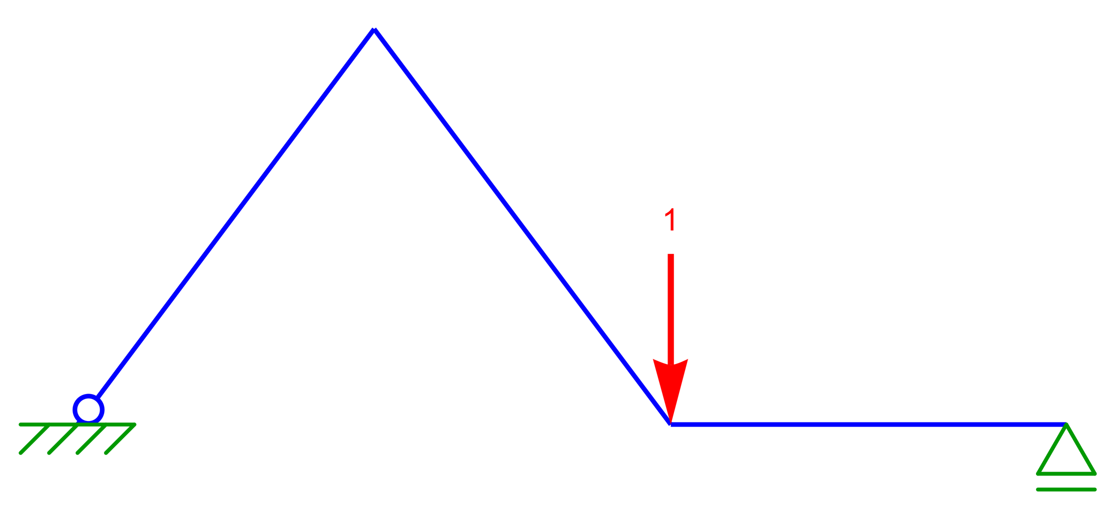
– od q_2:



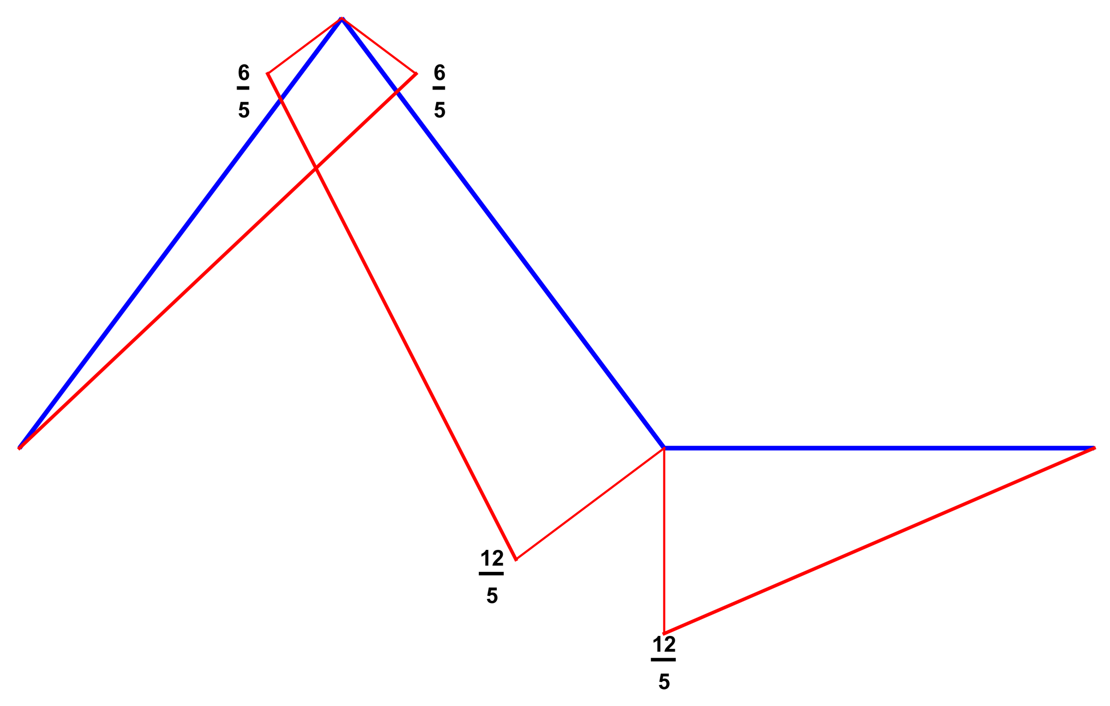
$M_2\,[l]$:



Wykresy momentów zginających od jednostkowego obciążenia wymuszającego:



$M_{01}\,[l]$:



