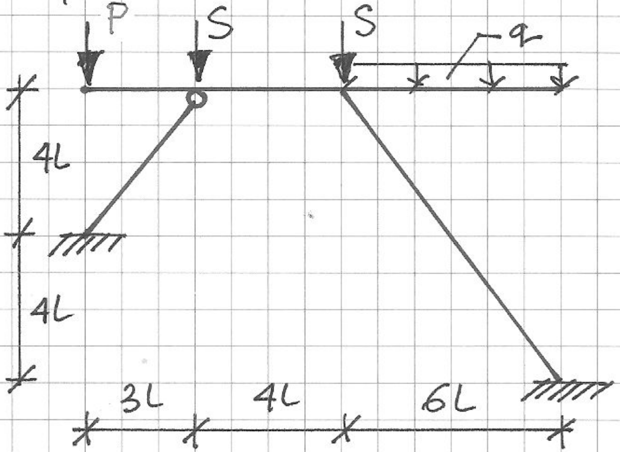
Kolokwium z MK2, 1.1., r.ak. 2013/14

Zapisać układ równań metody przemieszczeń. $EJ = const.$



$$S = \frac{1}{100}\frac{EJ}{L^2}$$

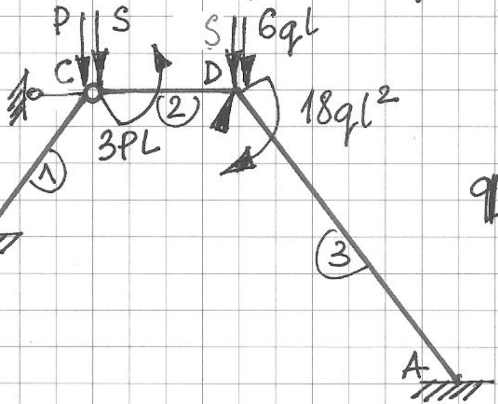
Wartości dużych sił osiowych i parametrów $\sigma^{(k)}$ w prętach:

$$S^{(1)} = \frac{5}{4}S \qquad \sigma^{(1)} = \frac{\sqrt{5}}{4}$$

$$S^{(2)} = \frac{3}{4}S \qquad \sigma^{(2)} = \frac{\sqrt{3}}{5}$$

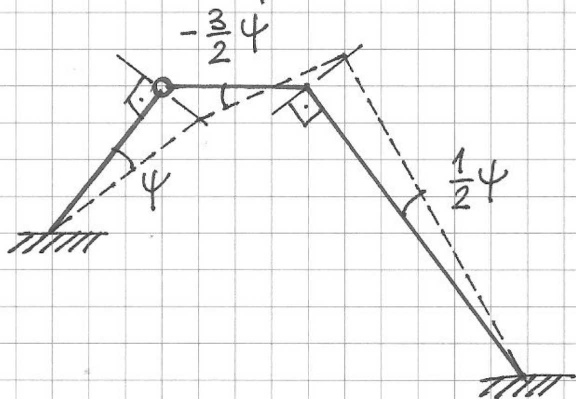
$$S^{(3)} = \frac{5}{4}S \qquad \sigma^{(3)} = \frac{\sqrt{5}}{2}$$

Schemat zredukowany geometrycznie wyznaczalny:



$$q = \begin{bmatrix} \varphi_D \\ \psi \end{bmatrix}$$

Plan przesunięć:



Równania równowagi:

$$\Phi_D^{(2)} + \Phi_D^{(3)} - 18qL^2 = 0$$

$$\Phi_B^{(1)}\cdot\overline{\psi} + \Phi_D^{(2)}\cdot\left(-\frac{3}{2}\overline{\psi}\right) + \left[\Phi_A^{(3)} + \Phi_D^{(3)}\right]\cdot\frac{1}{2}\overline{\psi} + \frac{5}{4}S\cdot\psi\cdot 5L\cdot\overline{\psi}$$

$$+ \frac{3}{4}S\cdot\left(-\frac{3}{2}\psi\right)\cdot 4L\cdot\left(-\frac{3}{2}\overline{\psi}\right) + \frac{5}{4}S\cdot\frac{1}{2}\psi\cdot 10L\cdot\frac{1}{2}\overline{\psi}$$

$$+ P\cdot 3L\cdot\overline{\psi} + 3PL\cdot\frac{3}{2}\overline{\psi} - 6qL\cdot 6L\cdot\frac{1}{2}\overline{\psi} = 0$$

