EGZAMIN MK (MK3) KB studia mgr stacj. dnia 23 VI 2016 r.

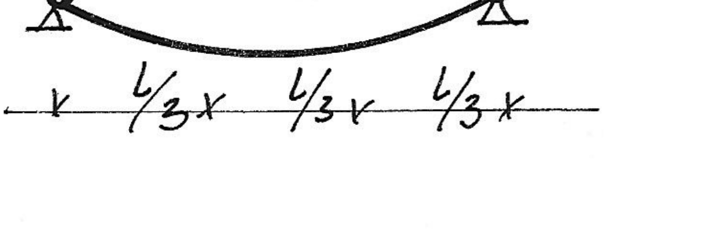
<table>
<tr><td colspan="3">NAZWISKO Imię</td></tr>
<tr><td colspan="2">Nr albumu</td><td>Ocena z ćwiczeń projektowych</td></tr>
<tr><td rowspan="2">ocena
zadania 1</td><td rowspan="2">ocena
zadania 2</td><td>Ocena z egzaminu po ustnym</td></tr>
<tr><td>Ocena łączna, data, podpis</td></tr>
</table>

Zadanie 1.

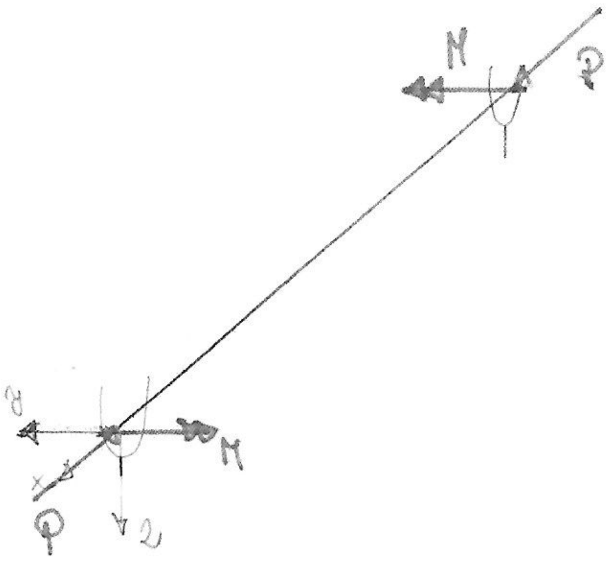
Rozważamy pręt o długości l=1 m, o przekroju w kształcie wąskiego prostokąta (h=8 cm, b=1 cm), podparty widełkowo na obu koncach i na obu końcach obciążony jednocześnie siłą ściskającą P i momentem zginającym M. Kierunek wektora momentu jest skierowany wzdłuż krótszego boku płaskownika. Przyjąć E=210GPa, współczynnik Poissona = 0.3. Znaleźć obszar stateczności (obszar bezpieczny) w płaszczyźnie P-M.

Zadanie 2.

Dane jest cięgno nierozciągliwe obciążone jak na rysunku. Obliczyć rzędną krzywej zwisu w środku cięgna.
Dane: L_0=1.05 l.



ZADANIE 1 EGZAMIN KB ST II 23 VI 2017



$$G = \frac{E}{2(1+\nu)} = 8077 \left[\frac{kN}{cm^2}\right]$$

$$E_1 = \frac{E}{1-\nu^2} = 23077 \left[\frac{kN}{cm^2}\right]$$

$$J_z = 0{,}667 \left[cm^4\right]$$

$$J_s = 2{,}667 \left[cm^4\right]$$

Równania

$$E_1 J_y w^{IV} + P w'' = 0 \qquad (1)$$

$$\begin{cases} E_1 J_z v^{IV} + P v'' - M \Theta^{IV} = 0 & (2) \\ G J_s \Theta'' + M v'' = 0 & (3) \end{cases}$$

Warunki brzegowe dla $x=0$, $x=l$

$\Theta'' = 0$ $\quad M_y = -M$

$\Theta = 0$ $\quad M_z = 0$

$v = 0$ $\quad B = 0$

$w = 0$

$$\begin{cases} \dfrac{d^4 v}{d\xi^4} - \dfrac{Ml^2}{E_1 J_z} \dfrac{d^2\Theta}{d\xi^2} + \dfrac{Pl^2}{E_1 J_z} \dfrac{d^2 v}{d\xi^2} = 0 \\ \dfrac{d^2\Theta}{d\xi^2} + \dfrac{M}{G J_s} \dfrac{d^2 v}{d\xi^2} = 0 \end{cases} \qquad \xi = \frac{x}{l}$$

