Exam on the Mechanics of Structures (MoS3 CES), 17.06.2019

LAST NAME, FIRST NAME (PLEASE, HANDWRITE VERY CLEARLY WITH CAPITAL LETTERS)			
index number			
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Problem 1.

Calculate the reactions at supports for a beam in Fig. 1

$$EJ = \text{const.}\ ,\ k = 4\ \frac{EJ}{l^4}$$

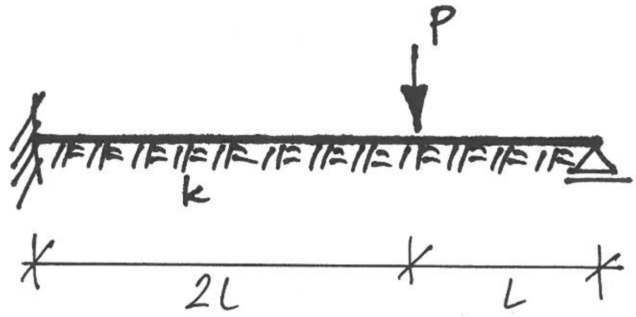

Fig. 1

Problem 2.

Calculate the frequency of natural vibrations for a curved bar in Fig. 2.

Next, calculate the reactions at supports for $t = 0$ sec. , $t = 5$ sec., and $t = 10$ sec.

Assume:

$$E = 205\,\text{GPa}\ ,\ G = 0.385\,E\ ,$$

$$J = \frac{b^4 - a^4}{12}\ ,\ J_s = 1.7\,J\ ,$$

and

$$R = 2\,\text{m}\ ,\ m = 100\,\text{kg}\ .$$

Initial conditions: $u(0) = 10\,\text{cm}$, $v(0) = 0\,\text{cm/sec}$.

Problem 3.

Derive equilibrium equations for a circular bar in bending and twisting.

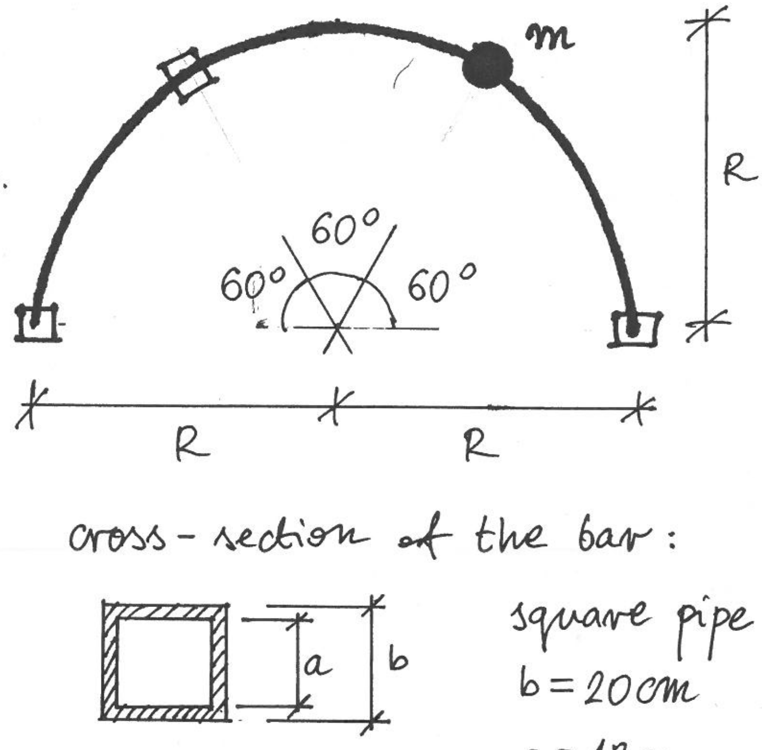

Fig. 2

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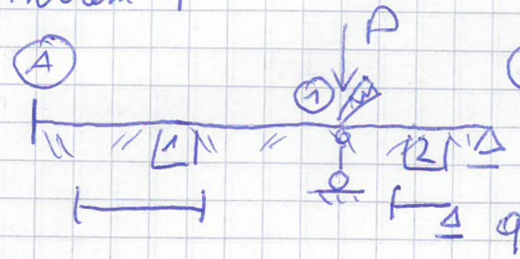
Problem 1

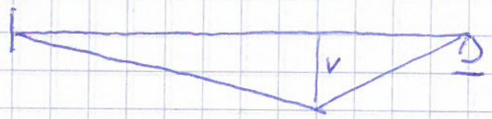


$$\lambda_i = l_i \sqrt[4]{\frac{k_i}{EJ_i}}$$

$$q_1 = \begin{bmatrix} \varphi_1 \\ \frac{v}{l} \end{bmatrix} \qquad \lambda_1 = 2l \sqrt[4]{\frac{4\frac{EJ}{l^4}}{4EJ}} = 2$$

$$\lambda_2 = 1$$

Translation Plan:



