Exam on the Mechanics of Structures (MoS3 CES), 26.06.2019

LAST NAME, FIRST NAME (PLEASE, HANDWRITE VERY CLEARLY WITH CAPITAL LETTERS)			
index number			
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu pisemnego

Problem 1.

Calculate the reactions at supports for a beam in Fig. 1

$$EJ = \text{const.}\ ,\ k = 2.6244\frac{EJ}{l^4}$$

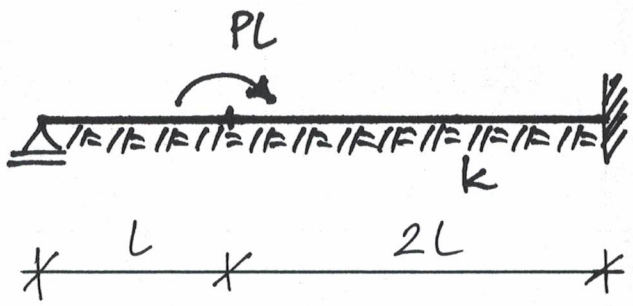


Fig. 1

Problem 2.

Calculate the frequency of natural vibrations for a curved bar in Fig. 2.

Next, calculate the reactions at supports for $t = 0$ sec. , $t = 5$ sec., and $t = 10$ sec.

Assume:

$$E = 205\,\text{GPa}\ ,\ G = 0.385\,E\ ,$$
$$J = \frac{b^4 - a^4}{12}\ ,\ J_s = 1.7\,J\ ,$$

and

$$R = 2\,\text{m}\ ,\ m = 100\,\text{kg}\ .$$

Initial conditions: $u(0) = 10\,\text{cm}$, $v(0) = 0\,\text{cm/sec.}$

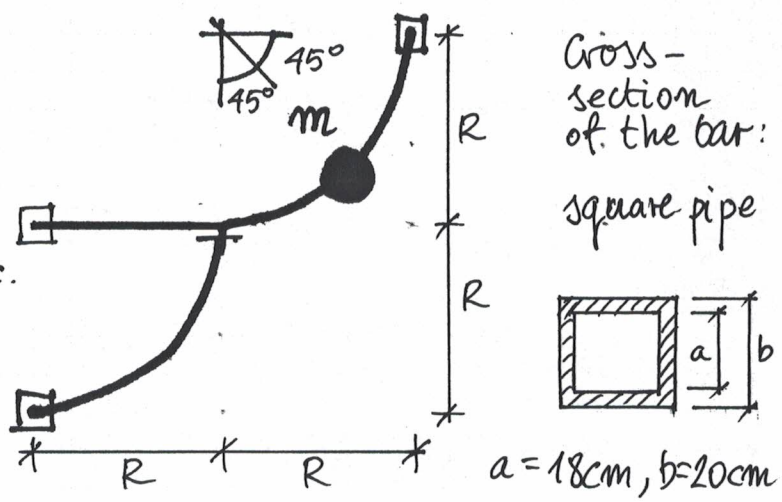


Fig. 2

Problem 3.

Derive the equation of motion for a structure in Fig. 3.

Assume E, G, J, J_s, R, m as in Problem 2, and $\theta = 0.8\,\omega$.

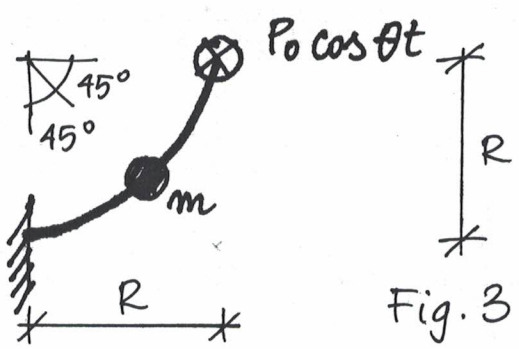
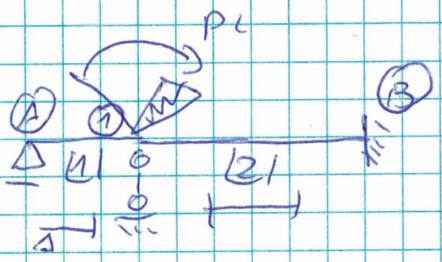