Macierz podatności:

$$d_{11} = \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot 4l\cdot 5\,l)\,(\tfrac{2}{3}\cdot 4l)\,]_1 + \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot 4l\cdot 5\,l)\,(\tfrac{2}{3}\cdot 4l)\,]_2 = \frac{160}{3}\,\frac{l^3}{EJ}$$

$$d_{12} = d_{21} = \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot 4l\cdot 5\,l)\,(\tfrac{2}{3}\cdot\tfrac{6}{5}l)\,]_1 + \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot 4l\cdot 5\,l)\,(\tfrac{2}{3}\cdot\tfrac{6}{5}l+\tfrac{1}{3}\cdot\tfrac{12}{5}l)\,]_2 = 24\,\frac{l^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{6}{5}l\cdot 5\,l)\,(\tfrac{2}{3}\cdot\tfrac{6}{5}l)\,]_1 + \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{6}{5}l\cdot 5\,l)\,(\tfrac{2}{3}\cdot\tfrac{6}{5}l+\tfrac{1}{3}\cdot\tfrac{12}{5}l)+(\tfrac{1}{2}\cdot\tfrac{12}{5}l\cdot 5\,l)\,(\tfrac{1}{3}\cdot\tfrac{6}{5}l+\tfrac{2}{3}\cdot\tfrac{12}{5}l)\,]_2 + \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{12}{5}l\cdot 4\,l)\,(\tfrac{2}{3}\cdot\tfrac{12}{5}l)\,]_3 = \frac{672}{25}\,\frac{l^3}{EJ}$$

$$\mathbf{D} = \frac{l^3}{EJ}\begin{pmatrix} \frac{160}{3} & 24 \\ 24 & \frac{672}{25} \end{pmatrix}$$

Przemieszczenia od jednostkowego obciążenia wymuszającego:

$$d_{10} = \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{6}{5}l\cdot 5\,l)\,(\tfrac{2}{3}\cdot 4l)\,]_1 + \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{6}{5}l\cdot 5\,l)\,(\tfrac{2}{3}\cdot 4l)+(\tfrac{1}{2}\cdot\tfrac{12}{5}l\cdot 5\,l)\,(\tfrac{1}{3}\cdot 4l)\,]_2 = 24\,\frac{l^3}{EJ}$$

$$d_{20} = \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{6}{5}l\cdot 5\,l)\,(\tfrac{2}{3}\cdot\tfrac{6}{5}l)\,]_1 + \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{6}{5}l\cdot 5\,l)\,(\tfrac{2}{3}\cdot\tfrac{6}{5}l+\tfrac{1}{3}\cdot\tfrac{12}{5}l)+(\tfrac{1}{2}\cdot\tfrac{12}{5}l\cdot 5\,l)\,(\tfrac{1}{3}\cdot\tfrac{6}{5}l+\tfrac{2}{3}\cdot\tfrac{12}{5}l)\,]_2 + \frac{1}{EJ}[\,(\tfrac{1}{2}\cdot\tfrac{12}{5}l\cdot 4\,l)\,(\tfrac{2}{3}\cdot\tfrac{12}{5}l)\,]_3 = \frac{672}{25}\,\frac{l^3}{EJ}$$

ZADANIE DRGAŃ HARMONICZNYCH

– poszukiwanie funkcji przemieszczeń postaci:

$$\mathbf{q}\,(t) = \mathbf{a}\sin(\theta\,t) = \mathbf{a}\sin((\frac{EJ}{l^3 m})^{0.500}\,t)$$

– równania ruchu:

$$(\,\mathbf{I} - \theta^2\,\mathbf{D}\,\mathbf{M}\,)\,\mathbf{a} = \mathbf{d_0}\,P$$

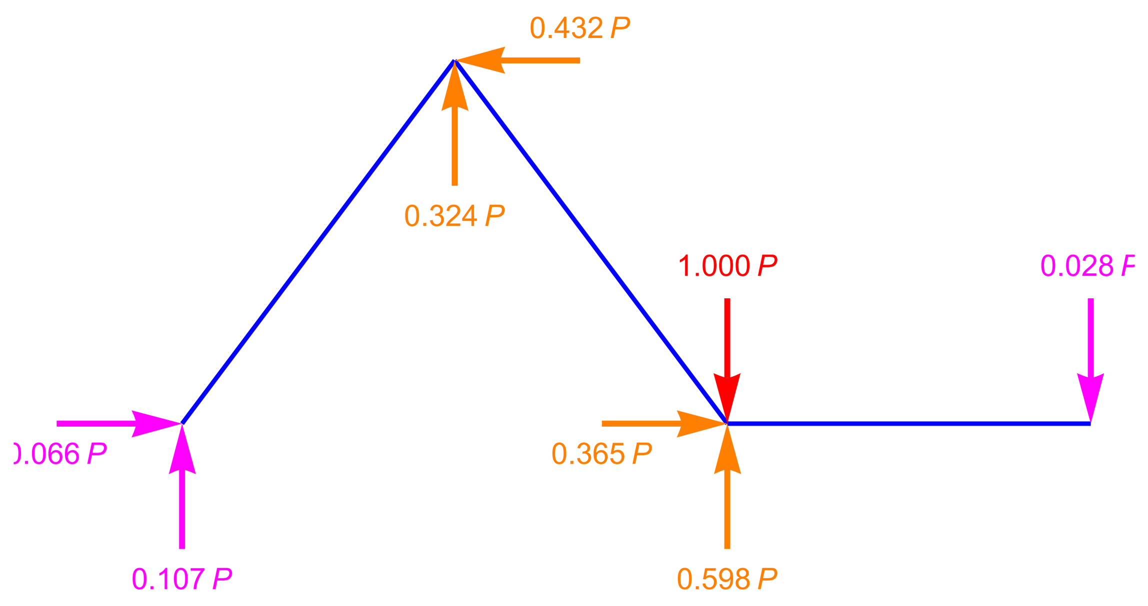
$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 1.0000\begin{pmatrix} \frac{160}{3} & 24 \\ 24 & \frac{672}{25} \end{pmatrix}\begin{pmatrix} \frac{57}{32} & \frac{25}{24} \\ \frac{25}{24} & \frac{43}{18} \end{pmatrix}\right)\mathbf{a} = \frac{l^3\,P}{EJ}\begin{pmatrix} 24 \\ \frac{672}{25} \end{pmatrix}$$

