Egzamin pisemny z Mechaniki Konstrukcji (KB) st. mgr stacj., 29 VI 2016 r.

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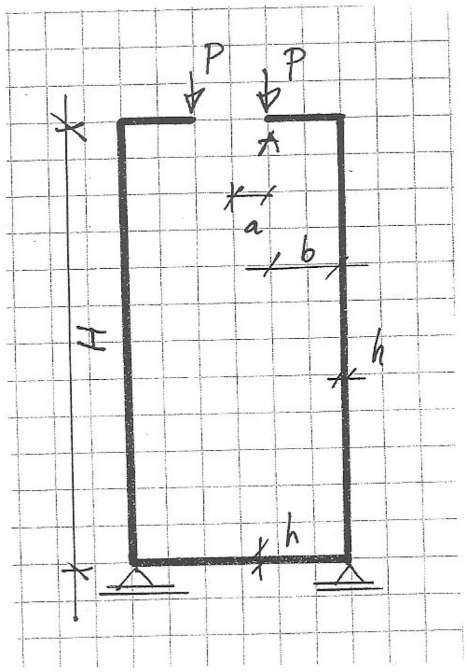
Zadanie 1

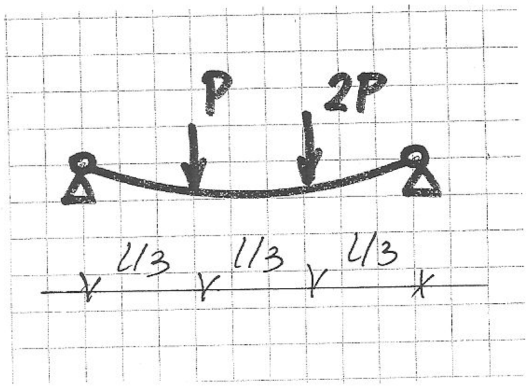
Dany zbiornik walcowy wzmocniony płytą pierścieniową na brzegu górnym, współpracujący z dnem (płyta kołowa tej samej grubości) jest poddany obciążeniu jak na rysunku. Obliczyć przemieszczenia wewnętrznego obwodu płyty górnej(pkt.A).
Dane:
E= 30 GPa;
współczynnik Poissona=0.2;
h=a/15; b=a; H=7a.



The given cylindrical container with a bottom (the circular plate of the same thickness) stiffened by the annular plate along the upper edge is subject to the load shown in the figure. Compute the displacements of the contour A of the upper plate.
Data: E= 30 GPa;
Poisson's ratio=0.2;
h=a/15; b=a; H=7a.

Zadanie 2

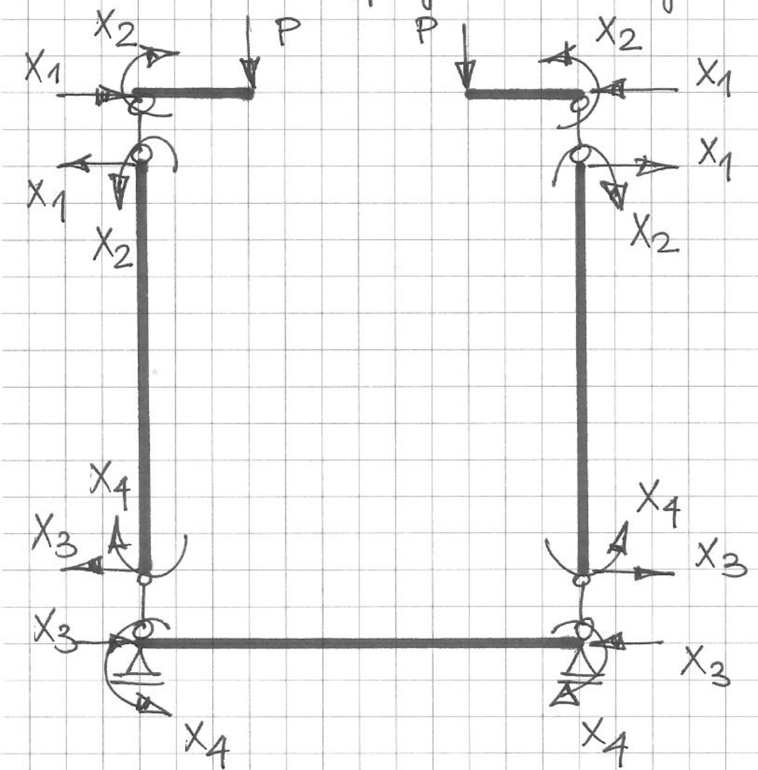
Dane jest cięgno obciążone jak na rysunku. Obliczyć rzędne krzywej zwisu w punktach przyłożenia sił skupionych. Cięgno jest nierozciągliwe o długości $L_0 = 1.05l$.



