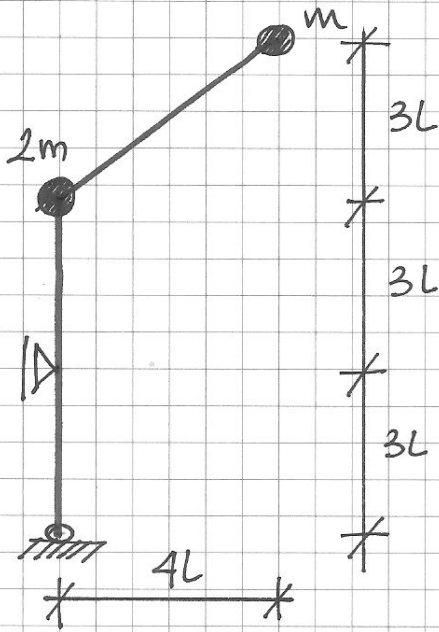
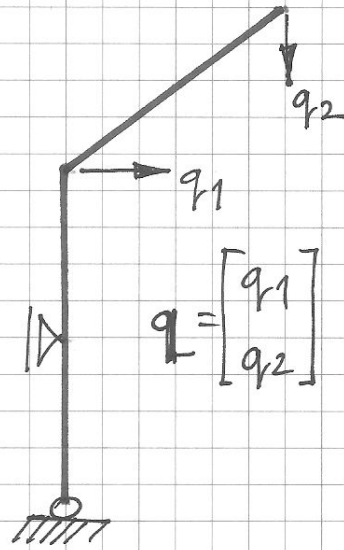


Kolokwium z MK2, 2.1a, r.ak. 2013/2014

Znaleźć częstotści drgań własnych. $EJ = \text{const.}$

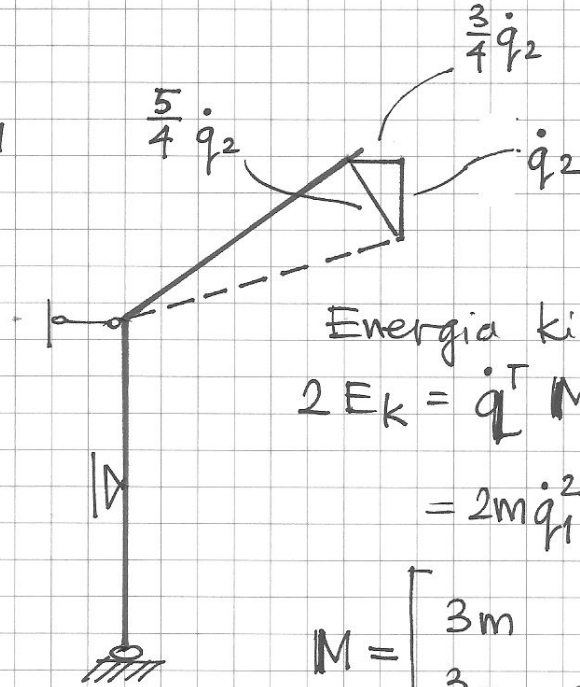
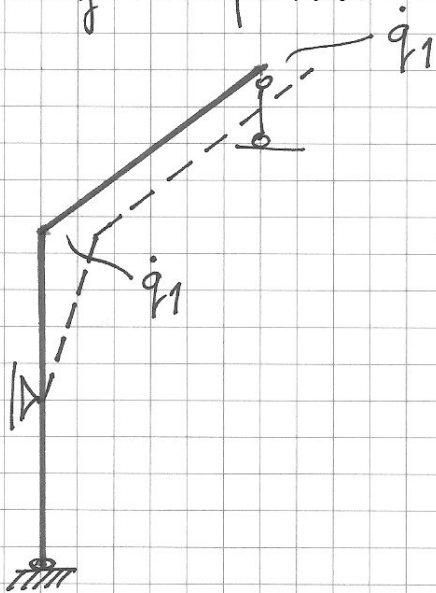


Współrzędne
Lagrange'a



$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad \dot{\mathbf{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

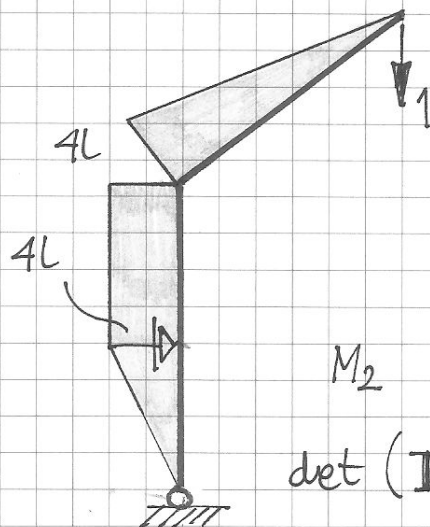
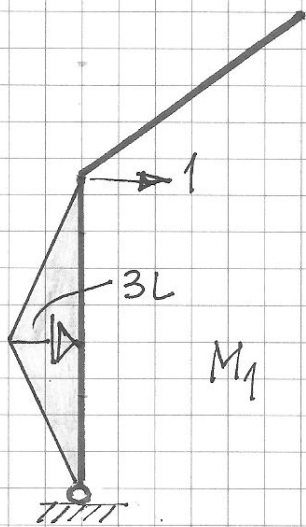
Plany amplitud



Energia kinetyczna:

$$2E_k = \dot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}} = 2m\dot{q}_1^2 + m\left[\dot{q}_2^2 + \left(\dot{q}_1 + \frac{3}{4}\dot{q}_2\right)^2\right]$$

$$\mathbf{M} = \begin{bmatrix} 3m & \frac{3}{4}m \\ \frac{3}{4}m & \frac{25}{16}m \end{bmatrix}$$