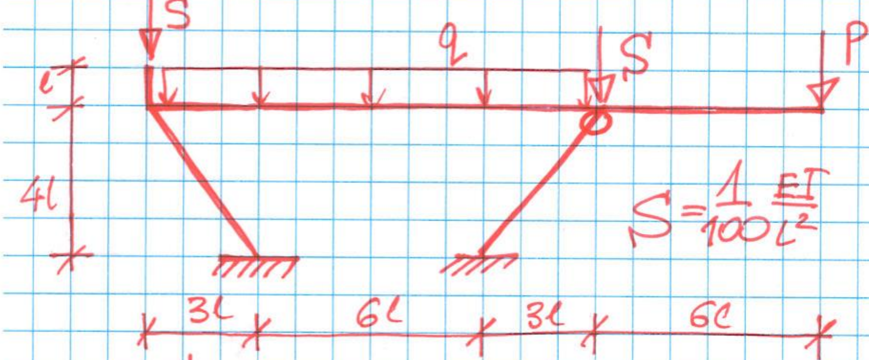
Wzory transformacyjne:

$$\Phi_B^{(1)} = \frac{EJ}{5L}\left[-\alpha'\left(\frac{\sqrt{5}}{4}\right)\psi\right]$$

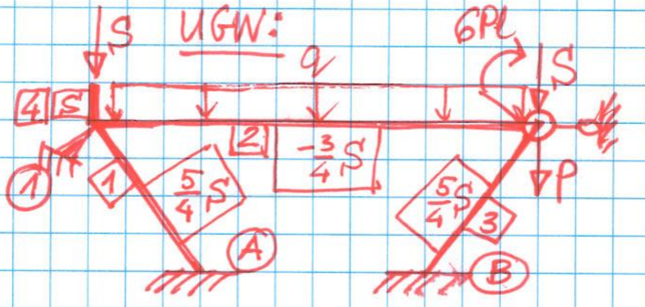
$$\Phi_D^{(2)} = \frac{EJ}{4L}\left[\alpha'\left(\frac{\sqrt{3}}{5}\right)\left(\varphi_D + \frac{3}{2}\psi\right)\right] - \frac{\beta\left(\frac{\sqrt{3}}{5}\right)}{\alpha\left(\frac{\sqrt{3}}{5}\right)}\cdot 3PL$$

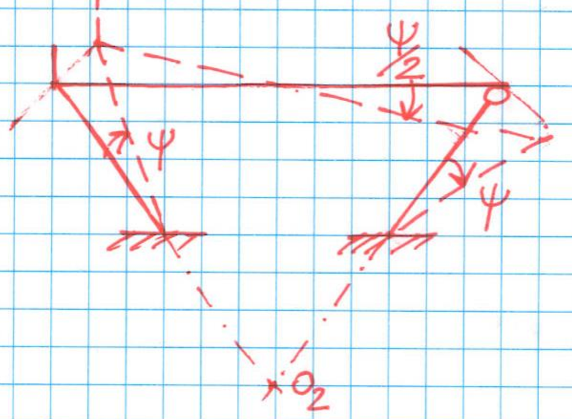
$$\Phi_D^{(3)} = \frac{EJ}{10L}\left[\alpha\left(\frac{\sqrt{5}}{2}\right)\varphi_D - \nu\left(\frac{\sqrt{5}}{2}\right)\cdot\frac{1}{2}\psi\right]$$

$$\Phi_A^{(3)} = \frac{EJ}{10L}\left[\beta\left(\frac{\sqrt{5}}{2}\right)\varphi_D - \nu\left(\frac{\sqrt{5}}{2}\right)\cdot\frac{1}{2}\psi\right]$$



$$S = \frac{1}{100}\frac{EI}{l^2}$$





$$S_1 \cos\alpha = S$$
$$S_1 = \frac{5}{4}S$$
$$S_2 + S_1 \sin\alpha = 0$$
$$S_2 = -\frac{3}{4}S$$

$l_1 = 5l \quad \psi_1 = \psi \quad \sigma_1 = 5l\sqrt{\frac{\frac{5}{4}S}{EI}} = \frac{\sqrt{5}}{4}$

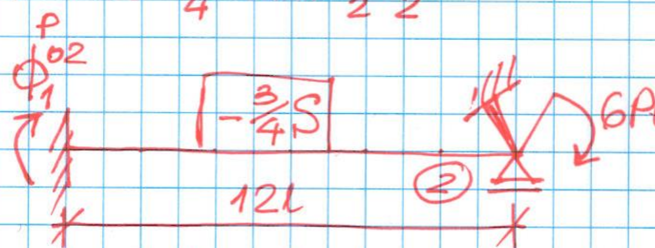
$l_2 = 12l \quad \psi_2 = \psi/2 \quad \sigma_2 = 12l\sqrt{\frac{-3/4\,S}{EI}} = \frac{3\sqrt{3}}{5}\,i$