Bar	$\overset{*}{w}$	w^*	λ
1	0	v	1
2	v	0	2

Equilibrium equasions:

$$\begin{cases} \Phi_1^1 + \Phi_1^2 = 0 \\ -(W_1^1 \bar{v} + W_1^2 \bar{v}) + P\bar{v} = 0 \end{cases}$$

$$\Phi_1^1 = \frac{EJ}{2l}\left(\alpha(2)\varphi_1 \ominus \vartheta(2)\frac{v}{2l}\right) = \frac{EJ}{l}\left(2{,}275\varphi_1 \ominus 2{,}268\frac{v}{l}\right)$$

$$\Phi_1^2 = \frac{EJ}{l}\left(\alpha'(1)\varphi_1 + \vartheta'(1)\frac{v}{l}\right) = \frac{EJ}{l}\left(3{,}075\varphi_1 + 3{,}338\frac{v}{l}\right)$$

$$W_1^1 = \ominus\frac{EJ}{(2l)^2}\left(\vartheta(2)\varphi_1 - \gamma(2)\frac{v}{2l}\right) = \frac{EJ}{l^2}\left(\ominus 2{,}268\varphi_1 + 4{,}305\frac{v}{l}\right)$$

$$W_1^2 = \frac{EJ}{l^2}\left(\vartheta'(1)\varphi_1 + \gamma'(1)\frac{v}{l}\right) = \frac{EJ}{l^2}\left(3{,}338\varphi_1 + 4{,}925\frac{v}{l}\right)$$

$$\frac{EJ}{l}\begin{bmatrix} 5{,}35 & 1{,}07 \\ 1{,}07 & 9{,}23 \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \frac{v}{l} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} Pl$$

$$\varphi_1 = \ominus 0{,}0222 \frac{Pl^2}{EJ}$$

$$\frac{v}{l} = 0{,}111 \frac{Pl^2}{EJ}$$

$$\Phi_A^1 = \frac{EJ}{2l}\left(\beta(2)\varphi_1 \ominus \delta(2)\frac{v}{2l}\right) = \ominus 0{,}136 Pl$$

$$W_A^1 = \frac{EJ}{(2l)^2}\left(\delta(2)\varphi_1 \ominus \varepsilon(2)\frac{v}{2l}\right) = \ominus 0{,}093 P$$

$$W_B^2 = \ominus\frac{EJ}{l^2}\left(\delta'(1)\varphi_1 + \varepsilon'(1)\frac{v}{l}\right) = \ominus 0{,}209 P$$

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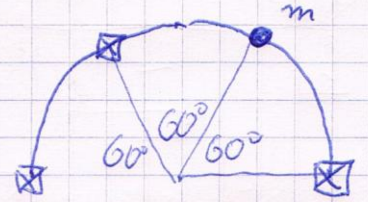
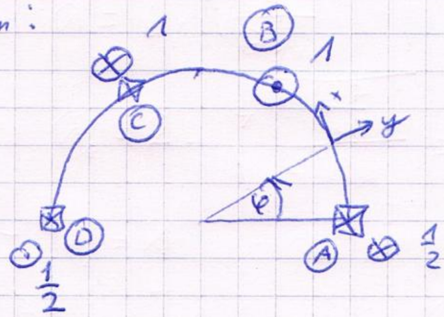
Problem 2



1 degree of freedom:



Torque:

$$\mathcal{M}(\varphi) = R \cdot \begin{cases} \frac{1}{2}(1 \ominus \cos\varphi) & , \varphi \in [0, \frac{\pi}{3}] \\ \frac{1}{2}(1 \ominus \cos\varphi) \ominus 1\left(1 \ominus \cos(\varphi \ominus \frac{\pi}{3})\right) & , \varphi \in [\frac{\pi}{3}, \frac{2}{3}\pi] \\ \frac{1}{2}(1 \ominus \cos\varphi) \ominus 1\left(1 \ominus \cos(\varphi \ominus \frac{\pi}{3})\right) + 1\left(1 \ominus \cos(\varphi \ominus \frac{2}{3}\pi)\right) & , \varphi \in [\frac{2}{3}\pi, \pi] \end{cases}$$

