Exam on the Mechanics of Structures (MoS3 CES), 27.06.2018

LAST NAME, First name (PLEASE HANDWRITE VERY CLEARLY WITH CAPITAL LETTERS)				
index number				
ocena zadania 1	ocena zadania 2		ocena egzaminu	ocena łączna

Problem # 1.
$EJ = GJ_s = const.$
Calculate circular frequencies of natural vibrations of the grillage in Fig. 1.

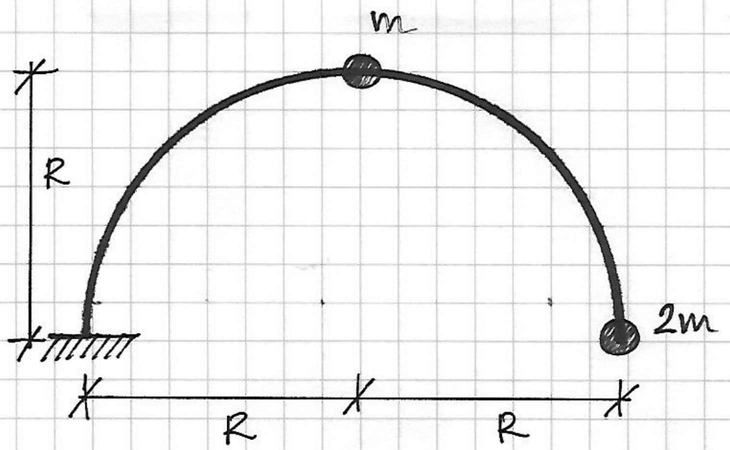


Fig. 1

Problem # 2.
Calculate the deflection at the middle of the bottom plate (point A) of a cylindrical vessel in Fig. 2.
Data:
$E = 30\ GPa, \nu = 0.2, h = \frac{1}{10}$

