Exam on the Mechanics of Structures (MoS3 CES), 2.09.2019

LAST NAME, FIRST NAME (PLEASE, HANDWRITE VERY CLEARLY WITH CAPITAL LETTERS)			
index number			
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Problem 1.

Calculate the reaction at support B for a beam in Fig. 1

$$EJ = \text{const.}\ ,\ k = 11.4244\frac{EJ}{l^4}$$

Problem 2.

Calculate the frequency of natural vibrations for a curved bar in Fig. 2.

Next, calculate the reactions at supports for $t = 0$ sec. , $t = 5$ sec. , and $t = 10$ sec.

Assume:

$$E = 205\,\text{GPa}\ ,\ G = 0.385\,E\ ,$$

$$J = \frac{b^4 - a^4}{12}\ ,\ J_s = 1.7\,J\ ,$$

$$R = 2\,\text{m}\ ,\ m = 100\,\text{kg}.$$

Initial conditions:

$$u(0) = 0\ \text{cm},\ \ v(0) = \ 10\ \text{cm/sec.}$$

Problem 3.

Derive equilibrium equations for a circular bar in bending and twisting.

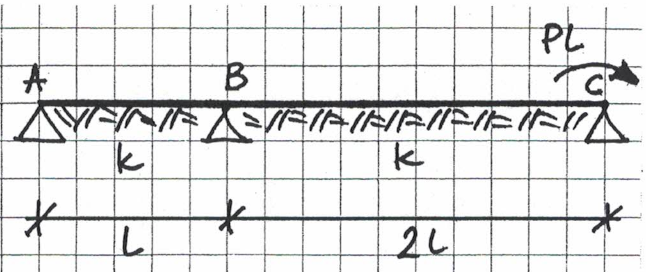

Fig.1

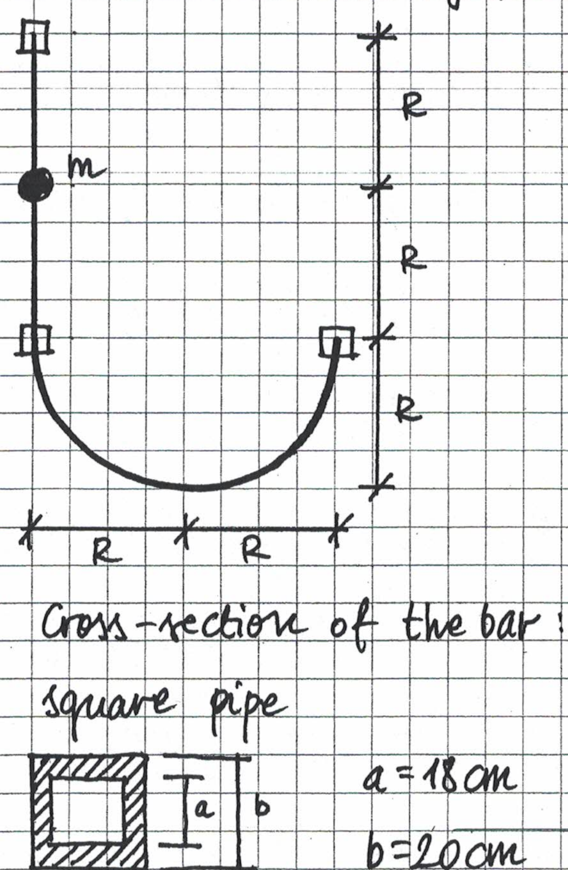

Fig.2

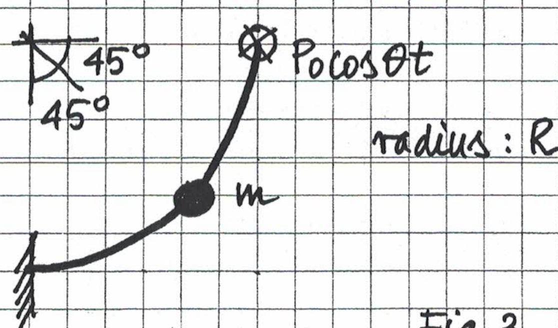

Fig.3

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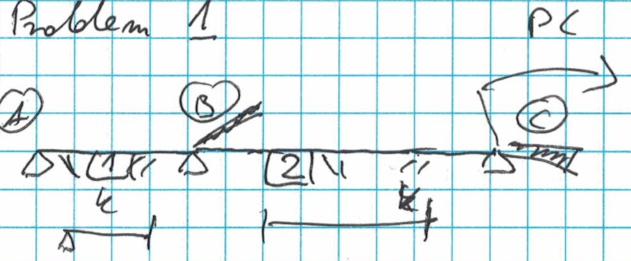
Problem 1



$$\lambda_i = l_i \sqrt[4]{\frac{k_i}{4EJ_i}}$$