$$\begin{pmatrix} -119 & -\frac{1016}{9} \\ -\frac{283}{4} & -\frac{6616}{75} \end{pmatrix}\mathbf{a} = \frac{l^3\,P}{EJ}\begin{pmatrix} 24 \\ \frac{672}{25} \end{pmatrix}$$

Wektor amplitud przemieszczeń:

$$\mathbf{a} = \frac{l^3\,P}{EJ}\begin{pmatrix} 0.365 \\ -0.598 \end{pmatrix}$$

Amplitudy sił działających na konstrukcje:



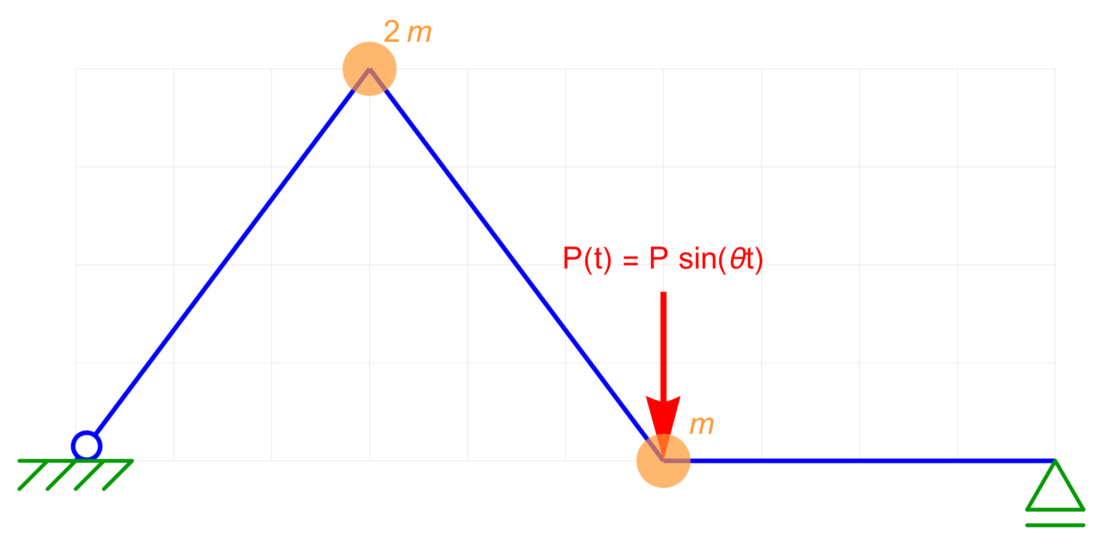
Rozwiązanie przygotował Karol Bołbotowski.

Kol 2.2 R.A. 2022/23

Obliczyć amplitudę reakcji pionowej w podporze przesuwnej.