Fig. 3

Exam on MoS 3 CES 26.06.2019

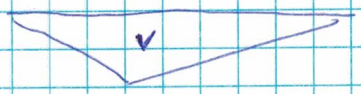
Problem 1



EF:

$$\begin{cases} \bar{\phi}_1^1 + \bar{\phi}_1^2 - Pl = 0 \\ -(W_1^1 \bar{v} + W_1^2 \bar{v}) = 0 \end{cases}$$

TD



Bar	$^*w$	w^*	λ
1	0	v	0,9
2	v	0	1,8

EJ/l

$$\begin{bmatrix} 5,247 & -1,204 \\ -1,204 & 7,642 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \frac{v}{l} \end{bmatrix} = \begin{bmatrix} Pl \\ 0 \end{bmatrix}$$

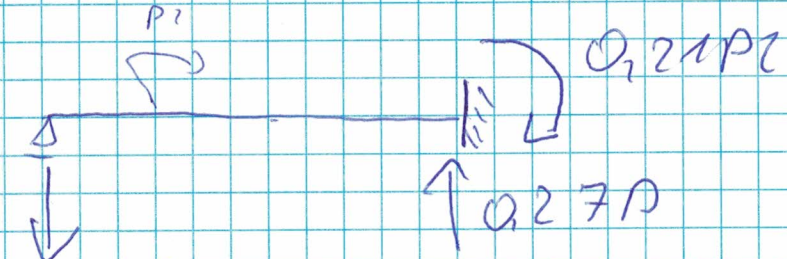
$$\varphi_1 = 0,198 \frac{Pl^2}{EJ}$$

$$\frac{v}{l} = 0,0312 \frac{Pl^2}{EJ}$$

$$W_A^1 = 0,49\,P$$

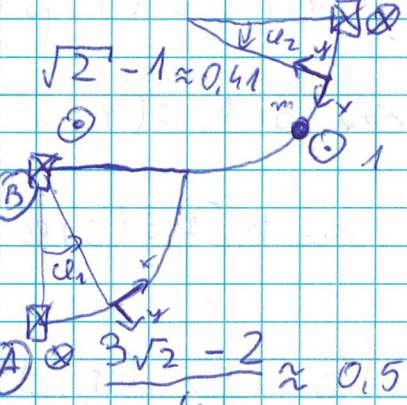
$$W_B^2 = -0,27\,P$$

$$\bar{\phi}_B^2 = 0,21\,Pl$$



Exam on MoS 3 CES 26.06.2019

Problem 2



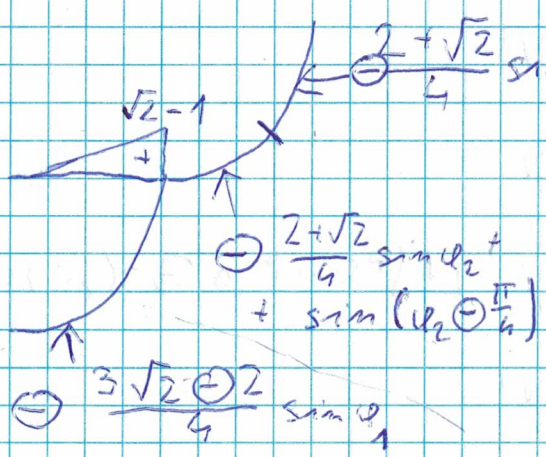
$$J = \frac{13756}{3}\,[cm^4] = \frac{13756}{3 \cdot 10^8}\,[m^4]$$

