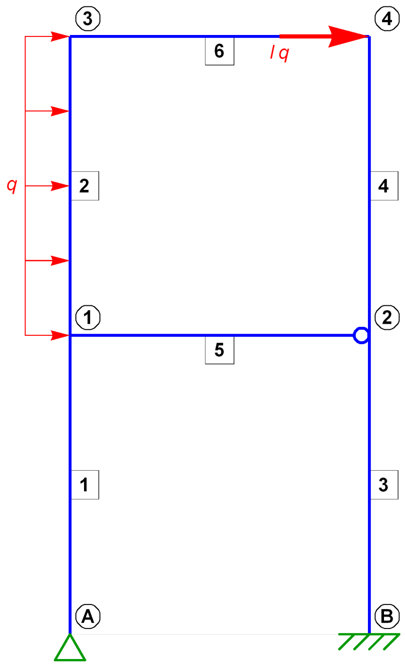
Kol 2.2.

Zapisać równania równowagi metody przemieszczeń w formie macierzowej.

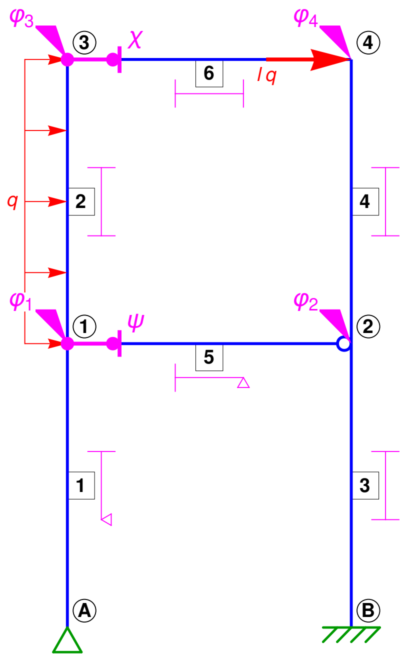
Geometria oraz obciążenia konstrukcji (wymiar oczka siatki - 1):



Wektor niewiadomych:

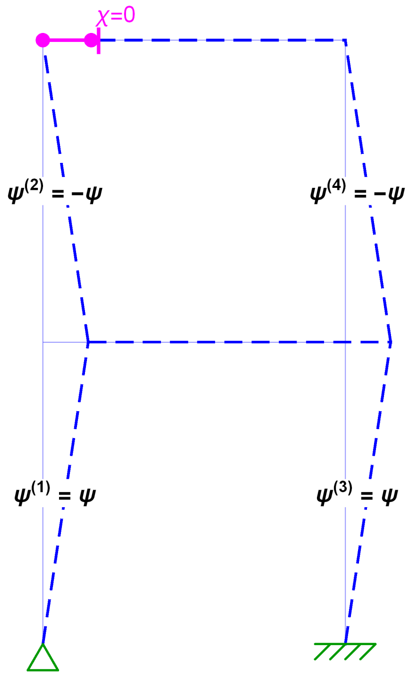
$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \psi \\ \chi \end{pmatrix}$$

Układ geometrycznie wyznaczalny:

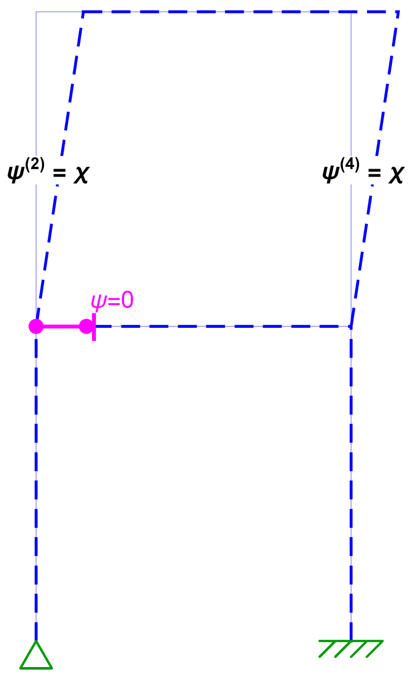


Plany przemieszczeń:

- plan przemieszczeń ψ:



- plan przemieszczeń χ:



Ostateczny plan przemieszczeń:

$\psi^{(1)} = \psi$

$\psi^{(2)} = -\psi + \chi$

$\psi^{(3)} = \psi$

$\psi^{(4)} = -\psi + \chi$

$\psi^{(5)} = 0$

$\psi^{(6)} = 0$

Momenty wyjściowe:

