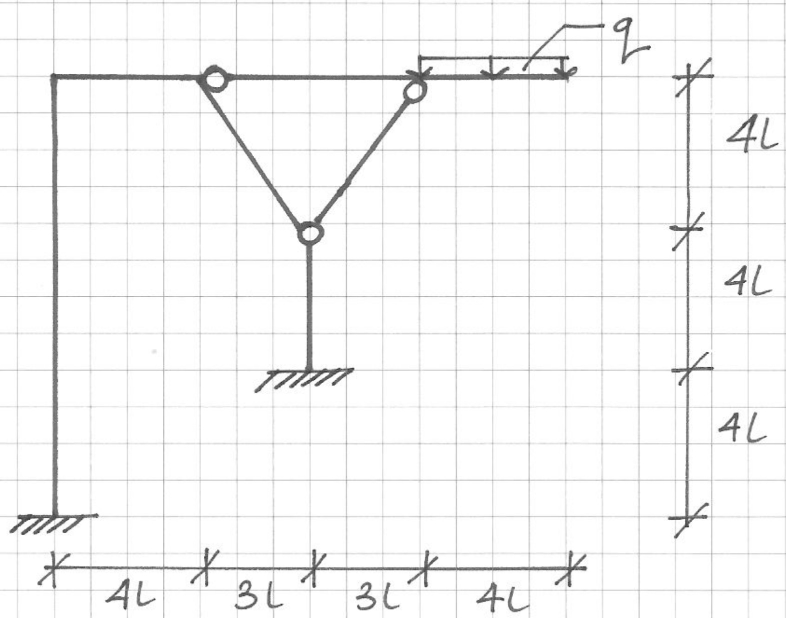
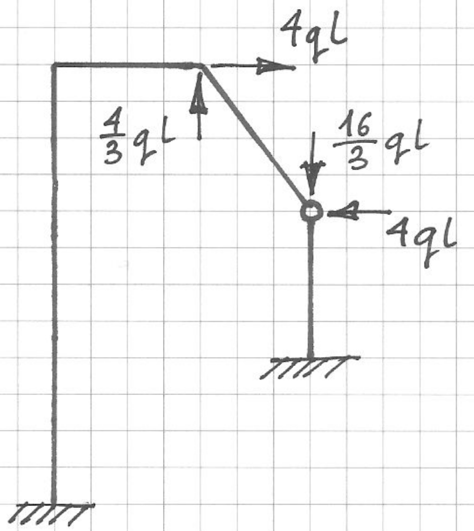
MK 1 kolokwium 2.2a, r.ak. 2013/2014

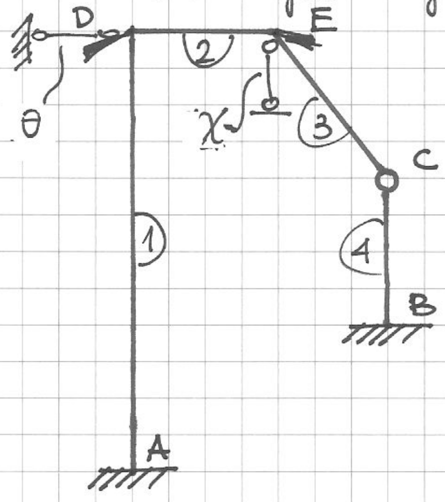
Narysować wykres M. $EJ = \text{const}.$ $EA = \infty$



Redukcja fragmentów statycznie wyznaczalnych

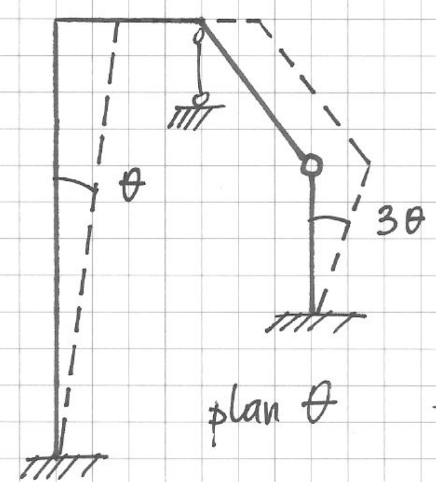


Schemat geometrycznie wyznaczalny:



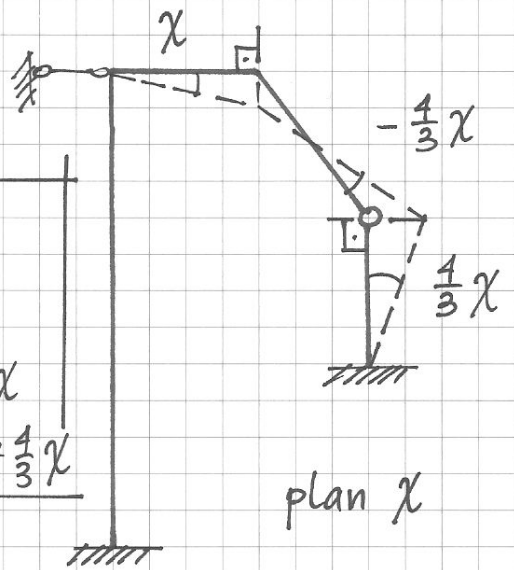
$$q = \begin{bmatrix} \varphi_D \\ \varphi_E \\ \theta \\ \chi \end{bmatrix}$$

Plany przesunięć:



plan θ

$$\psi^{(1)} = \theta$$
$$\psi^{(2)} = \chi$$
$$\psi^{(3)} = -\frac{4}{3}\chi$$
$$\psi^{(4)} = 3\theta + \frac{4}{3}\chi$$



plan χ

Równania równowagi:

$$\Phi_D^{(1)} + \Phi_D^{(2)} = 0$$

$$\Phi_E^{(2)} + \Phi_E^{(3)} = 0$$

$$\left[\Phi_A^{(1)} + \Phi_D^{(1)}\right]\cdot\overline{\theta} + \Phi_B^{(4)}\cdot 3\overline{\theta} = 0 \qquad \text{uwaga: } \overline{L}_\theta = 0$$

$$\left[\Phi_D^{(2)} + \Phi_E^{(2)}\right]\cdot\overline{\chi} + \Phi_E^{(3)}\cdot\left(-\tfrac{4}{3}\overline{\chi}\right) + \Phi_B^{(4)}\cdot\tfrac{4}{3}\overline{\chi} + \overline{L}_\chi = 0$$

$$\overline{L}_\chi = -\tfrac{4}{3}ql\cdot 4l\cdot\overline{\chi} - 4ql\cdot 4l\cdot\tfrac{4}{3}\overline{\chi} = -\tfrac{80}{3}ql^2\overline{\chi}$$

Wzory transformacyjne:

$$\Phi_A^{(1)} = \frac{2EJ}{12l}\left[\varphi_D - 3\theta\right] = -2{,}955\, ql^2$$

$$\Phi_D^{(1)} = \frac{2EJ}{12l}\left[2\varphi_D - 3\theta\right] = -4{,}412\, ql^2$$

$$\Phi_D^{(2)} = \frac{2EJ}{4l}\left[2\varphi_D + \varphi_E - 3\chi\right] = 4{,}412\, ql^2$$

$$\Phi_E^{(2)} = \frac{2EJ}{4l}\left[\varphi_D + 2\varphi_E - 3\chi\right] = 8{,}135\, ql^2$$

$$\Phi_E^{(3)} = \frac{3EJ}{5l}\left[\varphi_E + \tfrac{4}{3}\chi\right] = -8{,}135\, ql^2$$

$$\Phi_B^{(4)} = \frac{3EJ}{4l}\left[-3\theta - \tfrac{4}{3}\chi\right] = 2{,}456\, ql^2$$

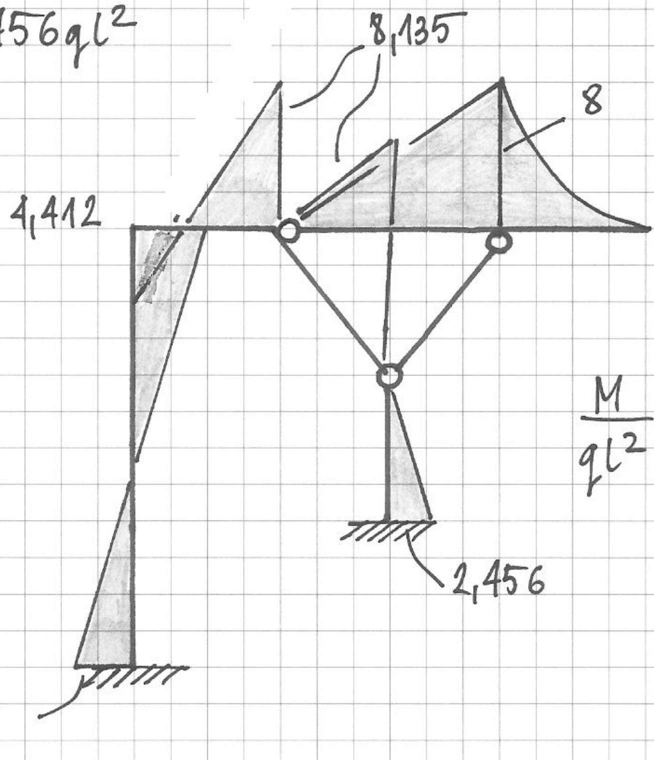
Po wstawieniu w.t. do r.r.