Bending:

$$M(\varphi) = R \cdot \begin{cases} \ominus \frac{1}{2}\sin\varphi & \varphi \in [0, \frac{\pi}{3}] \\ \ominus \frac{1}{2}\sin\varphi + 1 \sin(\varphi \ominus \frac{\pi}{3}) & \varphi \in [\frac{\pi}{3}, \frac{2}{3}\pi] \\ \ominus \frac{1}{2}\sin\varphi + 1 \sin(\varphi \ominus \frac{\pi}{3}) \ominus 1 \sin(\varphi \ominus \frac{2}{3}\pi) & \varphi \in [\frac{2}{3}\pi, \pi] \end{cases}$$

$$J = \frac{13756}{3}[cm^4] = \frac{13756}{3 \cdot 10^8}[m^2] \; ; \; E = 205\left[\frac{GN}{m^2}\right] = 2{,}05 \cdot 10^{11}\left[\frac{N}{m^2}\right]$$

$$J_s = 1{,}7 J = 7{,}8 \cdot 10^{-5}[m^2] \qquad G = 0{,}385 E = 7{,}89 \cdot 10^{10}\left[\frac{N}{m^2}\right]$$

$$d_{11} = \frac{1}{EJ}\left\{ \int_0^{\frac{\pi}{3}} \left(\ominus \frac{1}{2}\sin\varphi\right)^2 R^2 R\, d\varphi + \int_{\frac{\pi}{3}}^{\frac{2}{3}\pi} \left(\ominus \frac{1}{2}\sin\varphi + \sin(\varphi - \frac{\pi}{3})\right)^2 R^2 R\, d\varphi + \right.$$

$$\left. + \int_{\frac{2}{3}\pi}^{\pi} \left(\ominus \frac{1}{2}\sin\varphi + \sin(\varphi - \frac{\pi}{3}) - \sin(\varphi - \frac{2}{3}\pi)\right)^2 R^2 R \right\} +$$

$$+ \frac{1}{GJ_s}\left\{ \int_0^{\frac{\pi}{3}} \left(\frac{1}{2}(1 \ominus \cos\varphi)\right)^2 R^2 R\, d\varphi + \int_{\frac{\pi}{3}}^{\frac{2}{3}\pi} \left[\frac{1}{2}(1 \ominus \cos\varphi) \ominus \left(1 \ominus \cos(\varphi \ominus \frac{\pi}{3})\right)\right]^2 R^2 R\, d\varphi + \right.$$

$$\left. \int_{\frac{2}{3}\pi}^{\pi} \left[\frac{1}{2}(1 \ominus \cos\varphi) \ominus \left(1 \ominus \cos(\varphi \ominus \frac{\pi}{3})\right) + \left(1 \ominus \cos(\varphi \ominus \frac{2}{3}\pi)\right)\right]^2 R^2 R\, d\varphi \right. =$$

$$= \ldots = 0{,}221 \frac{R^3}{EJ} + 0{,}141 \frac{R^3}{GJ_s} = 3{,}71 \cdot 10^{-7}\left[\frac{s^2}{kg}\right]$$

$$\omega = \frac{1}{\sqrt{d_{11} m}} = 164\left[\frac{1}{s}\right]$$

$$q(t) = A_1 \sin(\omega t) + A_2 \cos(\omega t) \qquad q(0) = 0{,}1[m] \Rightarrow A_2 = 0{,}1[m]$$

$$\dot{q}(t) = A_1 \omega \cos(\omega t) - A_2 \omega \sin(\omega t) \qquad \dot{q}(0) = 0\left[\frac{m}{s}\right] \Rightarrow A_1 = 0\,[m]$$

$$q(t) = 0{,}1 \cos(164 t)$$

$$B(t) = -m\,\ddot{q}(t) = m\omega^2 A_1 \cos(\omega t) = 2{,}7 \cdot 10^5 \cdot \cos(164 t)\,[N]$$

$B(0_s) = 2{,}7 \cdot 10^5\,[N]$

$$V_A(0_s) = -\tfrac{1}{2} B(0) = -1{,}35 \cdot 10^5\,[N]$$
$$V_C(0_s) = -B(0) = -2{,}7 \cdot 10^5\,[N]$$
$$V_D(0_s) = \tfrac{1}{2} B(0) = 1{,}35 \cdot 10^5\,[N]$$

$B(5_s) = -2{,}4 \cdot 10^5\,[N]$

$$V_A(5_s) = -\tfrac{1}{2} B(5_s) = 1{,}2 \cdot 10^5\,[N]$$
$$V_C(5_s) = -B(5_s) = 2{,}4 \cdot 10^5\,[N]$$
$$V_D(5_s) = \tfrac{1}{2} B(5_s) = -1{,}2 \cdot 10^5\,[N]$$

