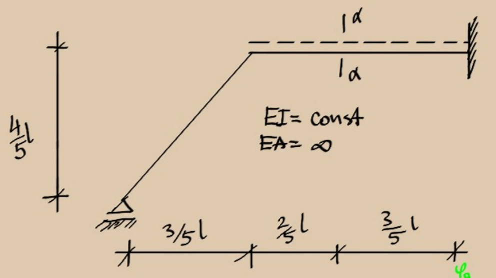
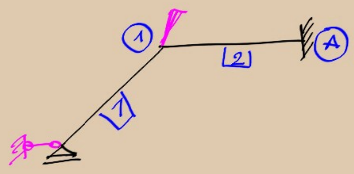
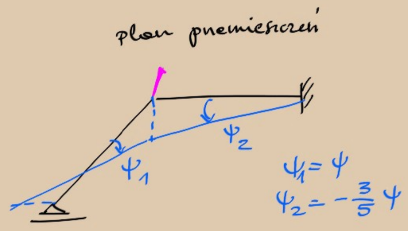
Kolokwium 1.1 18.03.2021



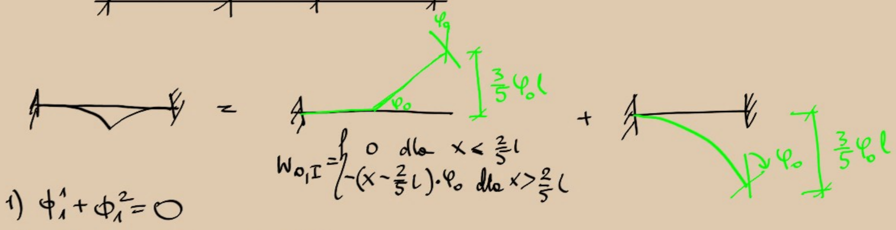
$M_\alpha(y) = ?$





$\psi_1 = \psi$

$\psi_2 = -\frac{3}{5}\psi$



$$W_{0,I} = \begin{cases} 0 & \text{dla } x < \frac{2}{5}l \\ -(x-\frac{2}{5}l)\cdot\varphi_0 & \text{dla } x > \frac{2}{5}l \end{cases}$$

$$\phi_1^{02} = \frac{2EI}{l}\left(\varphi_0 - 3\cdot\frac{3}{5}\varphi_0\right) = -\frac{8}{5}\frac{\varphi_0 EI}{l}$$

$$\phi_A^{02} = \frac{2EI}{l}\left(2\varphi_0 - 3\frac{3}{5}\varphi_0\right) = \frac{2}{5}\frac{\varphi_0 EI}{l}$$

1) $\phi_1^1 + \phi_1^2 = 0$

2) $\phi_1^1\cdot\overline{\psi} - (\phi_1^2 + \phi_A^2)\cdot\frac{3}{5}\overline{\psi} = 0$

$$\phi_1^1 = \frac{3EI}{l}(\varphi_1 - \psi)$$

$$\phi_1^2 = \frac{2EI}{l}\left(2\varphi_1 + \frac{9}{5}\psi\right) - \frac{8}{5}\frac{\varphi_0 EI}{l}$$

$$\phi_A^2 = \frac{2EI}{l}\left(\varphi_1 + \frac{9}{5}\psi\right) + \frac{2}{5}\frac{\varphi_0 EI}{l}$$

$$\frac{EI}{l}\begin{bmatrix} 7 & \frac{3}{5} \\ \frac{3}{5} & \frac{183}{25} \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix} = \begin{bmatrix} \frac{8}{5} \\ \frac{18}{25} \end{bmatrix}\frac{\varphi_0 EI}{l}$$

$$\varphi_1 = \frac{47}{212}\varphi_0 = 0.222\,\varphi_0$$

$$\psi = \frac{17}{212}\varphi_0 = 0.080\,\varphi_0$$

$$W(x) = W_{0,I}(x) + W_{spr}(x)$$

$$W_{spr}(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 \qquad \varphi_{spr}(x) = A_1 + 2A_2 x + 3A_3 x^2$$

