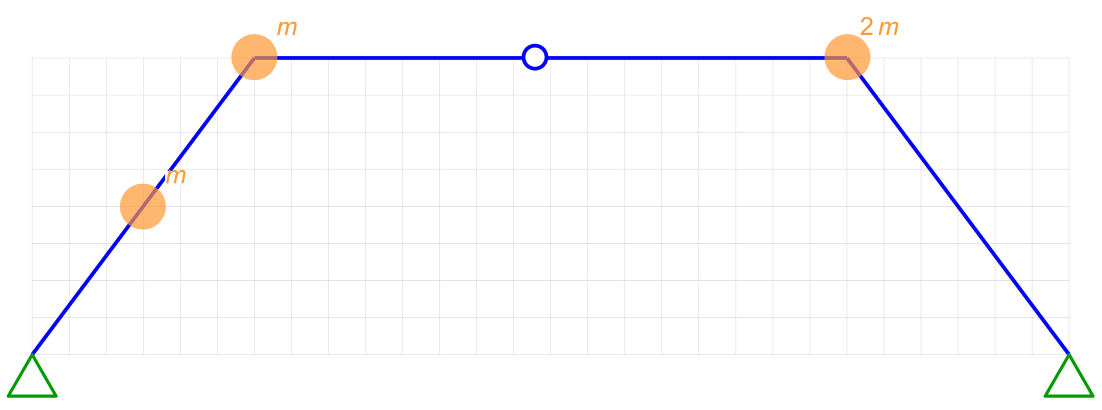
Kol 2.1 R.A. 2016/17

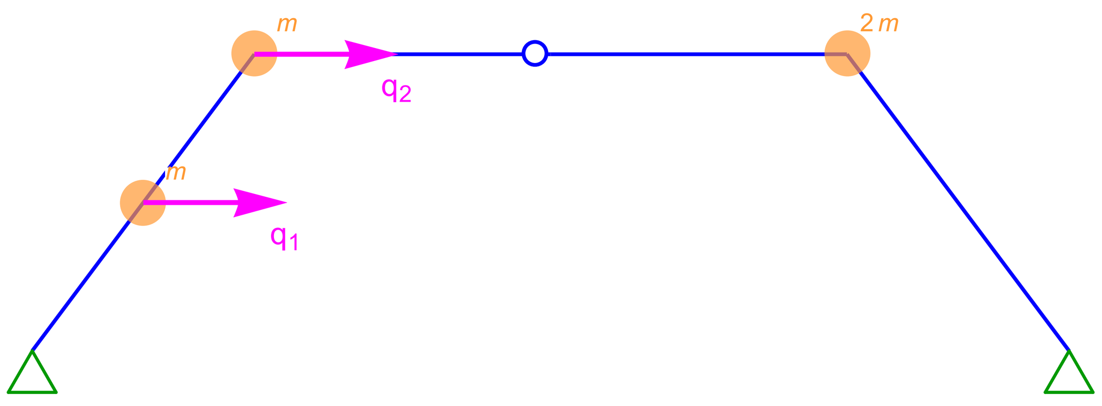
Obliczyć częstości drgań własnych.

Geometria oraz obciążenia konstrukcji (wymiar oczka siatki – l):