Podstawiamy $\quad v = l\, C_1 \sin(\pi\xi) \quad \Theta = C_2 \sin(\pi\xi)$

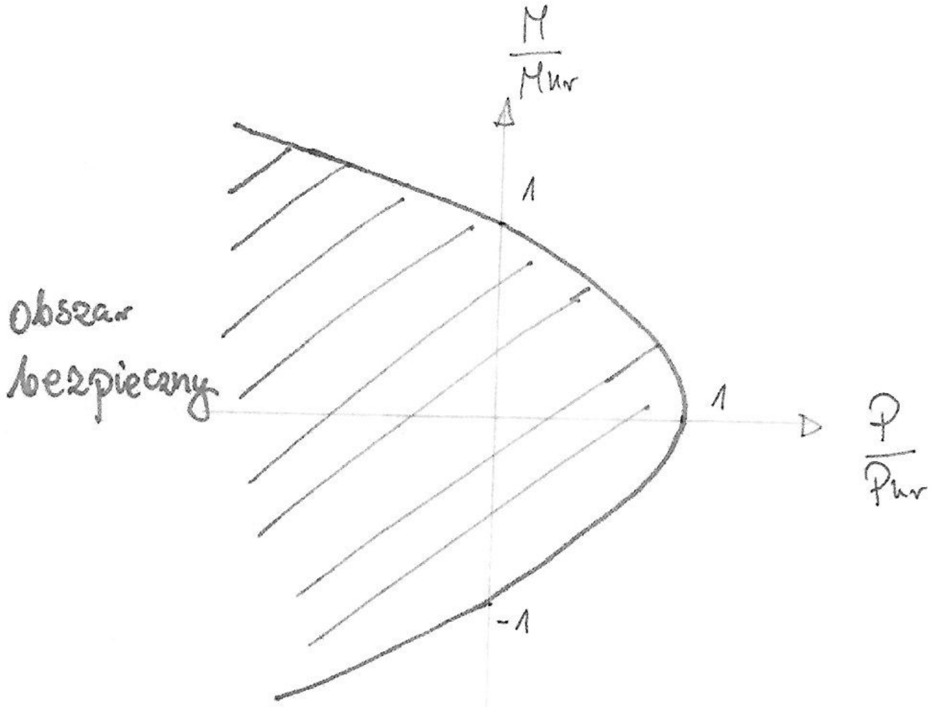
$$\begin{bmatrix} l\pi^2 - \dfrac{Pl^3}{E_1 J_z} & \dfrac{Ml^2}{E_1 J_z} \\ \dfrac{Ml}{G J_s} & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Z tego wynika wzór na granicę obszaru bezpiecznego (wyznacznik)

$$\frac{Pl^2}{\pi^2 E_1 J_z} + \frac{M^2 l^2}{\pi^2 E_1 J_z G J_s} = 1 \qquad \frac{P}{P_{kr}} + \left(\frac{M}{M_{kr}}\right)^2 = 1$$

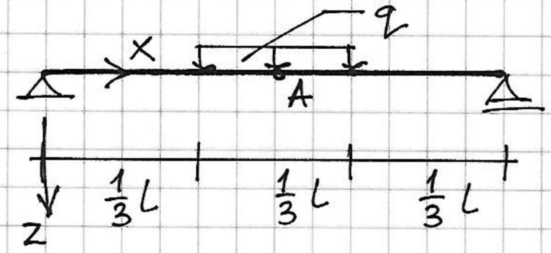
$$P_{kr} = \frac{E_1 J_2 \pi^2}{l^2} = 15{,}184\ [kN]$$

$$M_{kr} = \frac{\pi}{l}\sqrt{E_1 J_2 G J_s} = 571{,}87\ [kN\,cm]$$

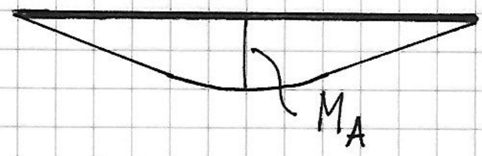


Zadanie 2

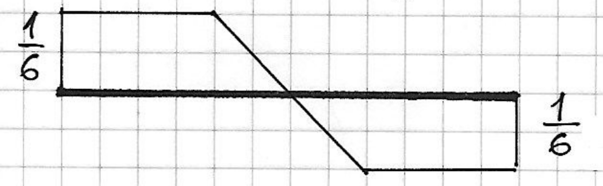
Belka zastępcza



$$\xi = \frac{x}{L}$$



M $[ql^2]$



T $[ql]$

$$z_A = \frac{M_A}{H} \quad , \quad H = Q\sqrt{\frac{\lambda_0}{2(1-\lambda_0)}} \quad , \quad \lambda_0 = \frac{L}{L_0}$$