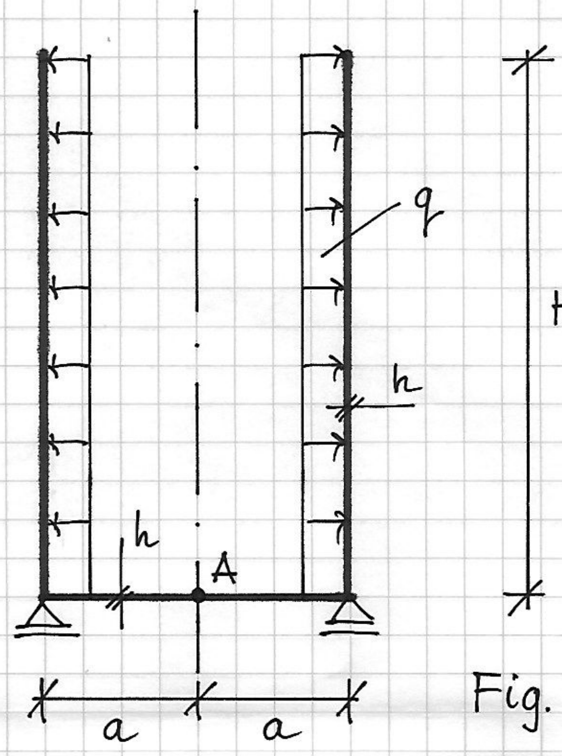


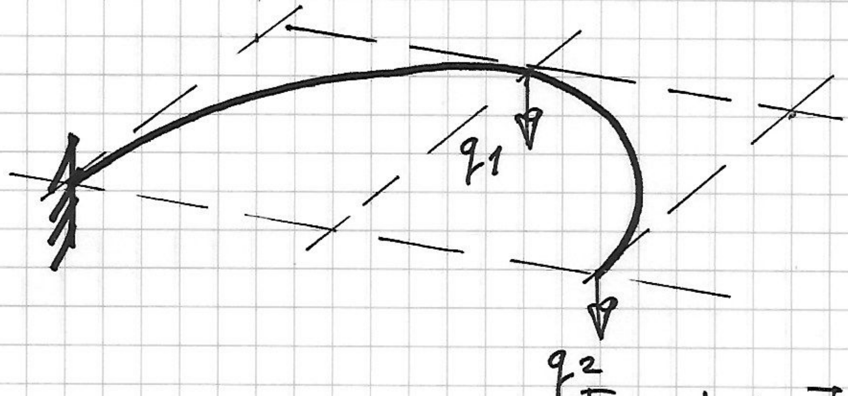
Fig. 2

Problem #1

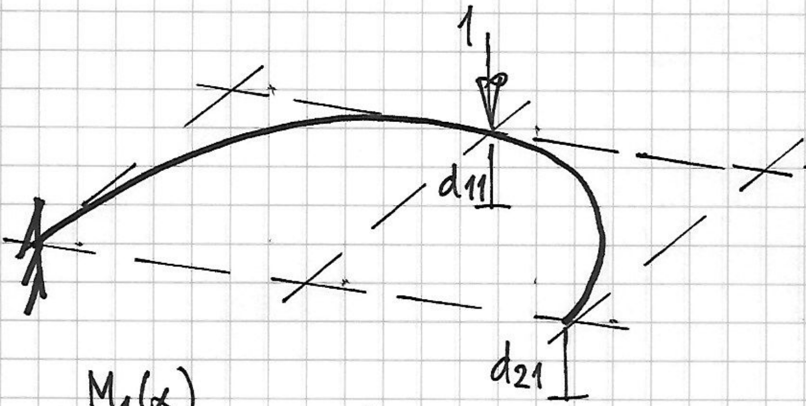
Our goal is to determine ω from the equation

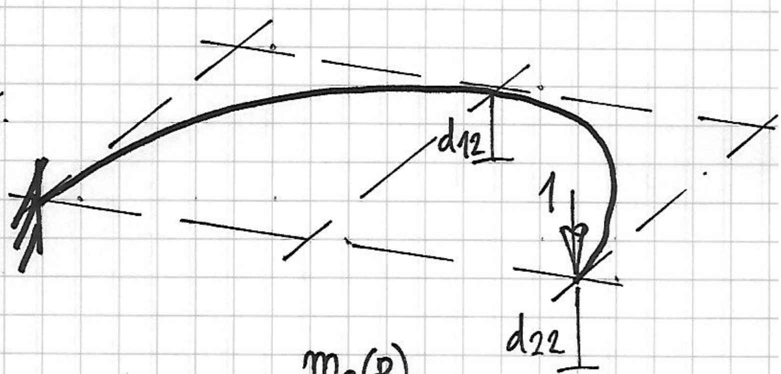
$$(\mathbb{I} - \omega^2 \mathbb{D}\mathbb{M})\, q = 0$$

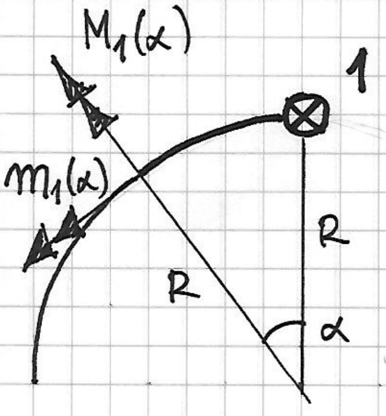
where $q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, $\mathbb{I} = \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}\right]$, $\mathbb{M} = \left[\begin{array}{c|c} m & 0 \\ \hline 0 & 2m \end{array}\right]$

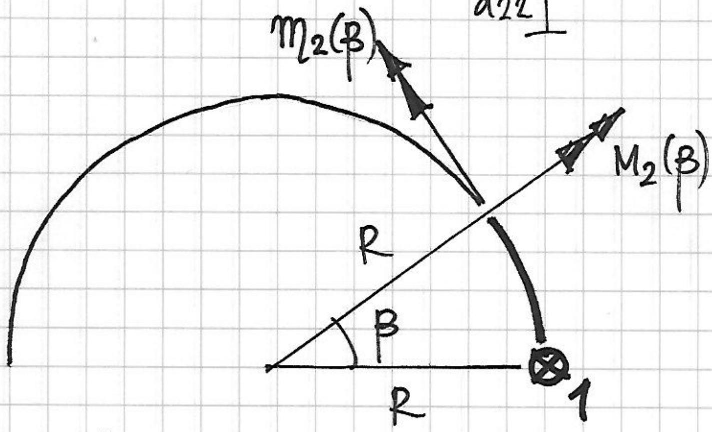


Now, we calculate $\mathbb{D} = \left[\begin{array}{c|c} d_{11} & d_{12} \\ \hline d_{21} & d_{22} \end{array}\right]$, $d_{12} = d_{21}$