$\Phi_1^{02} = -\frac{1}{12} l^2 q$

$\Phi_3^{02} = \frac{1}{12} l^2 q$

Wzory transformacyjne:

$\Phi^1_1 = \frac{EJ}{l}\left[\,3\,\varphi_1 - 3\,\psi\,\right]$

$\Phi^2_1 = \frac{EJ}{l}\left[\,4\,\varphi_1 + 2\,\varphi_3 + 6\,\psi - 6\,\chi\,\right] - \frac{1}{12}\,l^2\,q$

$\Phi^2_3 = \frac{EJ}{l}\left[\,2\,\varphi_1 + 4\,\varphi_3 + 6\,\psi - 6\,\chi\,\right] + \frac{1}{12}\,l^2\,q$

$\Phi^3_B = \frac{EJ}{l}\left[\,2\,\varphi_2 - 6\,\psi\,\right]$

$\Phi^3_2 = \frac{EJ}{l}\left[\,4\,\varphi_2 - 6\,\psi\,\right]$

$\Phi^4_2 = \frac{EJ}{l}\left[\,4\,\varphi_2 + 2\,\varphi_4 + 6\,\psi - 6\,\chi\,\right]$

$\Phi^4_4 = \frac{EJ}{l}\left[\,2\,\varphi_2 + 4\,\varphi_4 + 6\,\psi - 6\,\chi\,\right]$

$\Phi^5_1 = \frac{EJ}{l}\left[\,3\,\varphi_1\,\right]$

$\Phi^6_3 = \frac{EJ}{l}\left[\,4\,\varphi_3 + 2\,\varphi_4\,\right]$

$\Phi^6_4 = \frac{EJ}{l}\left[\,2\,\varphi_3 + 4\,\varphi_4\,\right]$

Równania równowagi:

$\Phi^1_1 + \Phi^2_1 + \Phi^5_1 = 0$

$\Phi^3_2 + \Phi^4_2 = 0$

$\Phi^2_3 + \Phi^6_3 = 0$

$\Phi^4_4 + \Phi^6_4 = 0$

$\Phi^1_1 \cdot \overline{\psi} + (\Phi^2_1 + \Phi^2_3)\,(-\overline{\psi}) + (\Phi^3_B + \Phi^3_2)\,\overline{\psi} + (\Phi^4_2 + \Phi^4_4)\,(-\overline{\psi}) + l\,q \cdot \frac{1}{2}\,l\,\overline{\psi} = \overline{0}$

$(\Phi^2_1 + \Phi^2_3)\,\overline{\chi} + (\Phi^4_2 + \Phi^4_4)\,\overline{\chi} + l\,q \cdot l\,\overline{\chi} + l\,q \cdot \frac{1}{2}\,l\,\overline{\chi} = \overline{0}$

$$\frac{EJ}{l}\begin{pmatrix} 10 & 0 & 2 & 0 & 3 & -6 \\ 0 & 8 & 0 & 2 & 0 & -6 \\ 2 & 0 & 8 & 2 & 6 & -6 \\ 0 & 2 & 2 & 8 & 6 & -6 \\ 3 & 0 & 6 & 6 & 39 & -24 \\ -6 & -6 & -6 & -6 & -24 & 24 \end{pmatrix}\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \psi \\ \chi \end{pmatrix} = l^2\,q\begin{pmatrix} \frac{1}{12} \\ 0 \\ -\frac{1}{12} \\ 0 \\ \frac{1}{2} \\ \frac{3}{2} \end{pmatrix}$$

Rozwiązanie metody przemieszczeń:

$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \psi \\ \chi \end{pmatrix} = \frac{l^3\,q}{EJ}\begin{pmatrix} 0.292 \\ 0.520 \\ 0.137 \\ 0.075 \\ 0.400 \\ 0.718 \end{pmatrix}$$

Momenty brzegowe:

$\Phi_1^1 = -0.323\, l^2 q$

$\Phi_1^2 = -0.552\, l^2 q$

$\Phi_3^2 = -0.695\, l^2 q$

$\Phi_B^3 = -1.358\, l^2 q$

$\Phi_2^3 = -0.319\, l^2 q$

$\Phi_2^4 = 0.319\, l^2 q$

$\Phi_4^4 = -0.571\, l^2 q$

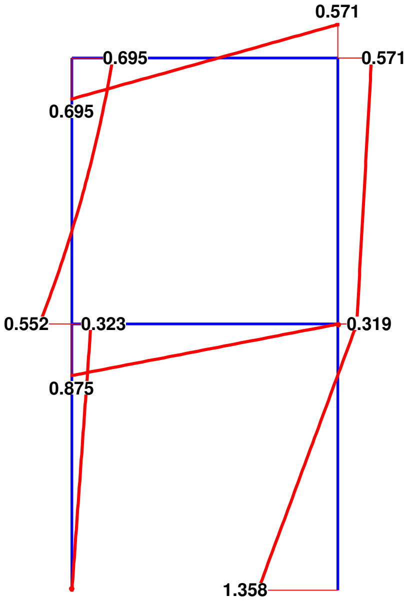
$\Phi_1^5 = 0.875\, l^2 q$

$\Phi_3^6 = 0.695\, l^2 q$

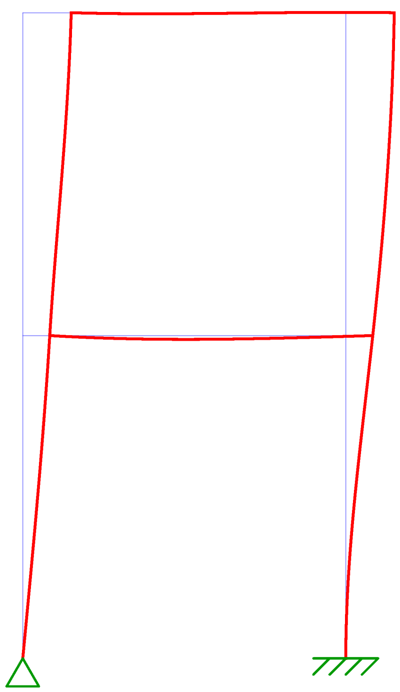
$\Phi_4^6 = 0.571\, l^2 q$

Wykres momentów zginających:

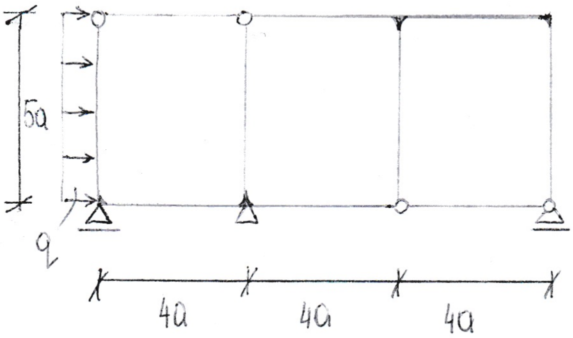
$M[l^2 q]$:



Deformacja konstrukcji:



KOLOKWIUM 2.2 R.A. 20?/2024

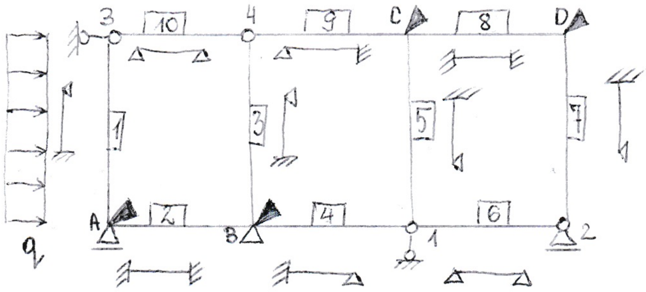


$EJ = const$

$EA = \infty$

$\mathbb{K} q = Q - ?$

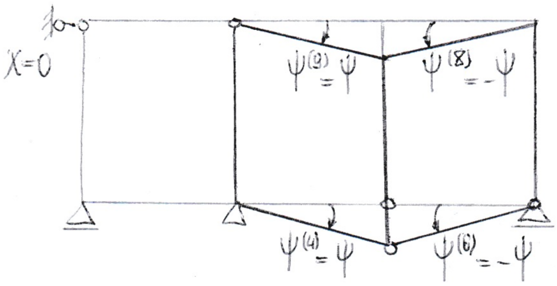
UKŁAD GEOMETRYCZNIE WYZNACZALNY:



Wektor niewiadomych:

$$q = \begin{bmatrix} \varphi_A \\ \varphi_B \\ \varphi_C \\ \varphi_D \\ \psi \\ \chi \end{bmatrix}$$

PLAN PRZEMIESZCZEŃ ψ:



$\psi^{(1)} = \chi$

$\psi^{(2)} = 0$

$\psi^{(3)} = \chi$

$\psi^{(4)} = \psi$

$\psi^{(5)} = \chi$

$\psi^{(6)} = -\psi$

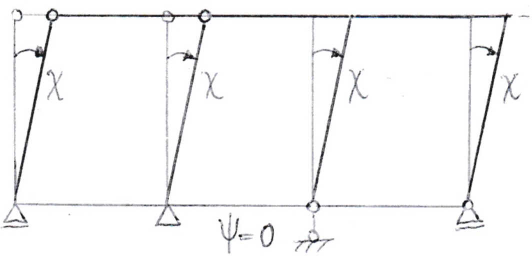
$\psi^{(7)} = \chi$

$\psi^{(8)} = -\psi$

$\psi^{(9)} = \psi$

$\psi^{(10)} = 0$

PLAN PRZEMIESZCZEŃ χ:



MOMENTY WYJŚCIOWE:

$$\Phi_A^1 = -\frac{q(5a)^2}{8} = -\frac{25}{8} qa^2$$

WZORY TRANSFORMACYJNE:

$$\Phi_A^1 = \frac{3EJ}{5a}\left[\varphi_A - \chi\right] = \frac{EJ}{a}\left[\frac{3}{5}\varphi_A - \frac{3}{5}\chi\right] - \frac{25}{8}qa^2 \qquad \Phi_B^3 = \frac{3EJ}{5a}\left[\varphi_B - \chi\right] = \frac{EJ}{a}\left[\frac{3}{5}\varphi_B - \frac{3}{5}\chi\right]$$

$$\Phi_A^2 = \frac{2EJ}{4a}\left[2\varphi_A + \varphi_B - 3\cdot 0\right] = \frac{EJ}{a}\left[\varphi_A + \frac{1}{2}\varphi_B\right] \qquad \Phi_B^4 = \frac{3EJ}{4a}\left[\varphi_B - \psi\right] = \frac{EJ}{a}\left[\frac{3}{4}\varphi_B - \frac{3}{4}\psi\right]$$

$$\Phi_B^2 = \frac{2EJ}{4a}\left[2\varphi_B + \varphi_A - 3\cdot 0\right] = \frac{EJ}{a}\left[\frac{1}{2}\varphi_A + \varphi_B\right] \qquad \Phi_C^5 = \frac{3EJ}{5a}\left[\varphi_C - \chi\right] = \frac{EJ}{a}\left[\frac{3}{5}\varphi_C - \frac{3}{5}\chi\right]$$

$$\Phi_D^7 = \frac{3EJ}{5a}\left[\varphi_D - \chi\right] = \frac{EJ}{a}\left[\frac{3}{5}\varphi_D - \frac{3}{5}\chi\right]$$

$$\Phi_C^9 = \frac{3EJ}{4a}\left[\varphi_C + \psi\right] = \frac{EJ}{a}\left[\frac{3}{4}\varphi_C + \frac{3}{4}\psi\right]$$

$$\Phi_C^8 = \frac{2EJ}{4a}\left[2\varphi_C + \varphi_D - 3\cdot(-\psi)\right] = \frac{EJ}{a}\left[\varphi_C + \frac{1}{2}\varphi_D + 3\psi\right]$$

$$\Phi_D^8 = \frac{2EJ}{4a}\left[2\varphi_D + \varphi_C - 3\cdot(-\psi)\right] = \frac{EJ}{a}\left[\frac{1}{2}\varphi_C + \varphi_D + 3\psi\right]$$

RÓWNANIA RÓWNOWAGI:

$\Phi_A^1 + \Phi_A^2 = 0 \qquad \frac{EJ}{a}\left[\frac{3}{5}\varphi_A - \frac{3}{5}\chi\right] + \frac{EJ}{a}\left[\varphi_A + \frac{1}{2}\varphi_B\right] = \frac{25}{8}qa^2$