$$Q^2 = \int_0^1 T^2(\xi)\,d\xi$$

$$\lambda_0 = \frac{L}{1{,}05L} = 0{,}952$$

$$Q^2 = \left[\frac{1}{6}qL \cdot \frac{1}{3} \cdot \frac{1}{6}qL + \frac{1}{2} \cdot \frac{1}{6}qL \cdot \frac{1}{6} \cdot \frac{2}{3} \cdot \frac{1}{6}qL\right] \cdot 2 = \frac{7}{324}q^2L^2$$

$$Q = 0{,}147\,qL$$

$$H = 0{,}465\,qL$$

$$M_A = \frac{1}{6}qL \cdot \frac{1}{2}L - \frac{1}{2}q\left(\frac{1}{2}L - \frac{1}{3}L\right)^2 = \frac{5}{72}qL^2 = 0{,}069\,qL^2$$

$$z_A = 0{,}148\,L$$

<table>
<tr><td colspan="3">LAST NAME, First Name</td></tr>
<tr><td colspan="2">Index Number</td><td>Project grade</td></tr>
<tr><td rowspan="2">Problem # 1
grade</td><td rowspan="2">Problem # 2
grade</td><td>Exam grade (Written part)</td></tr>
<tr><td>Overall grade</td></tr>
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Problem # 1.

Consider a bar of length l=1 m with thin, rectangular section (h=8 cm, b=1 cm). The bar is fork-supported at both ends. Calculate the critical values of loading in two following load cases.

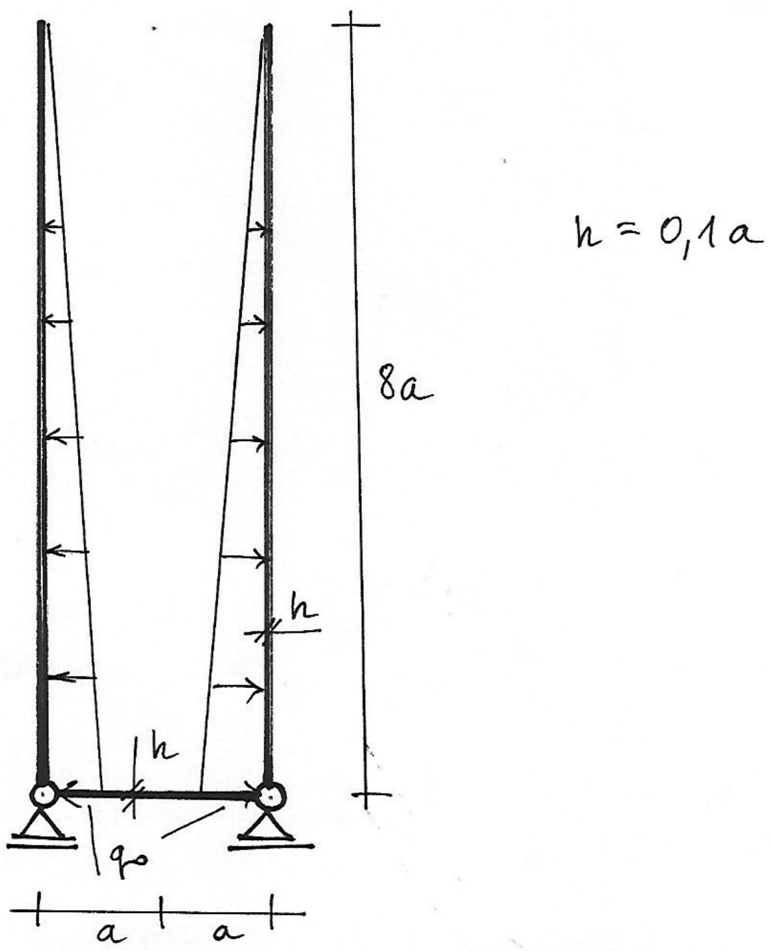
Case 1: The bar is subjected to a compressive force P applied at SC=S.

Case 2: The bar is subjected to a bending moment M applied at SC=S along axis y parallel to the shorter side of the rectangle.