$$\lambda_1 = l \sqrt[4]{\frac{11,4244 \frac{EJ}{l^4}}{4EJ}} = 1,3$$

$$\lambda_2 = 2,6$$

Bar	${}^*w$	w^*	λ
1	0	0	1,3
2	0	0	2,6

Equilibrium equations:

$$\ominus \Phi_B^1 \ominus \Phi_B^2 = 0$$

$$\ominus \Phi_C^2 + Pl = 0$$

$$\Phi_B^1 = \frac{EJ}{l}\left(\alpha'(1,3)\varphi_B\right) = \frac{EJ}{l}\left(3,209\varphi_B\right)$$

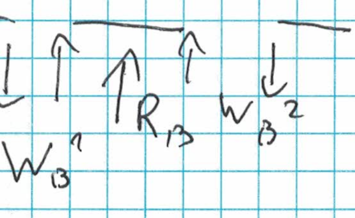
$$\Phi_B^2 = \frac{EJ}{2l}\left(\alpha(2,6)\varphi_B + \beta(2,6)\varphi_C\right) = \frac{EJ}{l}\left(2,6705\varphi_B + 0,5385\varphi_C\right)$$

$$\Phi_C^2 = \frac{EJ}{l}\left(0,5385\varphi_B + 2,6705\varphi_C\right)$$

$$\frac{EJ}{l}\begin{bmatrix} 5,8795 & 0,5385 \\ \text{sym} & 2,6705 \end{bmatrix}\begin{bmatrix} \varphi_B \\ \varphi_C \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} Pl$$

$$\varphi_B = \ominus 0,0349 \frac{Pl^2}{EJ} \qquad \varphi_C = 0,382 \frac{Pl^2}{EJ}$$

Node B:



$$\Rightarrow R_B = \ominus W_B^1 \ominus W_B^2$$

$$W_B^1 = \ominus \frac{EJ}{l^2}\left(\vartheta'(1,3)\varphi_B\right) = 0,138P$$

$$W_B^2 = \frac{EJ}{(2l)^2}\left(\vartheta(2,6)\varphi_B + \delta(2,6)\varphi_C\right) = 0,080P$$