$M_1(\alpha) = -R\sin\alpha$

$m_1(\alpha) = R(1-\cos\alpha)$

$\alpha \in (0, \frac{\pi}{2})$

$M_2(\beta) = -R\sin\beta$

$m_2(\beta) = R(1-\cos\beta)$

$\beta \in (0, \pi)$

$$d_{11} = \frac{1}{EJ}\int_0^{\frac{\pi}{2}}(M_1(\alpha))^2 R\,d\alpha + \frac{1}{GJ_s}\int_0^{\pi/2}(\mathcal{M}_1(\alpha))^2 R\,d\alpha =$$

$$= \frac{2R^3}{EJ}\left(\frac{\pi}{2}-1\right)$$

$$d_{12} = d_{21} = \frac{1}{EJ}\int_0^{\frac{\pi}{2}} M_1(\alpha)\,M_2\!\left(\alpha+\frac{\pi}{2}\right) R\,d\alpha + \frac{1}{GJ_s}\int_0^{\frac{\pi}{2}} \mathcal{M}_1(\alpha)\,\mathcal{M}_2\!\left(\alpha+\frac{\pi}{2}\right) R\,d\alpha =$$

$$= \frac{\pi}{2}\,\frac{R^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}\int_0^{\pi}(M_2(\beta))^2 R\,d\beta + \frac{1}{GJ_s}\int_0^{\pi}(\mathcal{M}_2(\beta))^2 R\,d\beta =$$

$$= 2\pi\,\frac{R^3}{EJ}$$

$$\mathbb{D} = \frac{R^3}{EJ}\begin{bmatrix} \pi-2 & \frac{\pi}{2} \\ \frac{\pi}{2} & 2\pi \end{bmatrix}$$

Let

$$\mathbb{A}(\omega) = \mathbb{I} - \omega^2 \mathbb{D}\mathbb{M}$$

$$\mathbb{A}(\omega) = \begin{bmatrix} 1-\omega^2(\pi-2)\frac{mR^3}{EJ} & -\omega^2\pi\frac{mR^3}{EJ} \\ -\omega^2\frac{\pi}{2}\frac{mR^3}{EJ} & 1-\omega^2\cdot 4\pi\frac{mR^3}{EJ} \end{bmatrix}$$

$$\det \mathbb{A}(\omega) = 1-(5\pi-2)a + \left(4\pi^2-8\pi-\frac{\pi^2}{2}\right)a^2$$