$$J_s = 7{,}8 \cdot 10^{-5}\,[m^4]$$

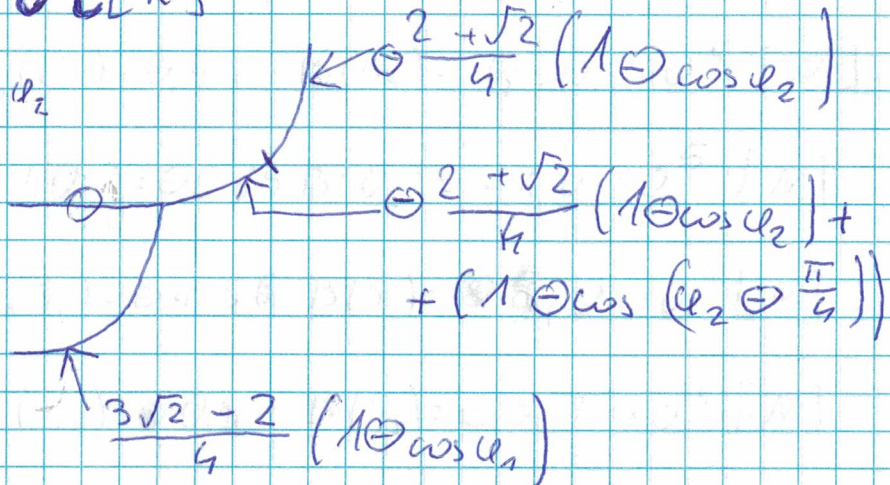
$$E = 205\left[\frac{GN}{m^2}\right] = 2{,}05 \cdot 10^{11}\left[\frac{N}{m^2}\right]$$

$$G = 7{,}89 \cdot 10^{10}\left[\frac{N}{m^2}\right]$$

$M[R]$



$M[R]$



$$\alpha_{11} = \frac{1}{EJ}\left\{\frac{1}{2} R(\sqrt{2}\ominus 1) R \frac{2}{3}(\sqrt{2}\ominus 1) R + \int_0^{\frac{\pi}{2}} \left(\ominus \frac{3\sqrt{2}-2}{4} \sin u_1\right)^2 R^3 \, du_1 + \right.$$

$$+ \int_0^{\frac{\pi}{4}} \left(\ominus \frac{2+\sqrt{2}}{4} \sin u_2\right)^2 R^3 \, du_2 + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\ominus \left(\frac{2+\sqrt{2}}{4} \sin u_2 + \sin\left(u_2 \ominus \frac{\pi}{4}\right)\right)\right]^2 R^3 \, du_2 \Big\}$$

$$+ \frac{1}{GJ_s}\left\{\int_0^{\frac{\pi}{2}} \left[\frac{3\sqrt{2}\ominus 2}{4}(1 \ominus \cos u_1)\right]^2 R^3 \, du_1 + \right.$$

$$+ \int_0^{\frac{\pi}{4}} \left[\ominus \frac{2+\sqrt{2}}{4}(1 \ominus \cos u_2)\right]^2 R^3 \, du_2 + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\ominus \frac{2+\sqrt{2}}{2}(1 \ominus \cos u_2) + (1 \ominus \cos(u_2 \ominus \tfrac{\pi}{4}))\right]^2 R^3$$

$$= \ldots = 0{,}545 \frac{R^3}{EJ} + 0{,}278 \frac{R^3}{GJ_s} = 8{,}25 \cdot 10^{-7}\left[\frac{s^2}{kg}\right]$$

$$\omega = \frac{1}{\sqrt{d_{11} m}} = 110 \left[\frac{1}{s}\right]$$

$q(t) = A_1 \sin(\omega t) + A_2 \cos(\omega t)$ $q(0) = 0{,}1\,[m] \Rightarrow A_2 = 0{,}1\,[m]$

$\dot{q}(t) = A_1 \omega \cos(\omega t) \ominus A_2 \omega \sin(\omega t)$ $\dot{q}(0) = 0 \left[\frac{m}{s}\right] \Rightarrow A_1 = 0\,[m]$

$q(t) = 0{,}1 \cos(110 t)\,[m]$

$B(t) = \ominus m \ddot{q}(t) = m \omega^2 A_1 \cos(\omega t) = 1{,}2 \cdot 10^5 \cos[110 t]\,[N]$