1) $W_{spr}(0) = \psi_1\cdot\frac{3}{5}l = \frac{3}{5}\psi l \rightarrow A_0 = \frac{3}{5}\psi l$

2) $W_{spr}(l) = \frac{3}{5}\varphi_0 l$

3) $\varphi_{spr}(0) = \varphi_1 \longrightarrow A_1 = \varphi_1$

4) $\varphi_{spr}(l) = \varphi_0$

2) $A_0 + A_1 l + A_2 l^2 + A_3 l^3 = \frac{3}{5}\varphi_0 l$

$A_2 l^2 + A_3 l^3 = \frac{3}{5}\varphi_0 l - A_0 - A_1 l = \frac{35}{106}\varphi_0 l$

$A_2 l + A_3 l^2 = \frac{35}{106}\varphi_0$

4) $A_1 + 2A_2 l + 3A_3 l^2 = \varphi_0$

$2A_2 l + 3A_3 l^2 = \varphi_0 - A_1$

$2A_2 l + 3A_3 l^2 = \frac{165}{212}\varphi_0$

odejmując stronami

$-A_3 l^2 = -\frac{25}{212}\varphi_0$

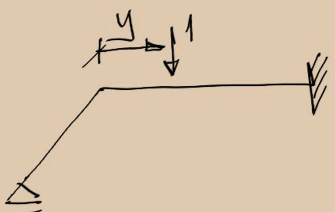
$A_3 = \frac{25}{212}\frac{\varphi_0}{l^2}$

$A_2 = \frac{35}{106}\frac{\varphi_0}{l} - A_3\cdot l = \frac{45}{212}\frac{\varphi_0}{l}$

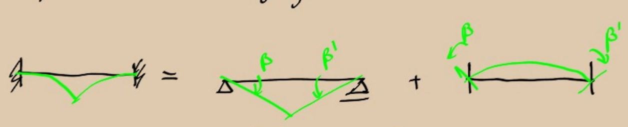
$$W(x) = \frac{1}{212}\cdot\varphi_0\left(\frac{51}{5}l + 47x + 45\frac{x^2}{l} + 25\frac{x^3}{l^2}\right) + \begin{cases} 0 & \text{dla } x \le \frac{2}{5}l \\ \frac{2}{5}l - x & \text{dla } x > \frac{2}{5}l \end{cases}$$

Z twierdzenia Bettiego

$$M_\alpha(y) = \frac{1}{\varphi_0}\cdot W(y)$$



Inny podział $W_0(x)$ prowadzi do tych samych momentów wyjściowych.
Podział $W_0(x)$ nie wpływa na rozwiązanie metody przemieszczeń, ale wymaga wprowadzenia innych warunków brzegowych



$\beta = \frac{3}{5}\varphi_0 \qquad \beta' = \frac{2}{5}\varphi_0$

$$\phi_1^{02} = \frac{2EI}{l}\left(-2\cdot\frac{3}{5}\varphi_0 + \frac{2}{5}\varphi_0\right) = -\frac{8}{5}\frac{\varphi_0 EI}{l}$$

$$\phi_A^{02} = \frac{2EI}{l}\left(-\frac{3}{5}\varphi_0 + 2\cdot\frac{2}{5}\varphi_0\right) = \frac{2}{5}\frac{\varphi_0 EI}{l}$$

$$W_{0,I}(x) = \begin{cases} \frac{3}{5}\varphi_0 x & \text{dla } x \le \frac{2}{5}l \\ (l-x)\frac{2}{5}\varphi_0 & \text{dla } x > \frac{2}{5}l \end{cases}$$

warunki brzegowe

$W_{spr}(0) = \frac{3}{5}l\cdot\psi_1$

$W_{spr}(l) = 0$

$\varphi_{spr}(0) = \varphi_1 - \frac{3}{5}\varphi_0$

$\varphi_{spr}(l) = \frac{2}{5}\varphi_0$