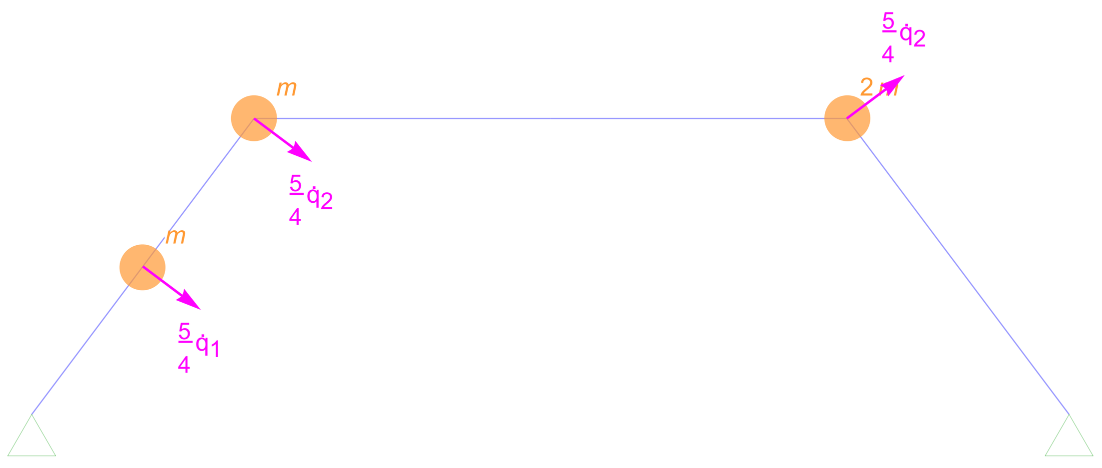


Zadanie statyki konstrukcji jest statycznie wyznaczalne.

Współrzędne Lagrange'a:



Plan predkosci:



Energia kinetyczna jako forma kwadratowa wektora $\dot{\mathbf{q}}$:

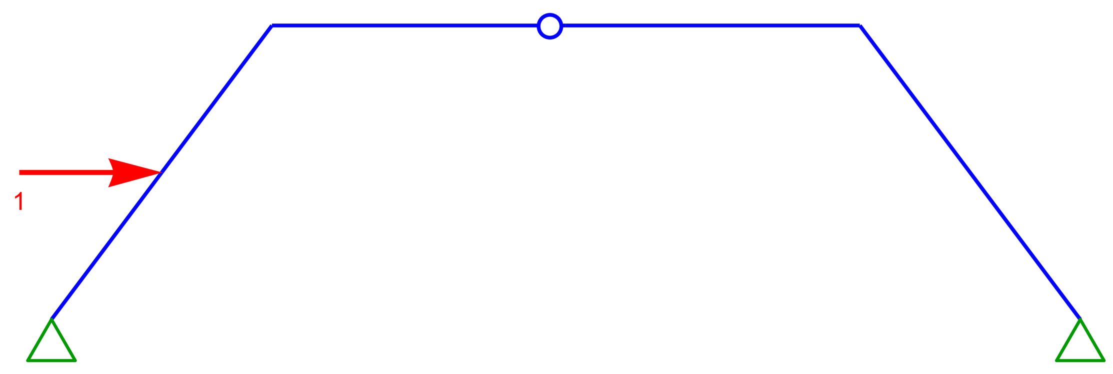
$$2\,E_k\,(\dot{\mathbf{q}}) = m\left(\frac{5}{4}\dot{q}_1\right)^2 + m\left(\frac{5}{4}\dot{q}_2\right)^2 + 2\,m\left(\frac{5}{4}\dot{q}_2\right)^2 = \frac{25}{16}\,m\,\dot{q}_1^{\,2} + \frac{75}{16}\,m\,\dot{q}_2^{\,2} = \dot{\mathbf{q}}^T\,\mathbf{M}\,\dot{\mathbf{q}}$$

Macierz mas:

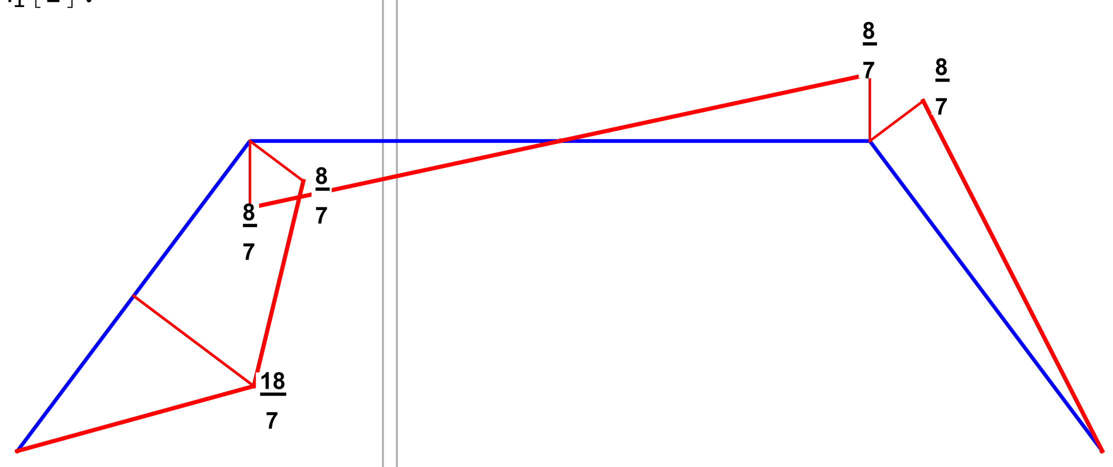
$$\mathbf{M} = m\begin{pmatrix} \frac{25}{16} & 0 \\ 0 & \frac{75}{16} \end{pmatrix}$$