In the calculations assume that E = 210GPa and ν = 0.3.

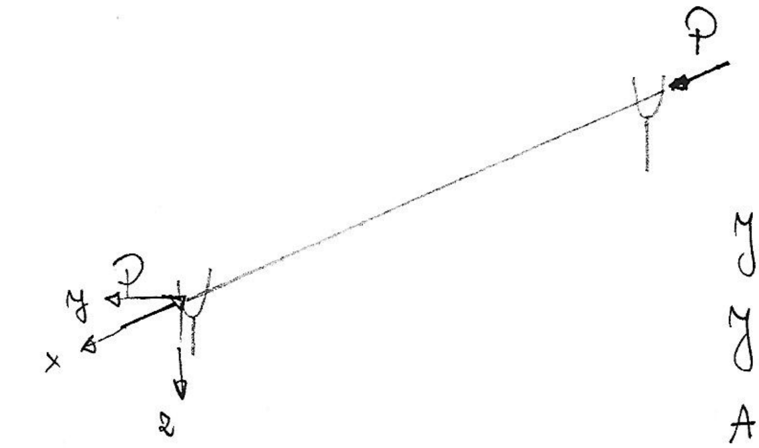
Problem # 2.

Calculate the diagram of the N_1 force in the cylindrical shell shown in the Figure below.
In the calculations assume that E = 210GPa and ν = 0.3.



ZADANIE 1 EGZAMIN KB ST II 23VI2017

A)



$$G = \frac{E}{2(1+\nu)} = 8077 \left[\frac{kN}{cm^2}\right]$$

$$E_1 = \frac{E}{1-\nu^2} = 23077 \left[\frac{kN}{cm^2}\right]$$

$$J_y = 42{,}667 \left[cm^4\right] \qquad J_s = 2{,}667\ cm^4$$

$$J_z = 0{,}667 \left[cm^4\right] \qquad J_\omega = 0$$

$$A = 8\ cm$$

$$r_0 = \sqrt{\frac{J_y + J_z}{A}} = 2{,}327 \left[cm\right] \qquad y_s = z_s = 0$$

$$\mathbb{A} = \begin{bmatrix} P-P_z & 0 & 0 \\ 0 & P-P_y & 0 \\ 0 & 0 & r_0^2(P-P_s) \end{bmatrix}$$

$$\det \mathbb{A} = 0 \qquad (P-P_z)(P-P_y)\, r_0^2\, (P-P_s) = 0$$

$$P_{kr} = \min\{P_y, P_z, P_s\}$$

$$P_y = \frac{\pi^2 E_1 J_y}{l^2} = 971{,}776 \left[kN\right]$$

$$P_z = \frac{\pi^2 E_1 J_z}{l^2} = 15{,}184 \left[kN\right]$$

$$P_s = \frac{G J_s}{r_0^2} = 3976{,}330 \left[kN\right]$$

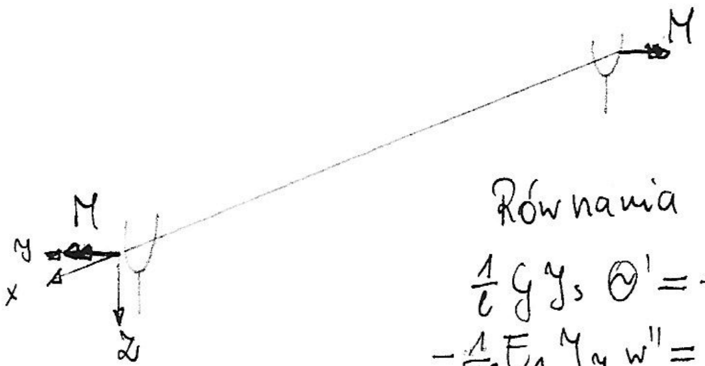
B)

$\xi = \frac{x}{l}$

$\hat{M}_x = 0$

$\hat{M}_y = M$

$\hat{M}_z = 0$



Równania

$\frac{1}{l} G J_s \Theta' = \frac{1}{l} M v'$ (1)

$-\frac{1}{l^2} E_1 J_y w'' = M$ (2) $\rightarrow$ (1) + (3)