$$\varphi_D = -8{,}739\,\frac{ql^3}{EJ}$$

$$\varphi_E = -1{,}293 \;\text{—"—}$$

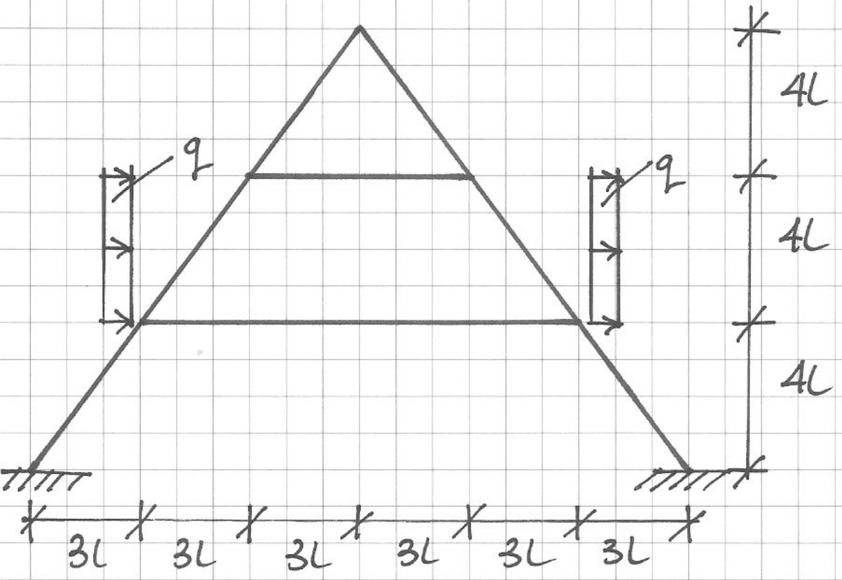
$$\theta = 2{,}997 \;\text{—"—}$$

$$\chi = -9{,}198 \;\text{—"—}$$

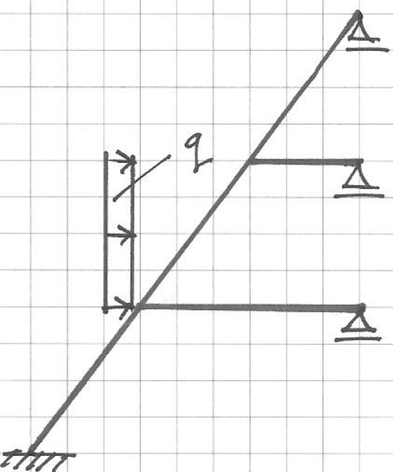


Kolokwium MK1 2.2b , r.ak. 2013/2014

Narysować wykres M $\qquad$ $EJ = \text{const.}$ $\quad EA = \infty$

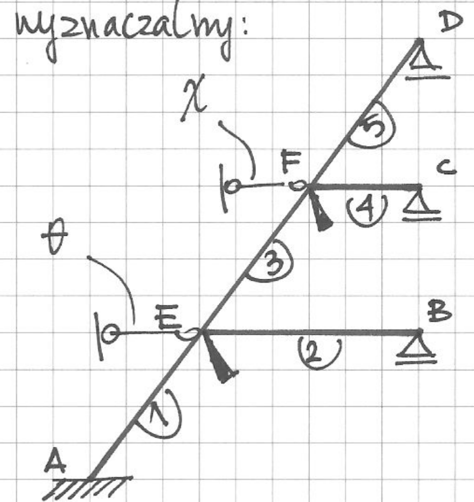


Po uwzględnieniu antysymetrii :

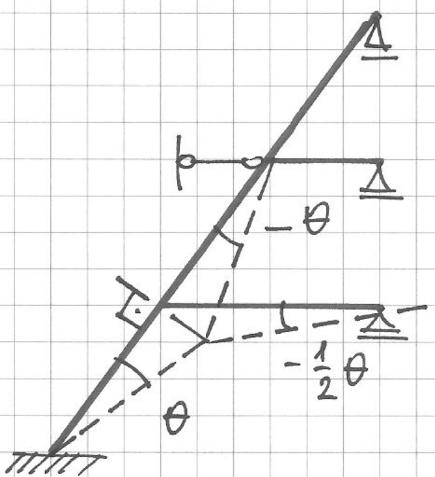


$$q = \begin{bmatrix} \varphi_E \\ \varphi_F \\ \theta \\ \chi \end{bmatrix}$$

Schemat geometrycznie wyznaczalny:

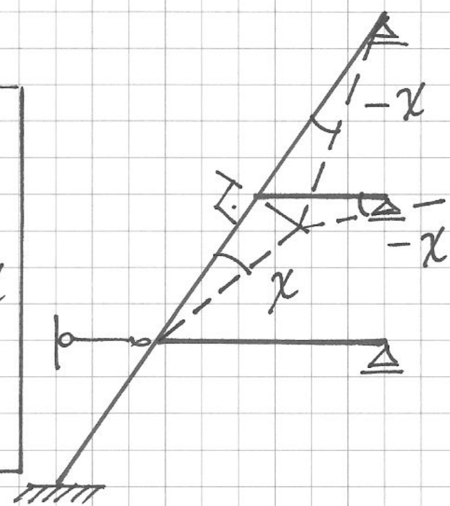


Plany przesunięć :



plan θ

$$\boxed{\begin{aligned} \psi^{(1)} &= \theta \\ \psi^{(2)} &= -\tfrac{1}{2}\theta \\ \psi^{(3)} &= -\theta + \chi \\ \psi^{(4)} &= -\chi \\ \psi^{(5)} &= -\chi \end{aligned}}$$



plan χ

Równania równowagi:

$$\Phi_E^{(1)} + \Phi_E^{(2)} + \Phi_E^{(3)} = 0$$

$$\Phi_F^{(3)} + \Phi_F^{(4)} + \Phi_F^{(5)} = 0$$

$$\left[\Phi_A^{(1)} + \Phi_E^{(1)}\right]\cdot \bar{\theta} + \left[\Phi_E^{(2)}\right]\cdot\left(-\tfrac{1}{2}\bar{\theta}\right) + \left[\Phi_E^{(3)} + \Phi_F^{(3)}\right]\cdot(-\bar{\theta}) + L_\theta = 0$$

$$\left[\Phi_E^{(3)} + \Phi_F^{(3)}\right]\cdot \bar{\chi} + \left[\Phi_F^{(4)}\right]\cdot(-\bar{\chi}) + \left[\Phi_F^{(5)}\right]\cdot(-\bar{\chi}) + L_\chi = 0$$

$$L_\theta = q\cdot 4l\cdot 2l\cdot \bar{\theta} = 8ql^2\bar{\theta}$$

$$L_\chi = q\cdot 4l\cdot 2l\cdot \bar{\chi} = 8ql^2\bar{\chi}$$

Wzory transformacyjne:

$$\Phi_A^{(1)} = \frac{2EJ}{5l}\left[\varphi_E - 3\theta\right] = -6{,}264\ ql^2$$

$$\Phi_E^{(1)} = \frac{2EJ}{5l}\left[2\varphi_E - 3\theta\right] = -3{,}824\ ql^2$$

$$\Phi_E^{(2)} = \frac{3EJ}{6l}\left[\varphi_E + \tfrac{1}{2}\theta\right] = 4{,}863\ ql^2$$

$$\Phi_E^{(3)} = \frac{2EJ}{5l}\left[2\varphi_E + \varphi_F + 3\theta - 3\chi\right] - \frac{1}{12}q(4l)^2 = -1{,}039\ ql^2$$

$$\Phi_F^{(3)} = \frac{2EJ}{5l}\left[\varphi_E + 2\varphi_F + 3\theta - 3\chi\right] + \frac{1}{12}q(4l)^2 = -3{,}481\ ql^2$$

$$\Phi_F^{(4)} = \frac{3EJ}{3l}\left[\varphi_F + \chi\right] = 2{,}175\ ql^2$$

$$\Phi_F^{(5)} = \frac{3EJ}{5l}\left[\varphi_F + \chi\right] = 1{,}305\ ql^2$$

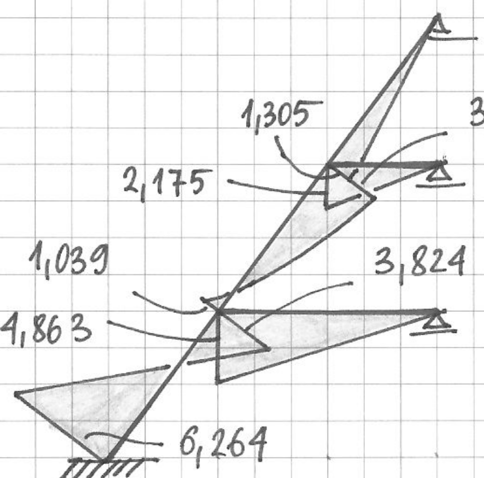
Rozwiązanie:

$$\varphi_E = 6{,}099\ \frac{ql^3}{EJ}$$

$$\varphi_F = -6{,}673\ \text{—"—}$$

$$\theta = 7{,}253\ \text{—"—}$$

$$\chi = 8{,}849\ \text{—"—}$$



$$\frac{M}{ql^2}$$