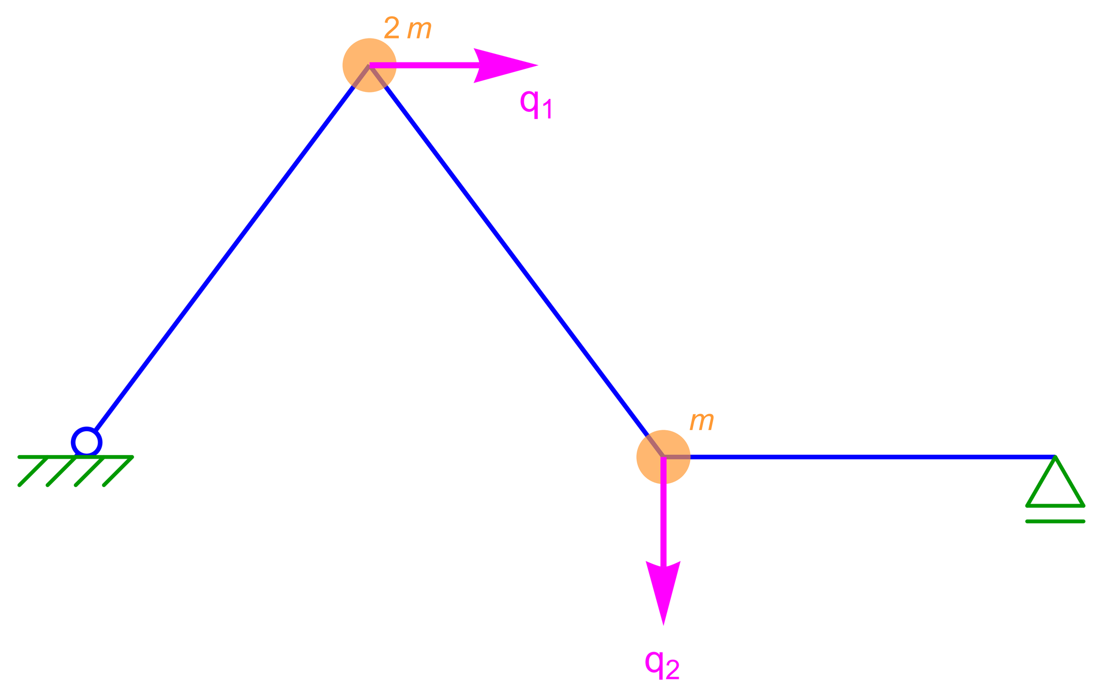
(Compute the amplitude of the vertical reaction in the roller support).

Geometria oraz obciążenia konstrukcji (wymiar oczka siatki - l, $\theta=\sqrt{\frac{EJ}{l^3 m}}$):

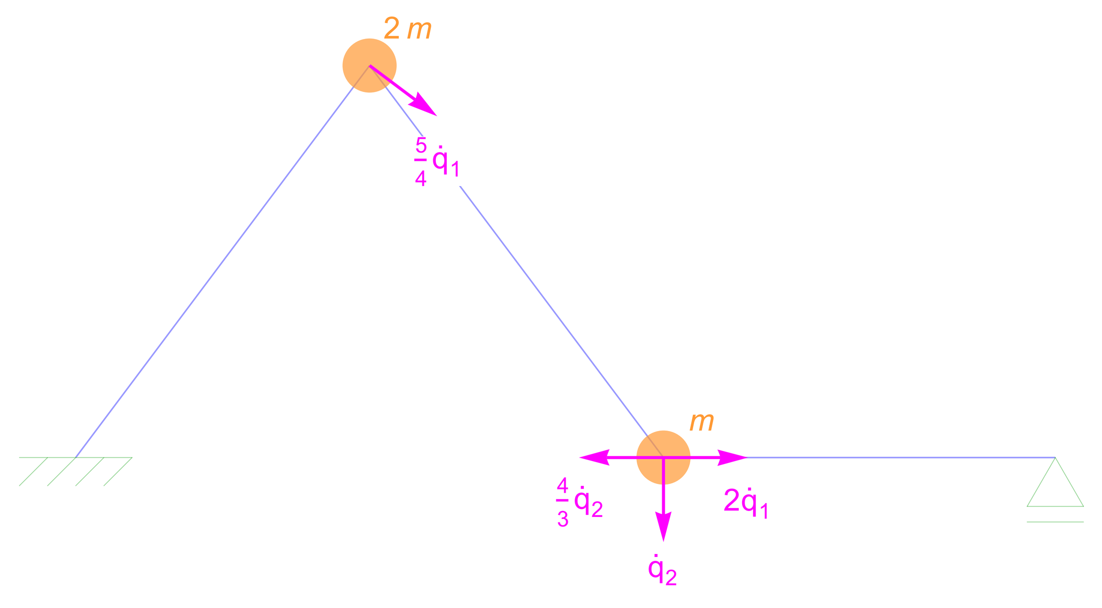


Zadanie statyki konstrukcji jest statycznie wyznaczalne.

Współrzędne Lagrange'a:



Plan predkosci:



Energia kinetyczna jako forma kwadratowa wektora $\dot{\mathbf{q}}$:

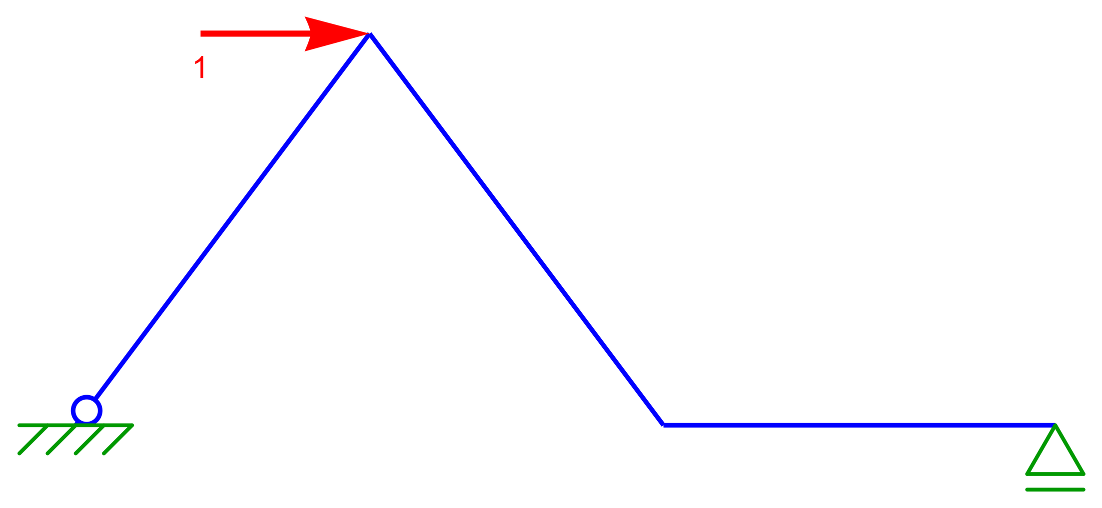
$$2\,E_k\,(\dot{\mathbf{q}}) = 2\,m\,\left(\frac{5}{4}\,\dot{q}_1\right)^2 + m\left[\left(2\,\dot{q}_1 - \frac{4}{3}\,\dot{q}_2\right)^2 + \dot{q}_2^{\,2}\right] = \frac{57}{8}\,m\,\dot{q}_1^{\,2} - \frac{8}{3}\,m\,\dot{q}_1\,\dot{q}_2 - \frac{8}{3}\,m\,\dot{q}_2\,\dot{q}_1 + \frac{25}{9}\,m\,\dot{q}_2^{\,2} = \dot{\mathbf{q}}^T\,\mathbf{M}\,\dot{\mathbf{q}}$$

Macierz mas:

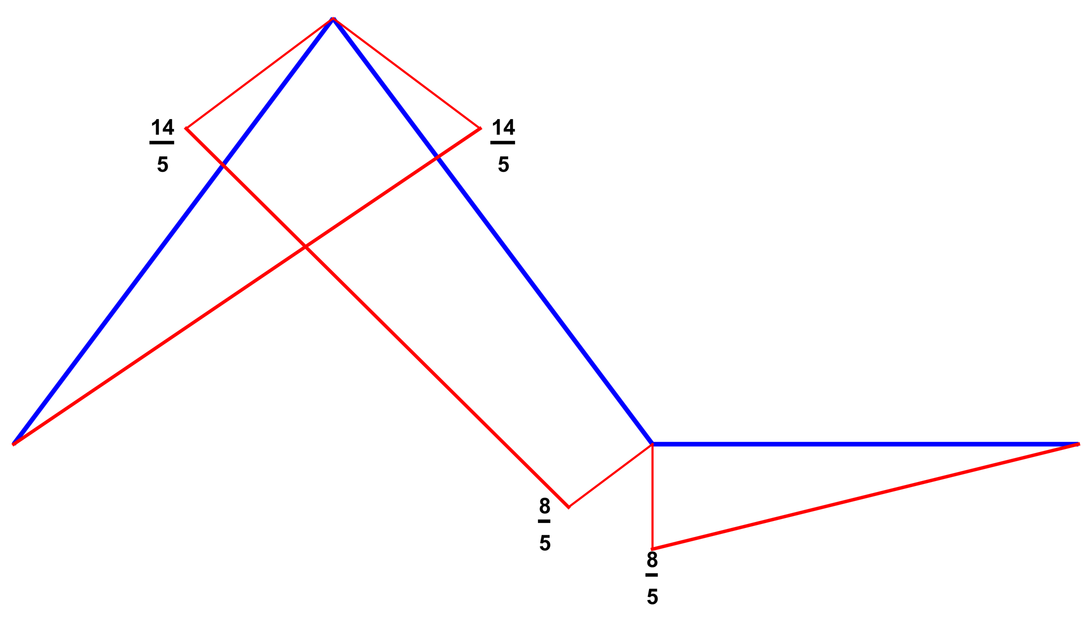
$$\mathbf{M} = m\begin{pmatrix} \frac{57}{8} & -\frac{8}{3} \\ -\frac{8}{3} & \frac{25}{9} \end{pmatrix}$$

Wykresy momentów zginających od jednostkowych sił bezwładności:

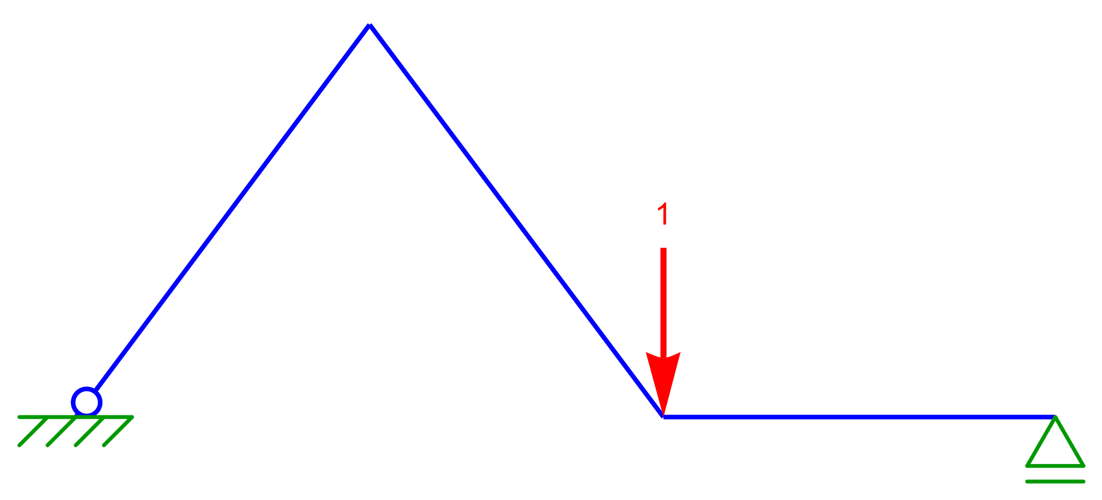
- od q_1:



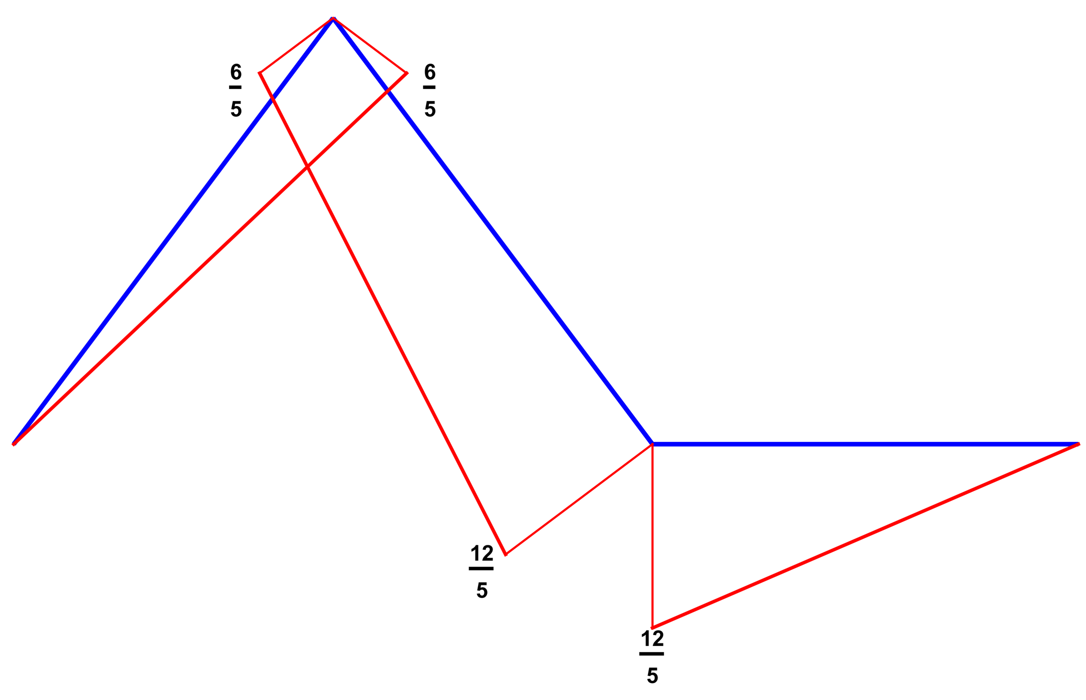
$M_1[l]$:



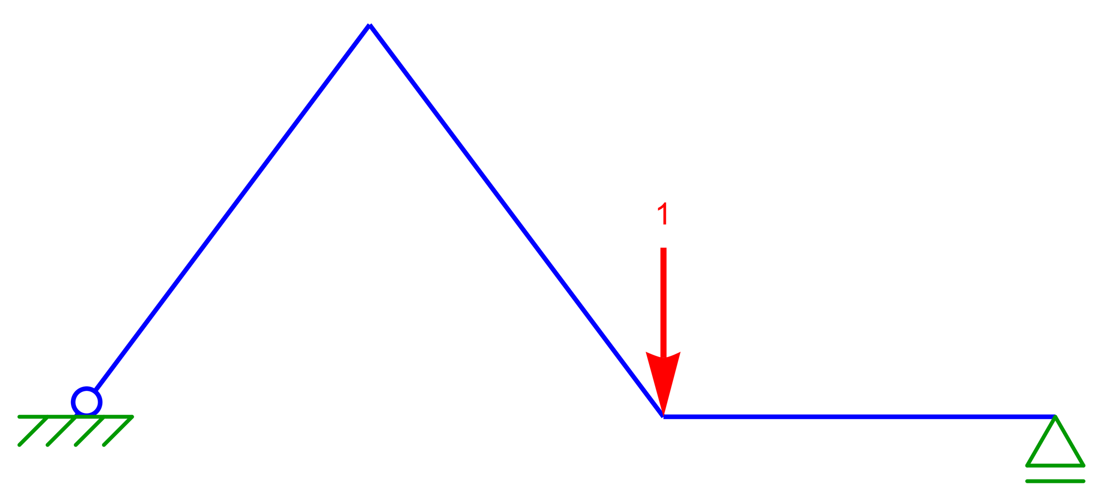
- od q_2:



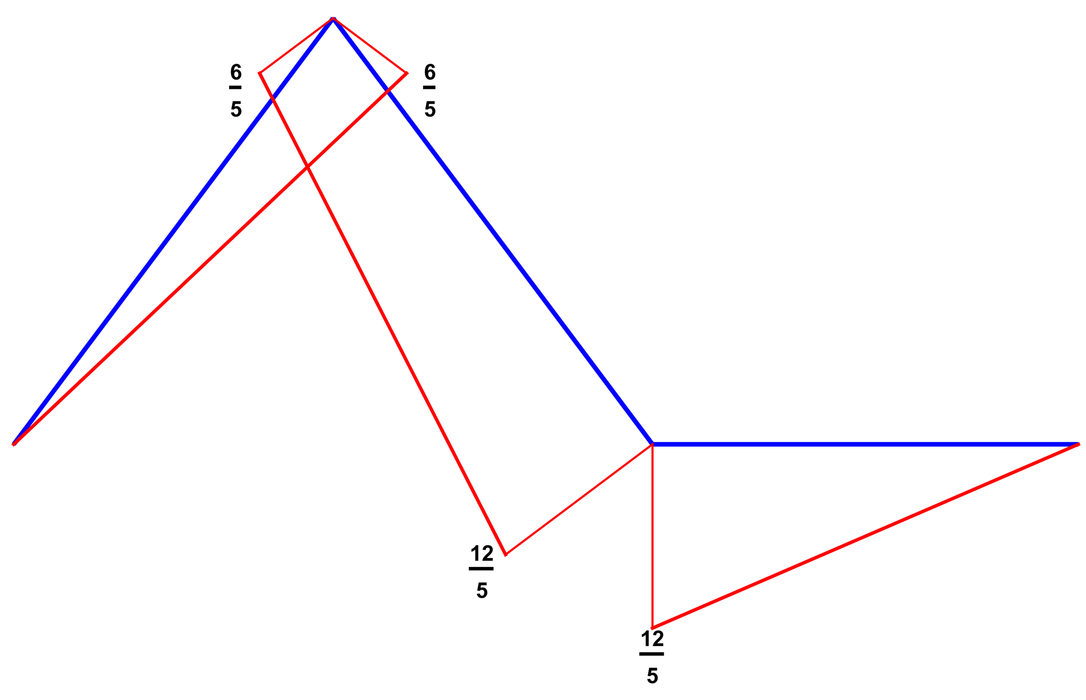
$M_2[l]$:



Wykresy momentów zginających od jednostkowego obciążenia wymuszającego:



$M_{01}[l]$:



Macierz podatności:

$$d_{11} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{14}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{14}{5}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{14}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{14}{5}l+\frac{1}{3}\cdot\frac{8}{5}l\right)+\left(\frac{1}{2}\cdot\frac{8}{5}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{14}{5}l+\frac{2}{3}\cdot\frac{8}{5}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{5}l\cdot 4\,l\right)\left(\frac{2}{3}\cdot\frac{8}{5}l\right)\right]_3 = \frac{1032}{25}\,\frac{l^3}{EJ}$$

$$d_{12} = d_{21} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{14}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{6}{5}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{14}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{6}{5}l+\frac{1}{3}\cdot\frac{12}{5}l\right)+\left(\frac{1}{2}\cdot\frac{8}{5}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{6}{5}l+\frac{2}{3}\cdot\frac{12}{5}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{5}l\cdot 4\,l\right)\left(\frac{2}{3}\cdot\frac{12}{5}l\right)\right]_3 = \frac{748}{25}\,\frac{l^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{6}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{6}{5}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{6}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{6}{5}l+\frac{1}{3}\cdot\frac{12}{5}l\right)+\left(\frac{1}{2}\cdot\frac{12}{5}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{6}{5}l+\frac{2}{3}\cdot\frac{12}{5}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{12}{5}l\cdot 4\,l\right)\left(\frac{2}{3}\cdot\frac{12}{5}l\right)\right]_3 = \frac{672}{25}\,\frac{l^3}{EJ}$$

$$\mathbf{D} = \frac{l^3}{EJ}\begin{pmatrix} \frac{1032}{25} & \frac{748}{25} \\ \frac{748}{25} & \frac{672}{25} \end{pmatrix}$$