$\Phi_B^2 + \Phi_B^3 + \Phi_B^4 = 0 \qquad \frac{EJ}{a}\left[\frac{1}{2}\varphi_A + \varphi_B\right] + \frac{EJ}{a}\left[\frac{3}{5}\varphi_B - \frac{3}{5}\chi\right] + \frac{EJ}{a}\left[\frac{3}{4}\varphi_B - \frac{3}{4}\psi\right] = 0$

$\Phi_C^5 + \Phi_C^8 + \Phi_C^9 = 0 \qquad \frac{EJ}{a}\left[\frac{3}{5}\varphi_C - \frac{3}{5}\chi\right] + \frac{EJ}{a}\left[\varphi_C + \frac{1}{2}\varphi_D + 3\psi\right] + \frac{EJ}{a}\left[\frac{3}{4}\varphi_C + \frac{3}{4}\psi\right] = 0$

$\Phi_D^7 + \Phi_D^8 = 0 \qquad \frac{EJ}{a}\left[\frac{3}{5}\varphi_D - \frac{3}{5}\chi\right] + \frac{EJ}{a}\left[\frac{1}{2}\varphi_C + \varphi_D + 3\psi\right] = 0$

- $\Phi_B^4 \cdot \bar{\psi}^{(4)} + (\Phi_C^8 + \Phi_D^8)\cdot\bar{\psi}^{(8)} + \Phi_C^9 \cdot \bar{\psi}^{(9)} = 0 \rightarrow \Phi_B^4 \cdot \bar{\psi} - \Phi_C^8\bar{\psi} - \Phi_D^8 \cdot \bar{\psi} + \Phi_C^9 \cdot \bar{\psi} = 0$

$\bar{\psi} = -1 \rightarrow \frac{EJ}{a}\left[-\frac{3}{4}\varphi_B + \frac{3}{4}\psi\right] + \frac{EJ}{a}\left[\varphi_C + \frac{1}{2}\varphi_D + 3\psi\right] + \frac{EJ}{a}\left[\frac{1}{2}\varphi_C + \varphi_D + 3\psi\right] + \frac{EJ}{a}\left[-\frac{3}{4}\varphi_C - \frac{3}{4}\psi\right] = 0$

- $\Phi_A^1 \cdot \bar{\psi}^{(1)} + \Phi_B^3 \cdot \bar{\psi}^{(3)} + \Phi_C^5 \cdot \bar{\psi}^{(5)} + \Phi_D^7 \cdot \bar{\psi}^{(7)} + q\cdot 5a \cdot \frac{5}{2}a \cdot \bar{\psi}^{(1)} = 0$

$\hookrightarrow \Phi_A^1 \cdot \bar{\chi} + \Phi_B^3 \cdot \bar{\chi} + \Phi_C^5 \cdot \bar{\chi} + \Phi_D^7 \cdot \bar{\chi} + \frac{25}{2}qa^2 \cdot \bar{\chi} = 0$

$\bar{\chi} = -1 \rightarrow \frac{EJ}{a}\left[-\frac{3}{5}\varphi_A + \frac{3}{5}\chi\right] + \frac{EJ}{a}\left[-\frac{3}{5}\varphi_B + \frac{3}{5}\chi\right] + \frac{EJ}{a}\left[-\frac{3}{5}\varphi_C + \frac{3}{5}\chi\right] + \frac{EJ}{a}\left[-\frac{3}{5}\varphi_D + \frac{3}{5}\chi\right] + \frac{25}{8}qa^2 - \frac{25}{2}qa^2 = 0$

$$\frac{EJ}{a}\begin{bmatrix} \frac{8}{5} & \frac{1}{2} & 0 & 0 & 0 & -\frac{3}{5} \\ \frac{1}{2} & \frac{47}{20} & 0 & 0 & -\frac{3}{4} & -\frac{3}{5} \\ 0 & 0 & \frac{47}{20} & \frac{1}{2} & \frac{3}{4} & -\frac{3}{5} \\ 0 & 0 & \frac{1}{2} & \frac{8}{5} & \frac{3}{2} & -\frac{3}{5} \\ 0 & -\frac{3}{4} & \frac{3}{4} & \frac{3}{2} & 6 & 0 \\ -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} & 0 & \frac{12}{5} \end{bmatrix} \cdot \begin{bmatrix} \varphi_A \\ \varphi_B \\ \varphi_C \\ \varphi_D \\ \psi \\ \chi \end{bmatrix} = \begin{bmatrix} \frac{25}{8} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{75}{8} \end{bmatrix} qa^2$$