The given cable is loaded as in the figure. Compute the values of the sag function at points of application of the point loads. The cable is inextensible of the length $L_0 = 1.05l$

Zadanie 1 / Problem #1

Schemat zastępczy / Primary structure



$$C = \frac{Eh}{1-\nu^2} = 2{,}083 \cdot 10^6 a \; \left[\frac{kN}{m}\right]$$

$$D = \frac{Eh^3}{12(1-\nu^2)} = 771{,}605\, a^3 \; [kNm]$$

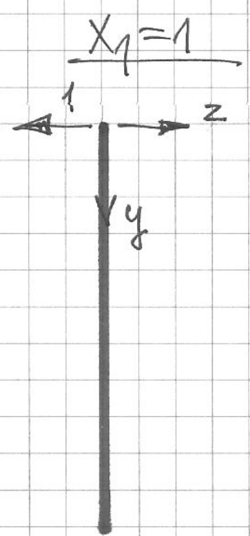
$$\lambda^4 = 3(1-\nu^2)\left(\frac{a+b}{h}\right)^2$$

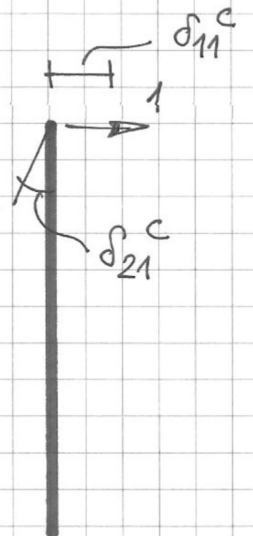
$$\lambda = 7{,}135$$

Uwaga / Remark:

Obciążenie zewnętrzne nie działa na płaszcz i płytę dolną.
W związku z tym siły $X_3 = X_4 = 0$.
The cylinder and bottom plate are free from the external load. Hence $X_3 = X_4 = 0$.

Ⓐ WALEC / CYLINDER



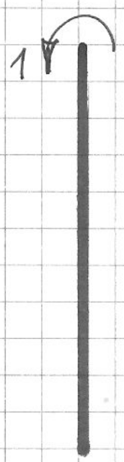


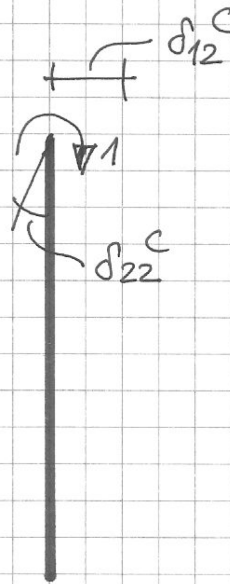
$$\xi = \frac{y}{a+b}$$

$$\delta_{11}^c = -w(0) = \frac{1}{2}\frac{(a+b)^3}{\lambda^3 D} = 1{,}427 \cdot 10^{-5} \; \left[\frac{m^2}{kN}\right]$$

$$\delta_{21}^c = \chi_2(0) = \frac{1}{2}\frac{(a+b)^2}{\lambda^2 D} = \frac{1}{a} \cdot 5{,}091 \cdot 10^{-5} \; \left[\frac{m}{kN}\right]$$

$\underline{X_2 = 1}$



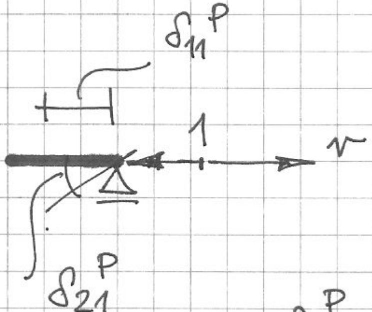


$$\delta_{12}^{C} = \delta_{21}^{C}$$

$$\delta_{22}^{C} = \frac{a+b}{\lambda D} = \frac{1}{a^2} \cdot 3{,}632 \cdot 10^{-4} \left[\frac{1}{kN}\right]$$
$$= \chi_2(0)$$