$l_3 = 5l \quad \psi_3 = \psi \quad \sigma_3 = \sqrt{5}/4$

$l_4 = l \quad \psi_4 = - \quad \sigma_4 = 1/10$

1) $\phi_1^1 + \phi_1^2 + \phi_1^4 = 0$

2) $(\phi_A^1 + \phi_1^1 + \phi_B^3)\bar{\psi} + \phi_1^2 \cdot \frac{1}{2}\bar{\psi} + \frac{5}{4}S \cdot 5l \cdot \psi \cdot \bar{\psi} \cdot 2 - \frac{3}{4}S \cdot 12l \cdot \frac{\psi}{2} \cdot \frac{\bar{\psi}}{2} +$
$+ 6Pl \cdot \frac{\bar{\psi}}{2} + P \cdot 3l \cdot \bar{\psi} = 0$



$$\phi_1^1 = \frac{EI}{5l}\left[\alpha(\sigma_1)\varphi_1 - \theta(\sigma_1)\psi\right]$$
$$\phi_A^1 = \frac{EI}{5l}\left[\beta(\sigma_1)\varphi_1 - \theta(\sigma_1)\psi\right]$$
$$\phi_1^2 = \frac{EI}{12l}\left[\alpha'(\sigma_2)\left(\varphi_1 - \frac{\psi}{2}\right)\right] + \overset{P}{\phi}{}_1^{02} + \overset{q}{\phi}{}_1^{02}$$
$$\phi_B^3 = \frac{EI}{5l}\left[\alpha'(\sigma_3)(-\psi)\right]$$
$$\phi_1^4 = \frac{EI}{l}\left[\alpha'''(\sigma_4)\varphi_1\right]$$

$$\phi_2^{02} - 6Pl = 0$$
$$\phi_2^{02} = \frac{EI}{12l}\left[\alpha(\sigma_2)\cdot\varphi_2\right]$$
$$\frac{EI}{12l}\cdot\alpha(\sigma_2)\cdot\varphi_2 = 6Pl$$
$$\varphi_2 = 6Pl\cdot\frac{12l}{EI}\cdot\frac{1}{\alpha(\sigma_2)}$$
$$\overset{P}{\phi}{}_1^{02} = \frac{EI}{12l}\cdot\beta(\sigma_2)\varphi_2 = 6Pl\cdot\frac{\beta(\sigma_2)}{\alpha(\sigma_2)}$$
$$\overset{q}{\phi}{}_1^{02} = -\gamma'(\sigma_2)\cdot q(12l)^2$$

$$\mathbb{P} = \begin{bmatrix} \frac{\beta\left(\frac{3\sqrt{3}}{5}i\right)}{\alpha\left(\frac{3\sqrt{3}}{5}i\right)}\cdot 6 \\ \frac{\beta\left(\frac{3\sqrt{3}}{5}i\right)}{\alpha\left(\frac{3\sqrt{3}}{5}i\right)}\cdot 3 + 6 \end{bmatrix} Pl \qquad \mathbb{Q} = \begin{bmatrix} -\gamma'\left(\frac{3\sqrt{3}}{5}i\right)\cdot 144 \\ -\gamma'\left(\frac{3\sqrt{3}}{5}i\right)\cdot 72 \end{bmatrix} ql^2$$

$$\mathbb{K} = \begin{bmatrix} \frac{1}{5}\alpha\left(\frac{\sqrt{5}}{4}\right) + \frac{1}{12}\alpha'\left(\frac{3\sqrt{3}}{5}i\right) + \alpha'''\left(\frac{1}{10}\right) & -\frac{1}{5}\theta\left(\frac{\sqrt{5}}{4}\right) - \alpha'\left(\frac{3\sqrt{3}}{5}i\right)\frac{1}{24} \\ \frac{1}{5}\beta\left(\frac{\sqrt{5}}{4}\right) + \frac{1}{5}\alpha\left(\frac{\sqrt{5}}{4}\right) + \frac{1}{24}\alpha'\left(\frac{3\sqrt{3}}{5}i\right) & -\frac{2}{5}\theta\left(\frac{\sqrt{5}}{4}\right) - \frac{1}{5}\alpha'\left(\frac{\sqrt{5}}{4}\right) - \frac{1}{48}\alpha'\left(\frac{6\sqrt{3}}{10}i\right) + \frac{41}{400} \end{bmatrix} \frac{EI}{l^2}$$

$$\mathbb{K}\mathbb{q} + \mathbb{P} + \mathbb{Q} = 0 \qquad \mathbb{q} = \begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix}$$