Exam on the Mechanics of Structures (MoS3 CES), 3.09.2018

LAST NAME, First name (PLEASE HANDWRITE VERY CLEARLY WITH CAPITAL LETTERS)				
index number				
ocena zadania 1	ocena zadania 2		ocena egzaminu	ocena łączna

Problem # 1.
$EJ = GJ_s = const.$
Calculate circular frequencies of natural vibrations of the grillage in Fig. 1.

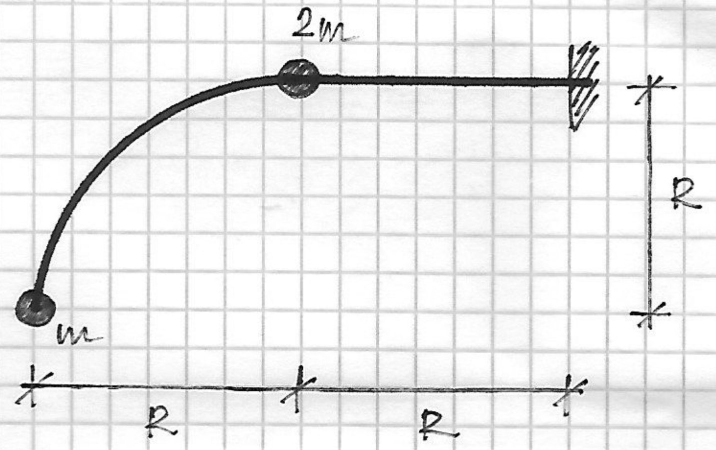


Fig. 1

Problem # 2.
Calculate the deflection at the middle of the span (point A) of an inextensible cable in Fig. 2.
Data:
$P = 5\ kN, l = 50\ m, L_0 = 53\ m$

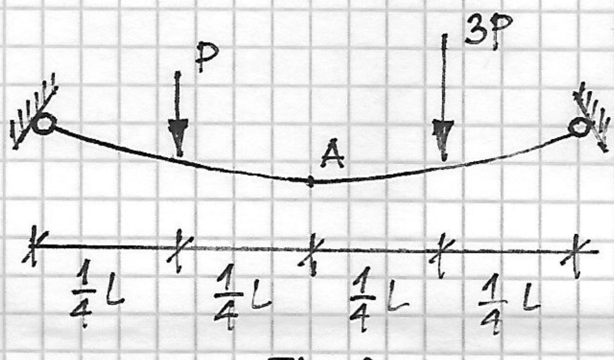


Fig. 2

Problem #1

The equilibrium equation reads

$$(\mathbb{I} - \omega^2 \mathbb{D}\mathbb{M})\, \mathbb{q} = \mathbb{0} \quad , \quad \omega - \text{circular frequency of natural vibrations}$$