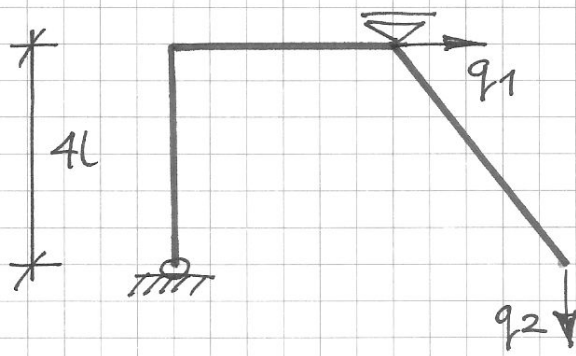
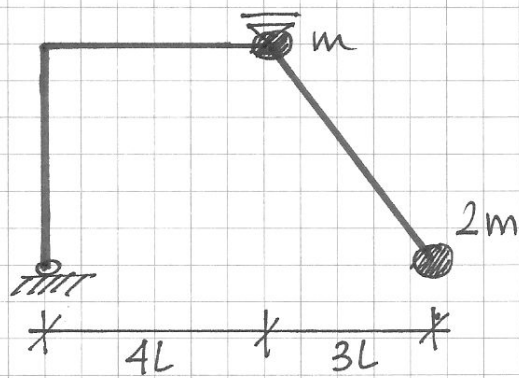


$$\mathbf{D} = \begin{bmatrix} 18 & 30 \\ 30 & \frac{272}{3} \end{bmatrix} \frac{L^3}{EJ}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(\mathbf{I} - \omega^2 \mathbf{D} \mathbf{M}) = 0 \rightarrow \begin{cases} \omega_1 = 0,066 \sqrt{\frac{EJ}{mL^3}} \\ \omega_2 = 0,274 \sqrt{\frac{EJ}{mL^3}} \end{cases}$$

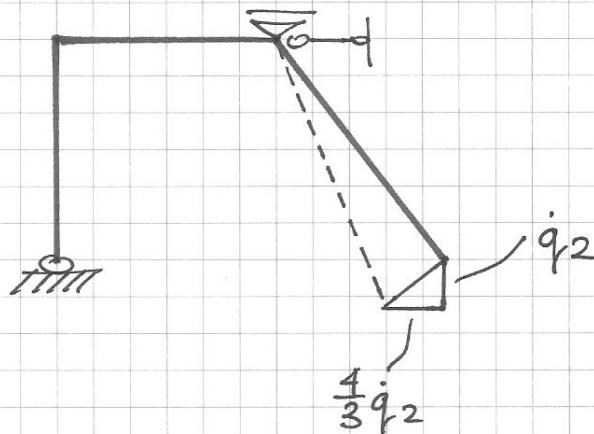
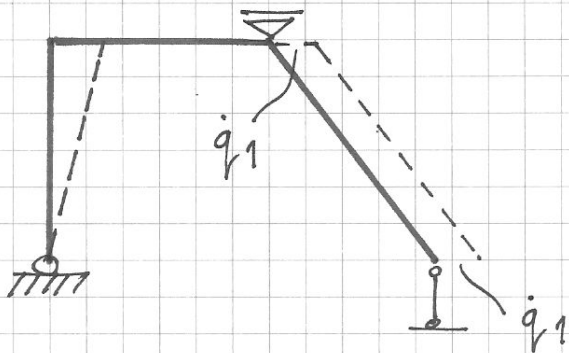
Znaleźć częstotści drgań własnych. $EJ = \text{const.}$



Współrzędne Lagrange'a:

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

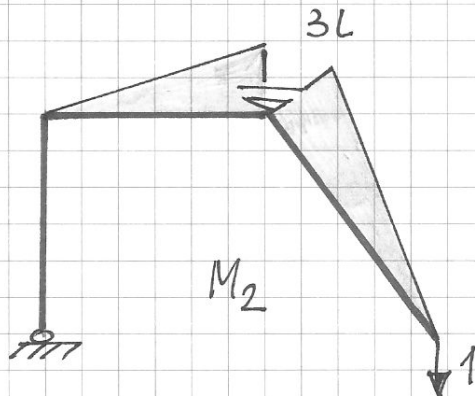
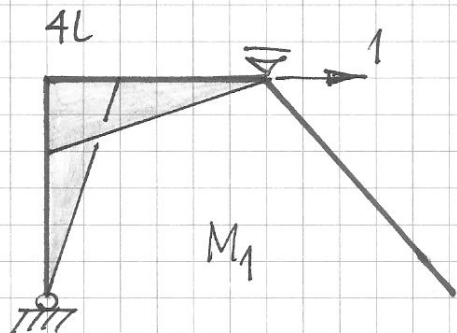
Plany amplitud:



Energia kinetyczna:

$$2 E_k = \dot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}} = m \dot{q}_1^2 + 2m \left[\dot{q}_2^2 + \left(\frac{4}{3} \dot{q}_2 - \dot{q}_1 \right)^2 \right]$$

$$\mathbf{M} = \begin{bmatrix} 3 & -\frac{8}{3} \\ -\frac{8}{3} & \frac{50}{9} \end{bmatrix} m$$



$$\mathbf{D} = \begin{bmatrix} \frac{128}{3} & -8 \\ -8 & 27 \end{bmatrix} \frac{L^3}{EJ}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(\mathbf{I} - \omega^2 \mathbf{D} \mathbf{M}) = 0 \rightarrow$$

$$\omega_1 = 0,059 \sqrt{\frac{EJ}{mL^3}}$$

$$\omega_2 = 0,165 \sqrt{\frac{EJ}{mL^3}}$$