$B(10_s) = 1{,}7 \cdot 10^5\,[N]$

$$V_A(10_s) = -\tfrac{1}{2} B(10_s) = -0{,}85 \cdot 10^5\,[N]$$
$$V_C(10_s) = -B(10_s) = -1{,}7 \cdot 10^5\,[N]$$
$$V_D(10_s) = \tfrac{1}{2} B(10_s) = 0{,}85 \cdot 10^5\,[N]$$

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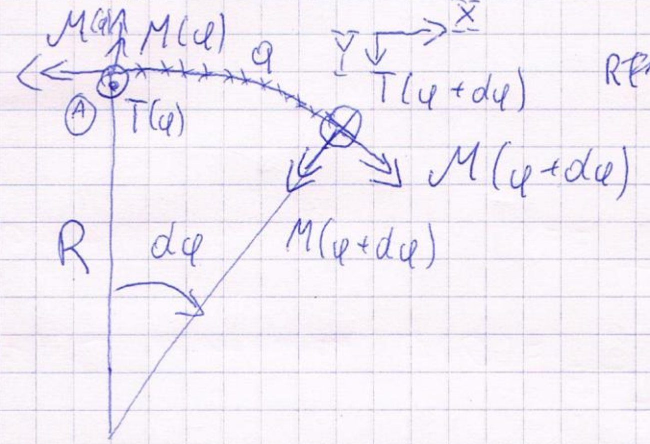
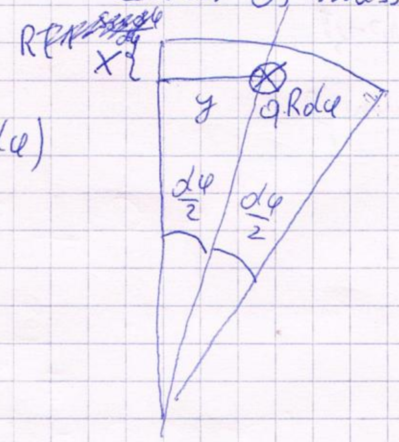
Problem 3



Center of mass of q:



$$x = R\left(1 - \frac{\sin d\varphi}{d\varphi}\right)$$

$$y = R\,\frac{1-\cos d\varphi}{d\varphi}$$

$\sum F_z = 0$

$T(\varphi + d\varphi) - T(\varphi) + qR\,d\varphi = 0$

$\dfrac{T(\varphi+d\varphi) - T(\varphi)}{d\varphi} + qR = 0 \qquad \lim d\varphi \to 0$

$$\boxed{\frac{dT(\varphi)}{d\varphi} + qR = 0}$$

$\sum M_{\underline{X}}^{A} = 0$

$M(\varphi+d\varphi)\cos d\varphi - M(\varphi+d\varphi)\sin d\varphi + T(\varphi+d\varphi)R(1-\cos d\varphi) +$
$+\, q\,d\varphi\, R\, R\left(1 - \dfrac{\sin d\varphi}{d\varphi}\right) = 0$

$\dfrac{M(\varphi+d\varphi)\cos d\varphi - M(\varphi)}{d\varphi} - M(\varphi+d\varphi)\dfrac{\sin d\varphi}{d\varphi} + T(\varphi+d\varphi)R\,\dfrac{1-\cos d\varphi}{d\varphi} + qR^2\left(1-\dfrac{\sin d\varphi}{d\varphi}\right) = 0$

$\lim d\varphi \to 0$

$$\boxed{\frac{dM(\varphi)}{d\varphi} + M(\varphi) = 0}$$

$\sum M_{\underline{Y}}^{A} = 0$

$M(\varphi+d\varphi)\cos d\varphi + M(\varphi+d\varphi)\sin d\varphi - M(\varphi) - T(\varphi+d\varphi)R\sin d\varphi +$

$+\, q\,d\varphi\, R R\left(\dfrac{1-\cos d\varphi}{d\varphi}\right) = 0$

$$\frac{M(\varphi+d\varphi)\cos d\varphi - M(\varphi)}{d\varphi} + M(\varphi+d\varphi)\frac{\sin d\varphi}{d\varphi} - T(\varphi+d\varphi)R\frac{\sin d\varphi}{d\varphi} +$$

$$+ qR^2\frac{1-\cos d\varphi}{d\varphi} = 0$$

$\lim d\varphi \to 0$

$$\boxed{\frac{dM(\varphi)}{d\varphi} + M(\varphi) - T(\varphi)R = 0}$$

Used Formulas:

$$\lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = \frac{\partial F(x)}{\partial x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1-\cos x}{x} = \lim_{x \to 0} \frac{\sin\frac{x}{2}}{\frac{x}{2}} \sin\frac{x}{2} = 1 \cdot 0 = 0$$