$\frac{1}{l^2} E_1 J_z v'' = -\Theta M$ (3)

$$\Theta'' + \alpha^2 \Theta = 0 \qquad \alpha = \sqrt{\frac{M^2 l^2}{E_1 J_z \, G J_s}}$$

$$\Theta(\xi) = C_1 \cos(\alpha \xi) + C_2 \sin(\alpha \xi)$$

Warunki brzegowe

$\Theta(0) = 0$ $\qquad$ $\Theta(1) = 0$

$B(0) = 0$ $\qquad$ $B(1) = 0$

$C_2 = 0$

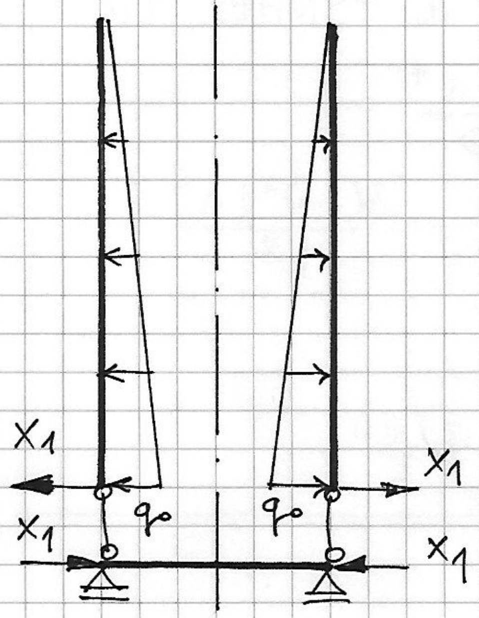
$C_1 \sin \alpha = 0 \Rightarrow \alpha = k\pi$

Dla $k = 1$

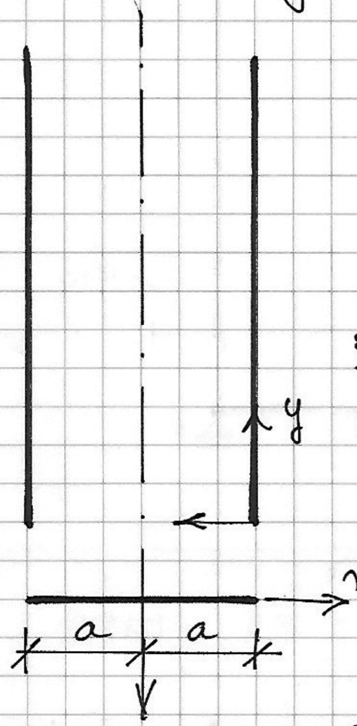
$$M_{kr} = \frac{\pi}{l} \sqrt{E_1 J_z \, G J_s} = 571{,}87 \, [\mathrm{kNcm}]$$

Problem #2

Primary structure



Coordinate systems



$\xi = \frac{y}{a}$

$0 \leq \xi \leq 8$

$\rho = \frac{r}{a}$

$0 \leq \rho \leq 1$

We know that

$$N_1 = -\frac{Eh}{a} w \quad \left[\frac{N}{m}\right]$$

where

w – deflection of the cylinder, $w = w(\xi)$

E – Young's modulus

$h = \frac{1}{10} a$

Hence, the goal is to calculate w from the superposition principle

$$w = \overset{0}{w} + \overset{1}{X} \cdot \overset{1}{w}$$

where

$\overset{0}{w}$ – deflection in the non-bending state

$\overset{1}{w}$ – deflection in the $\overset{1}{X} = 1$ state

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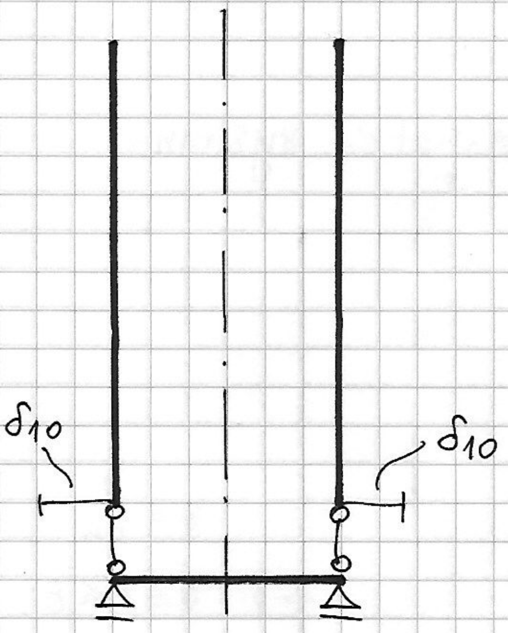
The non-bending state