$B(0_s) = 1{,}2 \cdot 10^5\,[N]$

$V_A(0_s) = \ominus 0{,}56 \cdot B(0_s) = \ominus 6{,}8 \cdot 10^4\,[N]$

$V_B(0_s) = 0{,}41 \cdot B(0_s) = 5{,}0 \cdot 10^4\,[N]$

$V_C(0_s) = \ominus 0{,}85\, B(0_s) = -1{,}0 \cdot 10^5\,[N]$

$B(5_s) = \ominus 8{,}3 \cdot 10^4\,[N]$

$V_A(5_s) = \ominus 0{,}56\, B(5_s) = \ominus 5{,}0 \cdot 10^4\,[N]$

$V_B(5_s) = \ominus 0{,}41\, B(5_s) = \ominus 3{,}7 \cdot 10^4\,[N]$

$V_C(5_s) = \ominus 0{,}85\, B(5_s) = 7{,}6 \cdot 10^4\,[N]$

$B(10_s) = 8{,}5 \cdot 10^3\,[N]$

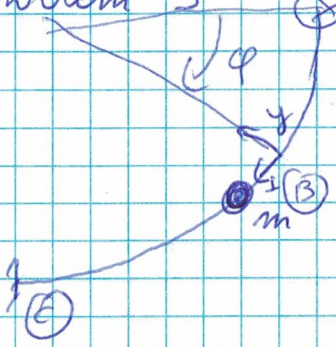
$V_A(10_s) = \ominus 0{,}56\, B(10_s) = \ominus 4{,}8 \cdot 10^3\,[N]$

$V_B(10_s) = 0{,}41\, B(10_s) = 3{,}5 \cdot 10^3\,[N]$

$V_C(10_s) = \ominus 0{,}85\, B(10_s) = \ominus 7{,}2 \cdot 10^3\,[N]$

Exam on MoS 3 CES 26.06.2018

Problem 3



$J = \frac{13756}{3} \cdot 10^{-8} \left[m^4\right]$ $J_s = 7{,}8 \cdot 10^{-5} \left[m\right.$

$E = 2{,}05 \cdot 10^{11} \left[\frac{N}{m^2}\right]$ $G = 7{,}89 \cdot 10^{10} \left[\frac{N}{m^2}\right]$

$\boxed{P = 2000\,N}$ - J assume it

q_1 state:

$$\frac{M_1(\varphi)}{R} = \begin{cases} 0 & \varphi \in [0, \frac{\pi}{4}] \\ \sin(\varphi - \frac{\pi}{4}) & \varphi \in [\frac{\pi}{4}, \frac{\pi}{2}] \end{cases}$$

$$\frac{M_1(\varphi)}{R} = \begin{cases} 0 & \varphi \in [0, \frac{\pi}{4}] \\ 1 - \cos(\varphi - \frac{\pi}{4}) & \varphi \in [\frac{\pi}{4}, \frac{\pi}{2}] \end{cases}$$

$$d_{11} = \frac{R^3}{EJ} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\sin(\varphi - \tfrac{\pi}{4})\right]^2 d\varphi + \frac{R^3}{GJ_s} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[1 - \cos(\varphi - \tfrac{\pi}{4})\right]^2 d\varphi =$$

O^4 state

$$d_{11} = 1{,}40 \cdot 10^{-7} \left[\frac{s^2}{kg}\right]$$

$$\omega = \sqrt{\frac{1}{d_{11} m}} = 268 \left[\frac{1}{s}\right] \quad \Theta = 0{,}8\,\omega = 214 \left[\frac{1}{s}\right]$$

$$\frac{M_0(\varphi)}{R} = \ominus \sin\varphi \qquad \frac{M_0(\varphi)}{R} = \cos\varphi - 1$$

$$d_{10} = \frac{R^3}{EJ} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\ominus \sin\varphi)\left(\sin(\varphi - \tfrac{\pi}{4})\right) d\varphi + \frac{R}{GJ_s} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos\varphi - 1)\left(1 - \cos(\varphi - \tfrac{\pi}{4})\right) d\varphi =$$

$$= \ominus 3{,}18 \cdot 10^{-7} \left[\frac{s^2}{kg}\right]$$

$$(1 \ominus \Theta^2 d_{11} m)\, A = d_{10} P$$

$$A = \frac{d_{10} P}{1 \ominus \Theta^2 d_{11} m} = \ominus 0{,}0177\,m$$

$$q(t) = A \cos \Theta t = \ominus 0{,}0177 \cos(214\,t) \left[m\right]$$