Przemieszczenia od jednostkowego obciążenia wymuszającego:

$$d_{10} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{6}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{14}{5}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{6}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{14}{5}l+\frac{1}{3}\cdot\frac{8}{5}l\right)+\left(\frac{1}{2}\cdot\frac{12}{5}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{14}{5}l+\frac{2}{3}\cdot\frac{8}{5}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{12}{5}l\cdot 4\,l\right)\left(\frac{2}{3}\cdot\frac{8}{5}l\right)\right]_3 = \frac{748}{25}\,\frac{l^3}{EJ}$$

$$d_{20} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{6}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{6}{5}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{6}{5}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{6}{5}l+\frac{1}{3}\cdot\frac{12}{5}l\right)+\left(\frac{1}{2}\cdot\frac{12}{5}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{6}{5}l+\frac{2}{3}\cdot\frac{12}{5}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{12}{5}l\cdot 4\,l\right)\left(\frac{2}{3}\cdot\frac{12}{5}l\right)\right]_3 = \frac{672}{25}\,\frac{l^3}{EJ}$$

ZADANIE DRGAŃ HARMONICZNYCH

- poszukiwanie funkcji przemieszczeń postaci:

$$\mathbf{q}\,(t) = \mathbf{a}\,\sin(\theta\,t) = \mathbf{a}\,\sin\left(\left(\frac{EJ}{l^3 m}\right)^{0.500} t\right)$$

- równania ruchu:

$$(\,\mathbf{I} - \theta^2\,\mathbf{D}\,\mathbf{M}\,)\,\mathbf{a} = \mathbf{d_0}\,P$$

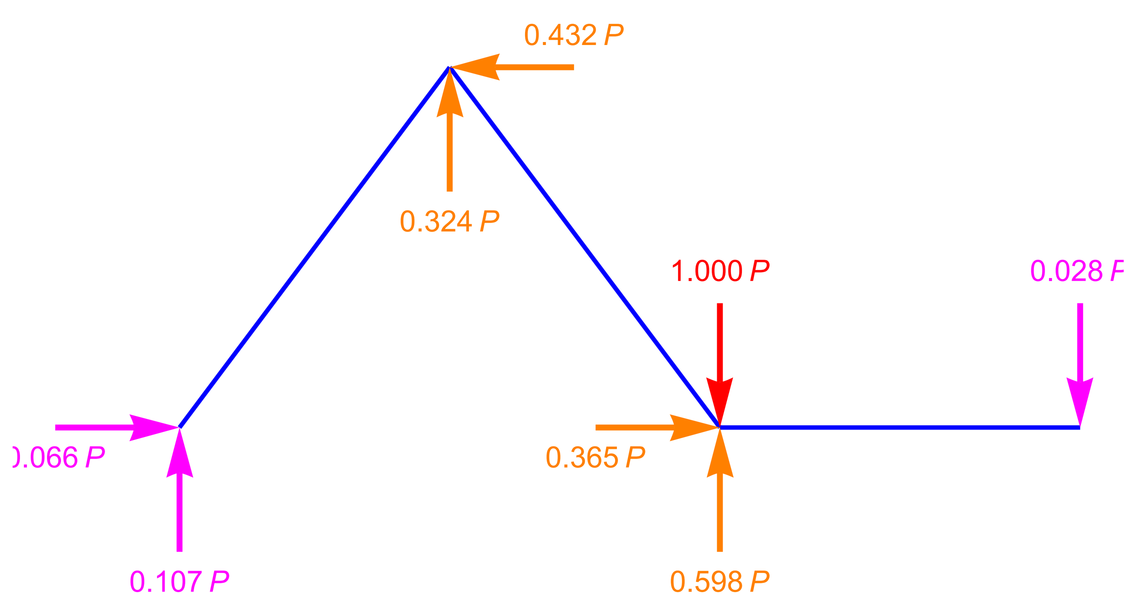
$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 1.0000\begin{pmatrix} \frac{1032}{25} & \frac{748}{25} \\ \frac{748}{25} & \frac{672}{25} \end{pmatrix}\begin{pmatrix} \frac{57}{8} & -\frac{8}{3} \\ -\frac{8}{3} & \frac{25}{9} \end{pmatrix}\right)\mathbf{a} = \frac{l^3 P}{EJ}\begin{pmatrix} \frac{748}{25} \\ \frac{672}{25} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{640}{3} & \frac{6068}{225} \\ -\frac{283}{2} & \frac{153}{25} \end{pmatrix}\mathbf{a} = \frac{l^3 P}{EJ}\begin{pmatrix} \frac{748}{25} \\ \frac{672}{25} \end{pmatrix}$$

Wektor amplitud przemieszczeń:

$$\mathbf{a} = \frac{l^3 P}{EJ}\begin{pmatrix} -0.216 \\ -0.598 \end{pmatrix}$$

Amplitudy sił działających na konstrukcje:



Rozwiązanie przygotował Karol Bołbotowski.