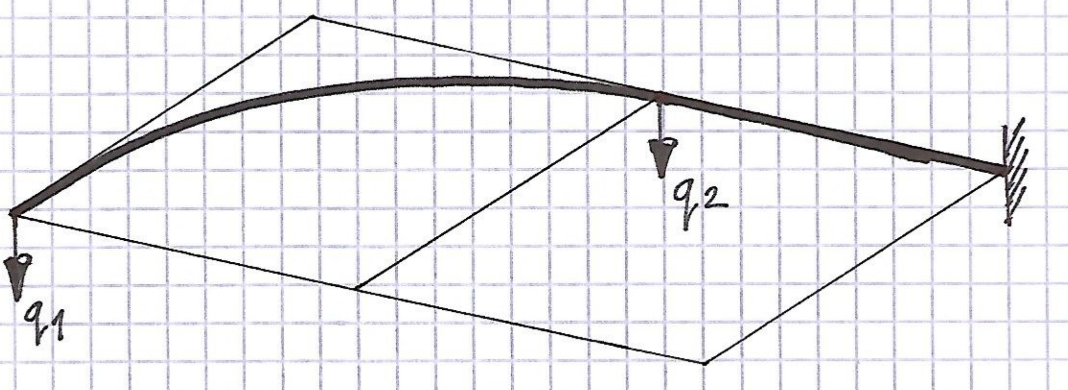
where

$$\mathbb{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

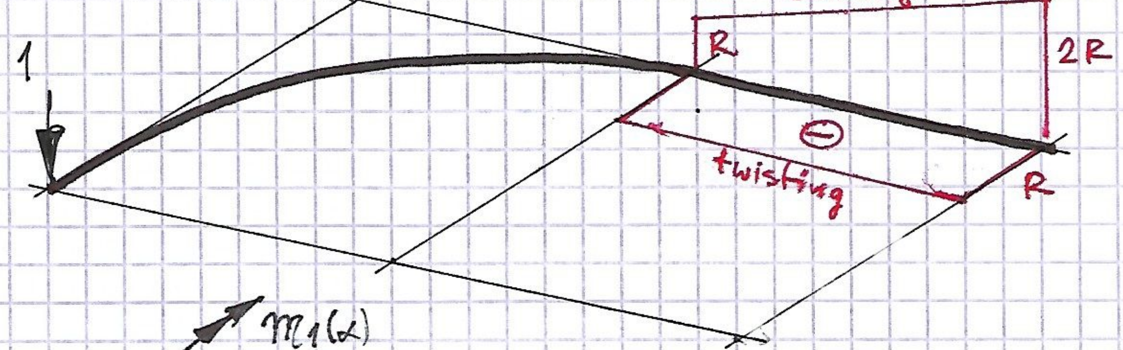
$$\mathbb{D} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \quad , \quad d_{12} = d_{22}$$

$$\mathbb{M} = \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}$$

in the Lagrange coordinate system $\mathbb{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ such that

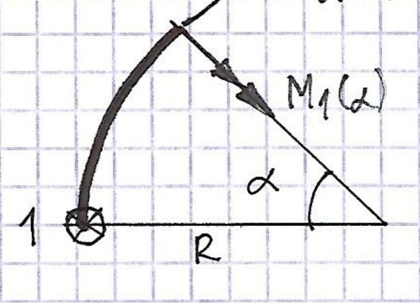


Now we calculate the coefficients $d_{11}, d_{12}, d_{21}, d_{22}$.

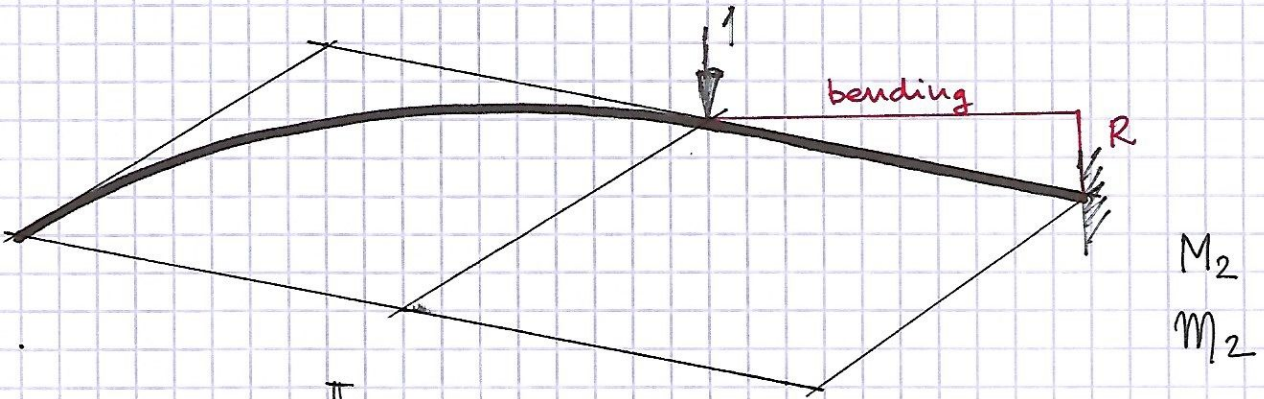


M_1

$\mathfrak{M}_1$



$$M_1(\alpha) + R\sin\alpha = 0$$
$$\mathfrak{M}_1(\alpha) + R(1-\cos\alpha) = 0 \qquad \alpha \in \left(0, \tfrac{\pi}{2}\right)$$



$$d_{11} = \frac{1}{EJ}\left[\int_0^{\frac{\pi}{2}} R^2 \sin^2\alpha \, R\, d\alpha + \frac{1}{2} R\cdot R\cdot\left(\frac{2}{3}R + \frac{1}{3}\cdot 2R\right) + \frac{1}{2}\cdot R\cdot 2R\cdot\left(\frac{1}{3}\cdot R + \frac{2}{3}\cdot 2R\right)\right]$$
$$+ \frac{1}{GJ_s}\left[\int_0^{\frac{\pi}{2}} R^2 (1-\cos\alpha)^2 R\, d\alpha + R\cdot R\cdot R\right] = 4{,}475\,\frac{R^3}{EJ}$$

$$d_{12} = d_{21} = \frac{1}{EJ}\left[\frac{1}{2}\cdot R\cdot R\cdot\left(\frac{1}{3}\cdot R + \frac{2}{3}\cdot 2R\right)\right] = 0{,}833\,\frac{R^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}\left[\frac{1}{2}\cdot R\cdot R\cdot \frac{2}{3}\cdot 2R\right] = 0{,}333\,\frac{R^3}{EJ}$$