$$\boxed{R_B = \ominus 0,218\ P}$$

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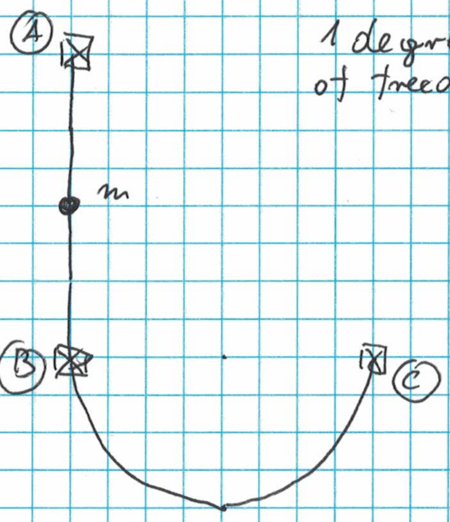
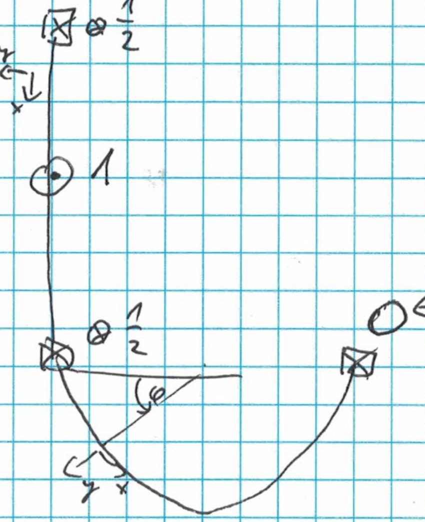
Problem 2

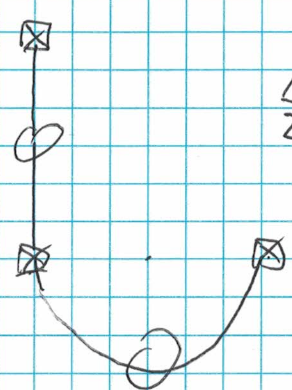


1 degree of freedom

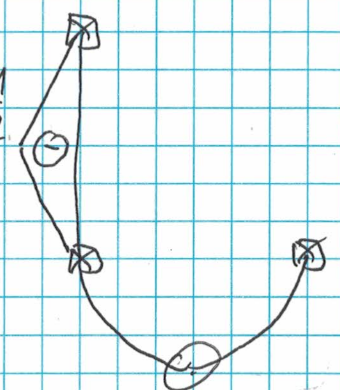


Since $R_C = 0$
Bending and Torsion Moment on the Arc $= 0$

$M[R]$



$M[R]$



$$\delta_{11} = \frac{1}{EJ} \cdot 2 \cdot \frac{1}{2} R \cdot R \cdot \frac{2}{3} \cdot \frac{1}{2} R = \frac{1}{3} \frac{R^3}{EJ}$$

$$J = \frac{13756}{3} [cm^4] = \frac{13756}{3 \cdot 10^8} [m^4]$$

$$E = 205 \left[\frac{GN}{m^2}\right] = 2{,}05 \cdot 10^{11} \left[\frac{N}{m^2}\right]$$

$$\delta_{11} = 2{,}84 \cdot 10^{-3} \left[\frac{s^2}{kg}\right]$$

$$\omega = \frac{1}{\sqrt{\delta_{11} m}} = 188 \left[\frac{1}{s}\right]$$

$$q(t) = A_1 \sin(\omega t) + A_2 \cos(\omega t) \qquad q(0) = 0 \Rightarrow A_2 = 0$$

$$\dot{q}(t) = A_1 \omega \cos(\omega t) - A_2 \omega \sin \omega t \qquad \dot{q}(0) = 0{,}1 \frac{m}{s} \Rightarrow A_1 = \frac{0{,}1}{\omega} = 5{,}3 \cdot 10^{-4} [m]$$

$$B(t) = \omega^2 m\, q(t) = 1{,}9 \sin(188 t) \; [kN]$$

$$R_A(t) = R_B(t) = -\frac{1}{2} B(t) \qquad R_C(t) = 0$$

$$B(0) = 0\,kN \Rightarrow R_A(0) = R_B(0) = R_C(0) = 0 \; [kN]$$

$$B(5_s) = 1 [kN] \Rightarrow R_A(5_s) = R_B(5_s) = -0{,}5 [kN] \qquad R_C = 0 [kN]$$

$$B(10_s) = -1{,}7 [kN] \Rightarrow R_A(10_s) = R_B(10_s) = 0{,}85 [kN] \qquad R_C = 0 [kN]$$