where $a = \omega^2\frac{mR^3}{EJ}$

$$= 1-13{,}71a+9{,}41a^2$$

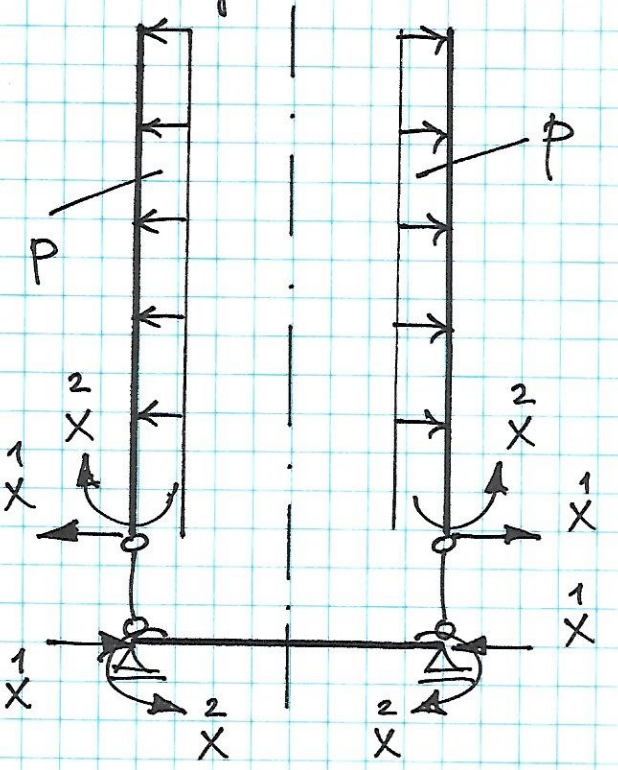
Hence,

$$a_1 = 0{,}077 \longrightarrow \omega_1 = 0{,}278\sqrt{\frac{mR^3}{EJ}}$$

$$a_2 = 1{,}38 \longrightarrow \omega_2 = 1{,}175\sqrt{\frac{mR^3}{EJ}}$$

Problem #2

Primary structure



We first calculate $\overset{1}{X}, \overset{2}{X}$ from

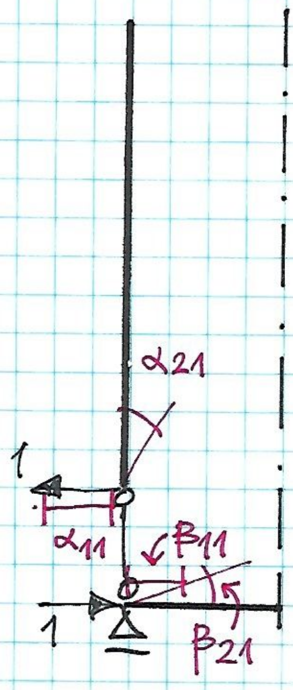
$$\begin{cases} \delta_{11}\overset{1}{X} + \delta_{12}\overset{2}{X} + \delta_{10} = 0 \\ \delta_{21}\overset{1}{X} + \delta_{22}\overset{2}{X} + \delta_{20} = 0 \end{cases}$$

where

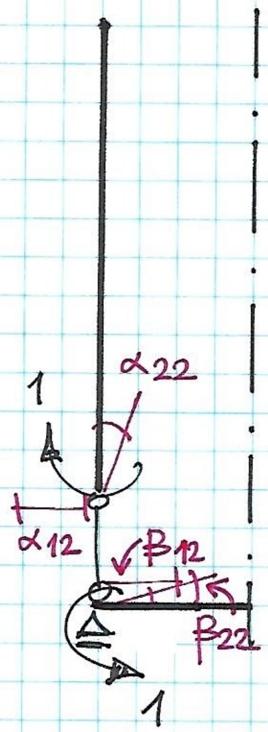
$$\delta_{ij} = \alpha_{ij} + \beta_{ij} \qquad i,j = 1,2$$

$$\delta_{i0} = \alpha_{i0} + \beta_{i0}$$

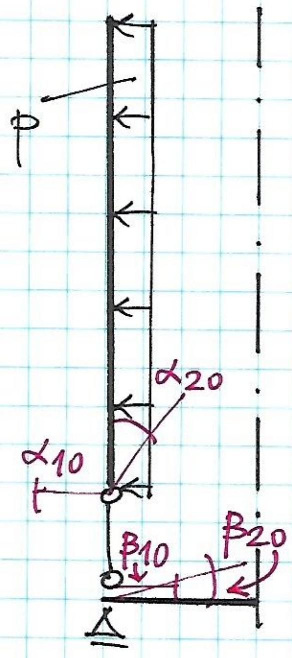
$\overset{1}{X} = 1$



$\overset{2}{X} = 1$



"0"

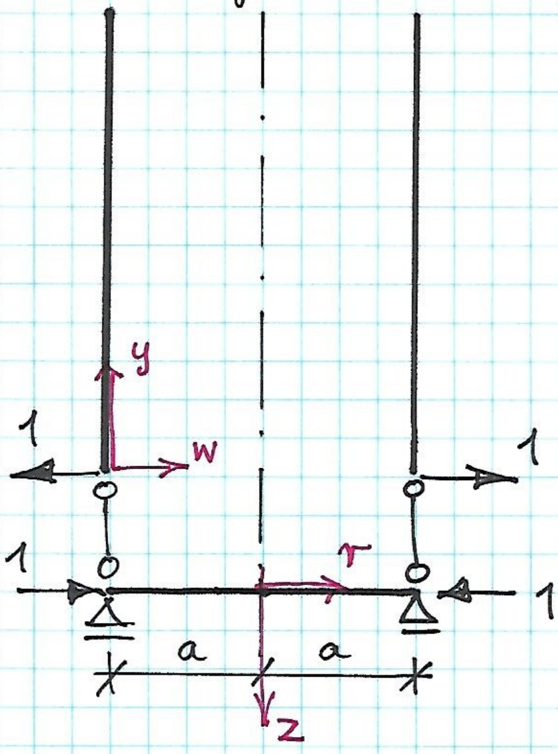


Bending and membrane deformations are independent (decoupled) hence: $\beta_{21} = 0$; $\beta_{12} = 0$;