$$q(\xi) = -\frac{1}{8} q_0 (8 - \xi) \qquad 0 \leq \xi \leq 8$$



$$\delta_{10} = -\overset{0}{w}(0)$$

where $\overset{0}{w} = \overset{0}{w}(\xi)$ is a PSNHE

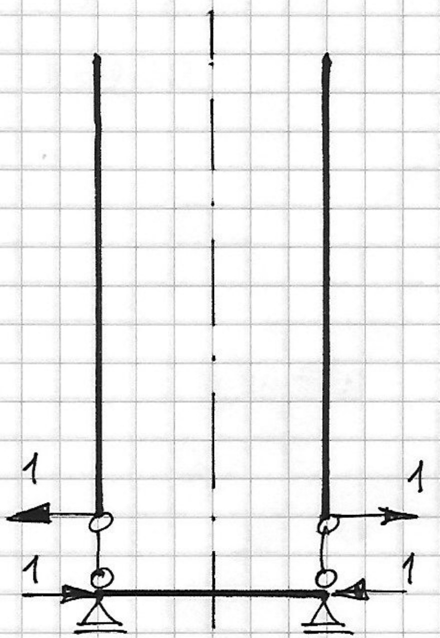
$$w^{IV} + 4\lambda^4 w = \frac{q\, a^4}{D}$$

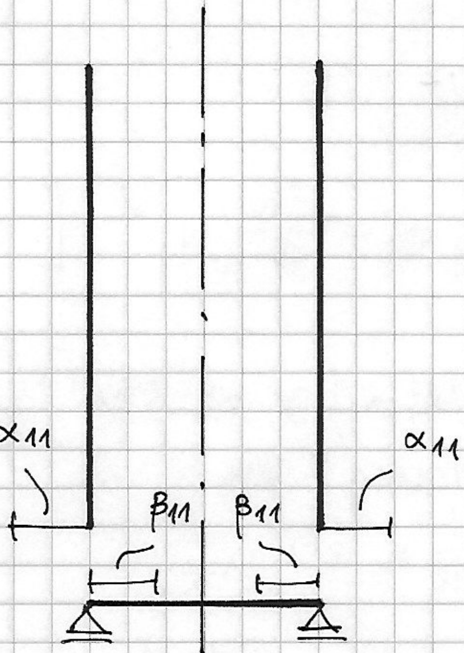
$$D = \frac{E h^3}{12(1-\nu^2)} \; ; \; \lambda^4 = \frac{3(1-\nu^2)\, a^2}{h^2}$$

hence

$$\overset{0}{w}(\xi) = \frac{a^4}{4\lambda^4 D}\, q(\xi)$$

The $\overset{1}{X} = 1$ state





$$\delta_{11} = \alpha_{11} + \beta_{11}$$

$\alpha_{11} = -\overset{1}{w}(0)$ where $\overset{1}{w} = \overset{1}{w}(\xi)$ is a GSHE $w^{IV} + 4\lambda^4 w = 0$.

It takes the form $\overset{1}{w}(\xi) = e^{-\lambda\xi}\left[A_1 \cos(\lambda\xi) + A_2 \sin(\lambda\xi)\right]$

Constants A_1, A_2 are calculated from boundary conditions

$Q_2(0) = 1$ where $Q_2 = Q_2(\xi) = -\dfrac{D}{a^3}\dfrac{d^3}{d\xi^3}\overset{1}{w}$

$M_2(0) = 0$ where $M_2 = M_2(\xi)$
$$= -\frac{D}{a^2}\frac{d^2}{d\xi^2}\overset{1}{w}$$

$\beta_{11} = -\overset{1}{u}(1)$ where $\overset{1}{u} = \overset{1}{u}(\xi)$ is a GSHE $\dfrac{d}{d\xi}\left(\dfrac{1}{\xi}\dfrac{d}{d\xi}(\xi\overset{1}{u})\right) = 0$.

It takes the form $\overset{1}{u}(\xi) = \frac{B_1}{\xi} + B_2 \xi$
with $B_1 = 0$.
Constant B_2 is calculated from boundary condition
$N_2(1) = -1$ where $N_2 = N_2(\xi) = \frac{c}{a}\left(\frac{d}{d\xi}\overset{1}{u} + \frac{\nu}{\xi}\overset{1}{u}\right)$

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the equation

$$\delta_{11} \overset{1}{X} + \delta_{10} = 0$$

gives

$$\overset{1}{X} = -\frac{\delta_{10}}{\delta_{11}}$$