Ⓑ PŁYTA / PLATE

$\underline{X_1 = 1}$



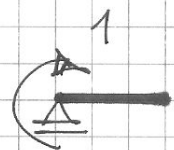
$$\rho = \frac{r}{a+b}$$

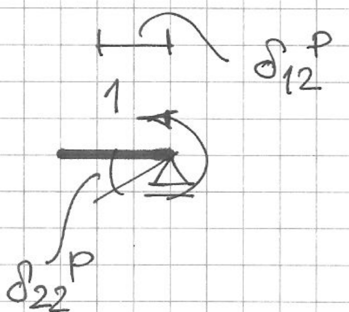
$$\alpha = \frac{a}{a+b}$$

$$\delta_{11}^{P} = -u(1) = \frac{(a+b)\left[\alpha^2(1+\nu)+(1-\nu)\right]}{C(1-\alpha^2)(1-\nu^2)}$$
$$= 1{,}467 \cdot 10^{-6} \left[\frac{m^2}{kN}\right]$$

$$\delta_{21}^{P} = 0$$

$\underline{X_2 = 1}$

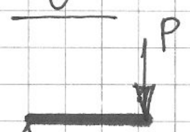


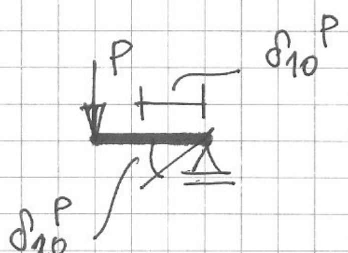


$$\delta_{12}^{P} = \delta_{21}^{P}$$

$$\delta_{22}^{P} = \chi_2(1) = \frac{(a+b)\left[\alpha^2(1+\nu)+(1-\nu)\right]}{D(1-\alpha^2)(1-\nu^2)}$$
$$= \frac{1}{a^2} \cdot 3{,}96 \cdot 10^{-3} \left[\frac{1}{kN}\right]$$

$\underline{\text{"0"}}$





$$\delta_{10}^{P} = 0$$

$$\delta_{20}^{P} = \chi_2(1) = \frac{1}{2}\alpha P (a+b)^2 \, \frac{(1-\nu)(1-\alpha^2) - 2(1+\nu)\alpha^2 \ln\alpha}{D(1-\alpha^2)(1-\nu^2)}$$
$$= \frac{P}{a} \cdot 1{,}829 \cdot 10^{-3} \left[-\right]$$

(C) OBLICZENIE X_1, X_2 / CALCULATION OF X_1, X_2

$$\delta_{11} = \delta_{11}^{c} + \delta_{11}^{P} = 1{,}573 \cdot 10^{-5} \left[\frac{m^2}{kN}\right]$$

$$\delta_{12} = \delta_{12}^{c} + \delta_{12}^{P} = \frac{1}{a} 5{,}091 \cdot 10^{-5} \left[\frac{m}{kN}\right]$$

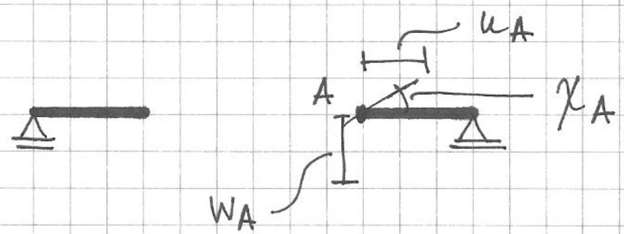
$$\delta_{22} = \delta_{22}^{c} + \delta_{22}^{P} = \frac{1}{a^2} 4{,}323 \cdot 10^{-3} \left[\frac{1}{kN}\right]$$

$$\delta_{10} = \delta_{10}^{c} + \delta_{10}^{P} = 0$$

$$\delta_{20} = \delta_{20}^{c} + \delta_{20}^{P} = \frac{P}{a} \cdot 1{,}829 \cdot 10^{-3} \; [-]$$

$$\left.\begin{aligned} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{10} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{20} = 0 \end{aligned}\right| \rightarrow \begin{aligned} X_1 &= 1{,}422\, P \left[\frac{kN}{m}\right] \\ X_2 &= -0{,}440\, Pa \left[\frac{kNm}{m}\right] \end{aligned}$$