Moreover, deformation in the "0" state is such that:

$$\alpha_{20} = 0\ ;\ \beta_{10} = 0\ ;\ \beta_{20} = 0 .$$

Calculating α_{11}, α_{21}, β_{11}



Cylinder:

$$\xi = \frac{y}{a}$$

$$\alpha_{11} = -w(\xi)\Big|_{\xi=0}$$

$$\alpha_{21} = \chi_2(\xi)\Big|_{\xi=0}$$

$$\chi_2(\xi) = \frac{1}{a}\frac{dw}{d\xi}(\xi)$$

$$w(\xi) = e^{-\lambda\xi}\left[A_1\cos(\lambda\xi) + A_2\sin(\lambda\xi)\right]$$

$$Q_2 = -\frac{D}{a^3}\frac{d^3w}{d\xi^3}$$

$$M_2 = -\frac{D}{a^2}\frac{d^2w}{d\xi^2}$$

Boundary conditions:

$$\left.\begin{array}{l} Q_2(0) = 1 \\ M_2(0) = 0 \end{array}\right| \rightarrow A_1, A_2$$

$$\alpha_{11} = \frac{2a\lambda}{hE} \; ; \; \alpha_{21} = \frac{2\lambda^2}{hE}$$

Plate:

$$\rho = \frac{r}{a}$$

$$u(\rho) = A_1\rho \qquad N_2 = \frac{C(1+\nu)}{a}\frac{du}{d\rho}$$

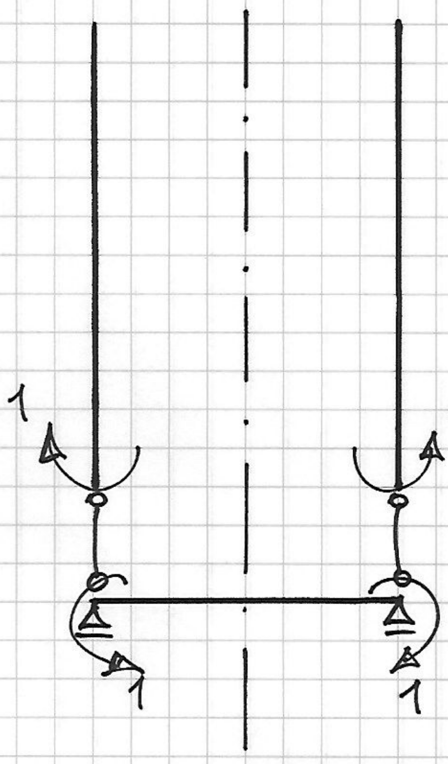
$\beta_{11} = -u(1)$ $u = u(\rho)$ – measures displacement in-plane of the plate

Boundary condition:

$$N_2(1) = -1 \longrightarrow A_1$$

$$\beta_{11} = \frac{a(1-\nu)}{hE}$$

Calculating α_{22}, β_{22}



We use the same coordinate systems as for calculations of $\alpha_{11}, \alpha_{21}, \beta_{11}$.

Cylinder:

$$\alpha_{22} = \chi_2(0)$$

Boundary conditions:

$$\left.\begin{aligned} Q_2(0) &= 0 \\ M_2(0) &= 1 \end{aligned}\right| \rightarrow A_1, A_2$$

$$\alpha_{22} = \frac{4\lambda^3}{ahE}$$

Plate:

$$\beta_{22} = \frac{1}{a} \frac{dw}{d\xi}(\xi) \Big|_{\xi=1}$$

$w = w(\xi)$ – measures deflection of the plate

$$w(\xi) = A_1 + A_2 \xi^2$$

Boundary conditions:

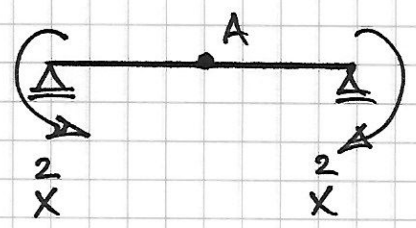
$$\left.\begin{aligned} M_1(1) &= -1 \\ w_1(1) &= 0 \end{aligned}\right| \rightarrow A_1, A_2$$

$$\beta_{22} = \frac{12a(1-\nu^2)}{h^3 E}$$

Calculating α_{10}

$$\alpha_{10} = -\frac{pa^2}{hE}$$

$\overset{1}{X} = -0{,}121\, pa$, $\overset{2}{X} = 0{,}003\, pa^2$



$w_A = w(0) = -17{,}28\, \frac{pa}{E}$

prepared by
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