Wykresy momentów zginających od jednostkowych sił bezwładności:

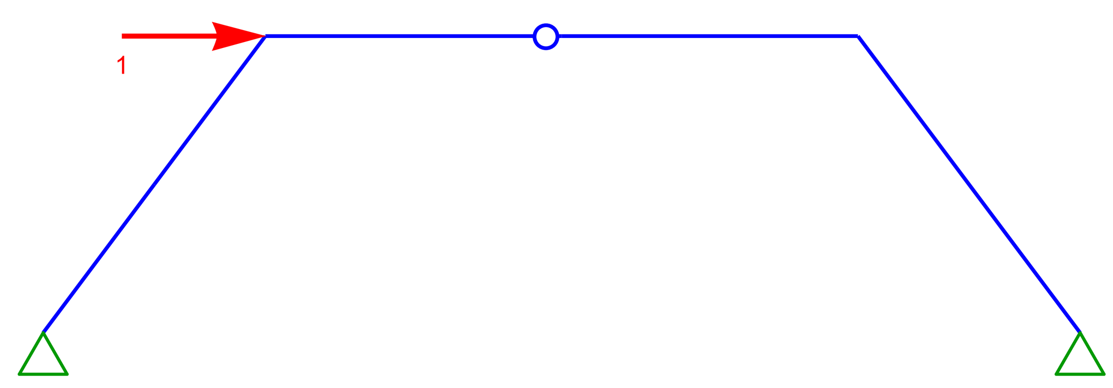
– od q_1:



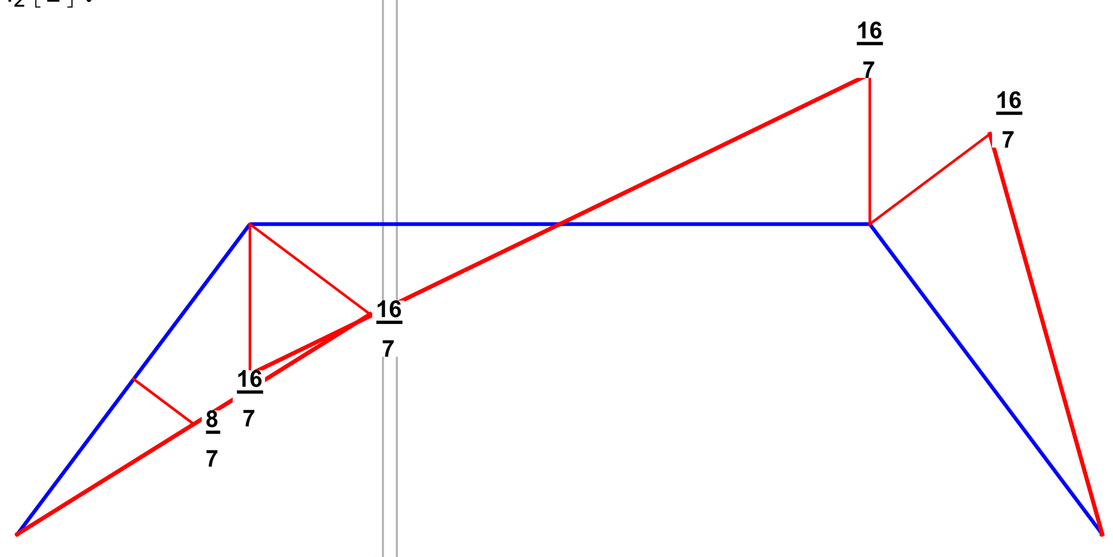
$M_1[l]$:



– od q_2:



$M_2[l]$:



Macierz podatności:

$$d_{11} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{18}{7}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{18}{7}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{18}{7}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{18}{7}l+\frac{1}{3}\cdot\frac{8}{7}l\right)+\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{18}{7}l+\frac{2}{3}\cdot\frac{8}{7}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 8\,l\right)\left(\frac{2}{3}\cdot\frac{8}{7}l\right)\right]_3 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 8\,l\right)\left(\frac{2}{3}\cdot\frac{8}{7}l\right)\right]_4 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 10\,l\right)\left(\frac{2}{3}\cdot\frac{8}{7}l\right)\right]_5 = \frac{5944}{147}\,\frac{l^3}{EJ}$$

$$d_{12} = d_{21} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{18}{7}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{8}{7}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{18}{7}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{8}{7}l+\frac{1}{3}\cdot\frac{16}{7}l\right)+\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{8}{7}l+\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 8\,l\right)\left(\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_3 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 8\,l\right)\left(\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_4 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 10\,l\right)\left(\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_5 = \frac{2096}{49}\,\frac{l^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{8}{7}l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{8}{7}l\cdot 5\,l\right)\left(\frac{2}{3}\cdot\frac{8}{7}l+\frac{1}{3}\cdot\frac{16}{7}l\right)+\left(\frac{1}{2}\cdot\frac{16}{7}l\cdot 5\,l\right)\left(\frac{1}{3}\cdot\frac{8}{7}l+\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{16}{7}l\cdot 8\,l\right)\left(\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_3 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{16}{7}l\cdot 8\,l\right)\left(\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_4 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot\frac{16}{7}l\cdot 10\,l\right)\left(\frac{2}{3}\cdot\frac{16}{7}l\right)\right]_5 = \frac{3072}{49}\,\frac{l^3}{EJ}$$

$$\mathbf{D} = \frac{l^3}{EJ}\begin{pmatrix} \frac{5944}{147} & \frac{2096}{49} \\ \frac{2096}{49} & \frac{3072}{49} \end{pmatrix}$$

ZADANIE DRGAŃ WŁASNYCH:

– poszukiwanie funkcji przemieszczeń postaci:

$$\mathbf{q}\,(t) = \mathbf{a}\,\sin(\omega t)$$

– zadanie własne:

$$(\,\mathbf{I} - \omega^2\,\mathbf{D}\,\mathbf{M}\,)\,\mathbf{a} = \mathbf{0}$$

– równanie charakterystyczne $\left(\lambda = \frac{\omega^2\,l^3\,m}{EJ}\right)$:

$$\det(\,\mathbf{I} - \omega^2\,\mathbf{D}\,\mathbf{M}\,) = 0$$

$$\det\left(\begin{pmatrix} 1-\frac{18575\,\lambda}{294} & -\frac{9825\,\lambda}{49} \\ -\frac{3275\,\lambda}{49} & 1-\frac{14400\,\lambda}{49} \end{pmatrix}\right) = 1 - \frac{104975\,\lambda}{294} + \frac{253125\,\lambda^2}{49} = 0$$

$\lambda^{(1)} = 0.00292$, $\lambda^{(2)} = 0.06619$

Częstości drgań własnych:

$\omega^{(1)} = 0.054\,\frac{1}{\sqrt{\frac{l^3\,m}{EJ}}}$, $\omega^{(2)} = 0.257\,\sqrt{\frac{EJ}{l^3\,m}}$

Postaci drgań własnych:

$\mathbf{a}^{(1)} = \begin{pmatrix} 0.719 \\ 1.000 \end{pmatrix}$, $\mathbf{a}^{(2)} = \begin{pmatrix} -4.171 \\ 1.000 \end{pmatrix}$

Zadanie przygotował Karol Bołbotowski.