(D) OBLICZENIE PRZEMIESZCZEŃ / CALCULATION OF DISPLACEMENTS



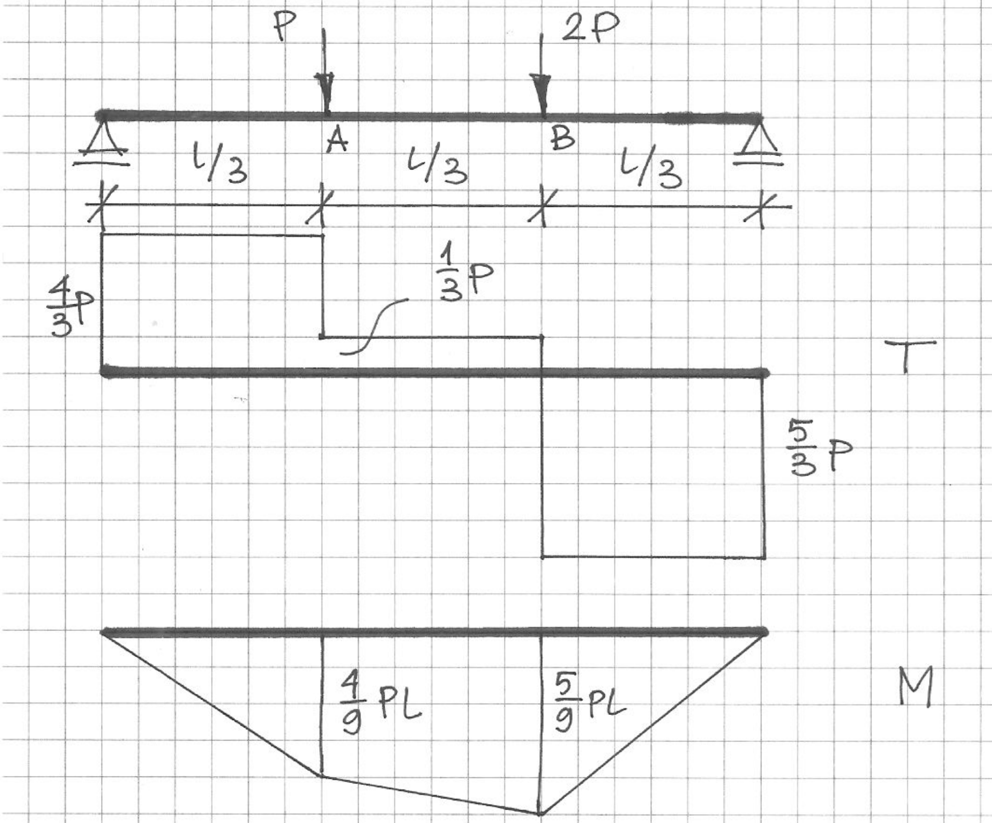
$$u_A = X_1 \cdot u(\alpha)\Big|_{X_1=1} = -1{,}897 \cdot 10^{-6} \cdot P \; [m]$$

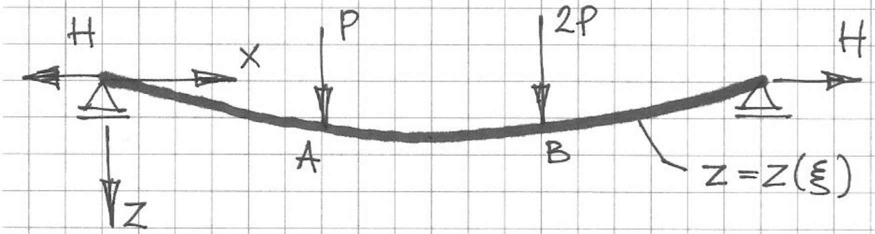
$$w_A = X_2 \cdot w(\alpha)\Big|_{X_2=1} + w(\alpha)\Big|_{P} = 3{,}105 \cdot 10^{-4} \cdot P \; [m]$$

$$\chi_A = X_2 \cdot \chi_2(\alpha)\Big|_{X_2=1} + \chi_2(\alpha)\Big|_{P} = 4{,}542 \cdot 10^{-4} \cdot \frac{P}{a} \; [-]$$

Zadanie 2 / Problem #2

Belka zastępcza / Equivalent beam





$$\xi = \frac{x}{L}$$

$$z(\xi) = \frac{M(\xi)}{H} \longrightarrow z_A = \frac{M_A}{H}, \quad z_B = \frac{M_B}{H}$$

$$M_A = \frac{4}{9}PL \qquad M_B = \frac{5}{9}PL$$

$$H = Q\sqrt{\frac{\lambda_0}{2(1-\lambda_0)}}, \quad \lambda_0 = \frac{L}{L_0} = 0{,}952$$

$$Q^2 = \int_0^1 T^2 d\xi = \frac{4}{3}P \cdot \frac{1}{3} \cdot \frac{4}{3}P + \frac{1}{3}P \cdot \frac{1}{3} \cdot \frac{1}{3}P + \frac{5}{3}P \cdot \frac{1}{3} \cdot \frac{5}{3}P$$

$$= \frac{2}{3}P^2 \longrightarrow Q = 0{,}816\,P$$

$$H = 2{,}57\,P \qquad z_A = 0{,}173\,L, \quad z_B = 0{,}216\,L$$