Now, we define $\mathbb{A}(\omega) = \mathbb{I} - \omega^2 \mathbb{D}\mathbb{M}$. Hence the equilibrium equation becomes

$$\mathbb{A}(\omega)\, \mathbf{q} = \mathbb{0}.$$

Circular frequencies ω are calculated from the requirement

$$\det \mathbb{A}(\omega) = 0$$

$$\mathbb{A}(\omega) = \begin{bmatrix} 1 - 4{,}475\,\omega^2 \frac{mR^3}{EJ} & -1{,}667\,\omega^2 \frac{mR^3}{EJ} \\ -0{,}833\,\omega^2 \frac{mR^3}{EJ} & 1 - 0{,}667\,\omega^2 \frac{mR^3}{EJ} \end{bmatrix}$$

From this, we find

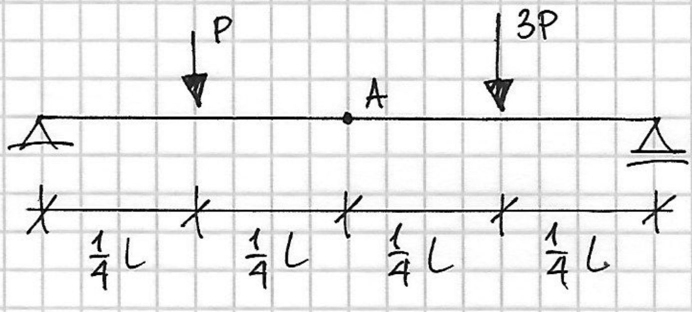
$$\det \mathbb{A}(\omega) = 1 - 5{,}142\,\omega^2 \frac{mR^3}{EJ} + 1{,}594\,\omega^4 \frac{m^2R^6}{EJ^2}$$

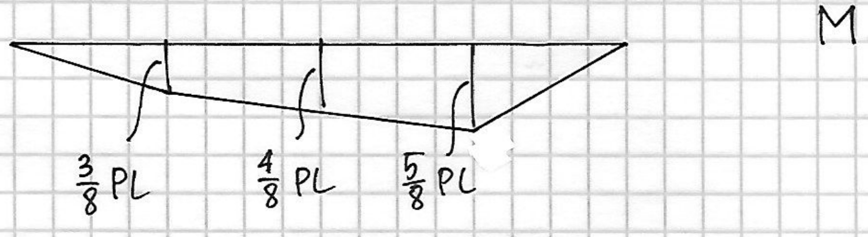
and

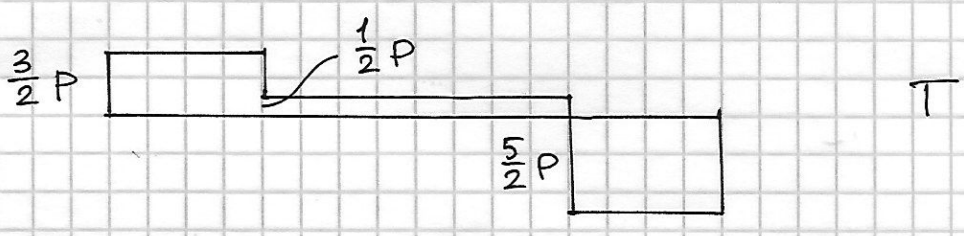
$$\omega_1 = 0{,}456\sqrt{\frac{EJ}{mR^3}}$$
$$\omega_2 = 1{,}737\sqrt{\frac{EJ}{mR^3}}$$

Problem #2

The auxiliary beam







$$M_A = \frac{1}{2}PL$$

$$\lambda_0 = \frac{L}{L_0} = 0{,}943$$

$$Q^2 = \frac{3}{2}P \cdot \frac{1}{4} \cdot \frac{3}{2}P + \frac{1}{2}P \cdot \frac{1}{2} \cdot \frac{1}{2}P + \frac{5}{2}P \cdot \frac{1}{4} \cdot \frac{5}{2}P = \frac{9}{4}P^2$$

$$Q = \frac{3}{2}P$$

$$H = Q\sqrt{\frac{\lambda_0}{2(1-\lambda_0)}} = \frac{5\sqrt{3}}{2}P = 4{,}33\,P$$

$$z_A = \frac{M_A}{H} = \frac{\sqrt{3}}{15}L = 0{,}115\,L = 5{,}77\ m$$

Solutions by

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