Egzamin pisemny z Mechaniki Konstrukcji (KB) st. mgr stacj., 20 VI 2016 r.

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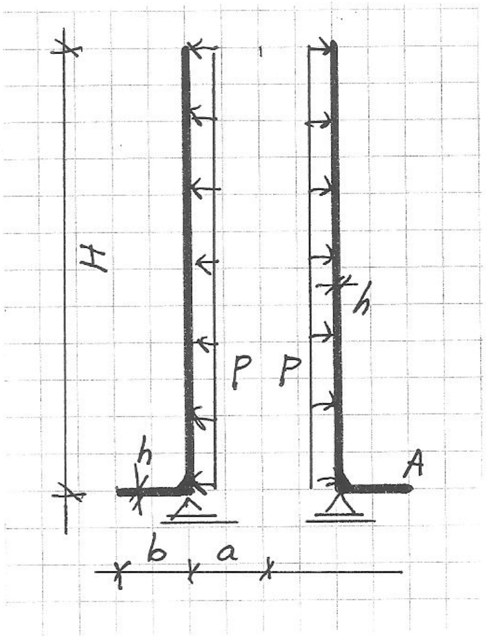
Zadanie 1

Dany zbiornik walcowy wzmocniony płytą pierścieniową jest poddany ciśnieniu wewnętrznemu o intensywności p. Obliczyć przemieszczenia zewnętrznego obwodu płyty dennej (pkt.A).
Dane:
E= 30 GPa;
współczynnik Poissona=0.2;
h=a/20; b=a; H=6a.

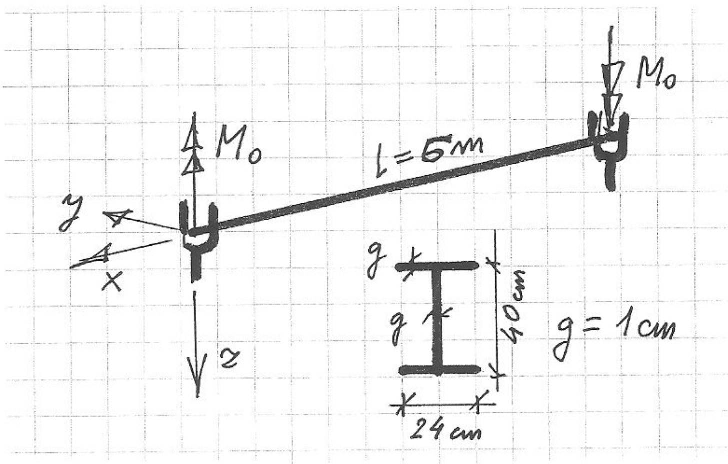
The given cylindrical container stiffened by the annular plate is subject to an internal pressure of intensity p. Compute the displacements of the contour A of the plate.
Data: E= 30 GPa;
Poisson's ratio=0.2;
h=a/20; b=a; H=6a.



Zadanie 2
Dany jest pręt cienkościenny o przekroju dwuteowym o długości l=6m, podparty widełkowo, obciążony momentami M_0 na końcach.
Obliczyć moment krytyczny.

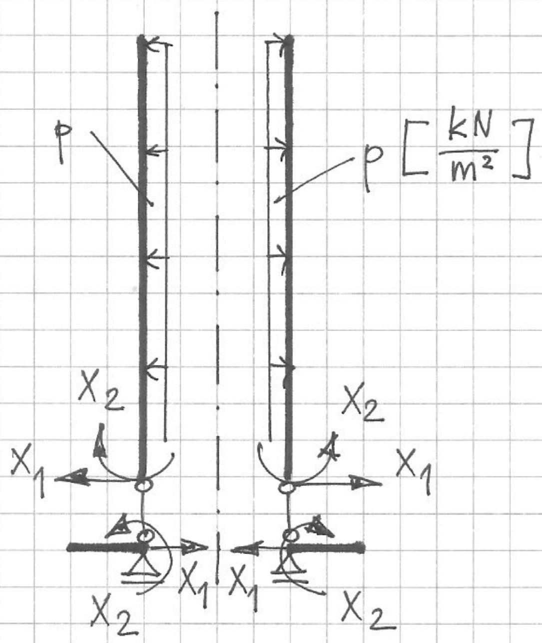
The given thin-walled bar of I section and length l=6m is fork supported and subject to moments M_0 at its ends.
Compute the critical moment.

Dane(*data*):
E=205GPa, ν =0.3,
J_y=24533cm^4, J_z=2304cm^4 ,
J_s=29.33cm^4
$J_\omega = 921600 cm^6$



Zadanie 1 / Problem #1

Schemat zastępczy / Primary structure



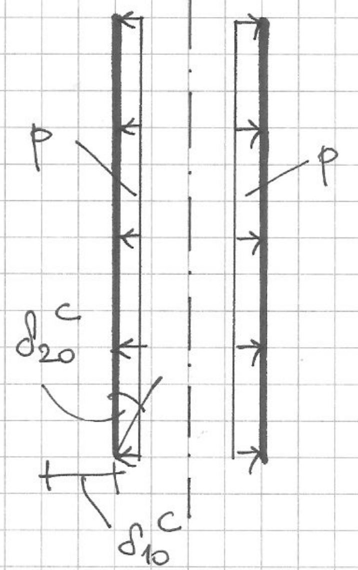
$$C = \frac{Eh}{1-\nu^2} = 1{,}56 \cdot 10^6 \cdot a \left[\frac{kN}{m}\right]$$

$$D = \frac{h^2}{12} \cdot C = 325{,}52 \cdot a^3 \, [kNm]$$

$$\lambda^4 = 3(1-\nu^2)\left(\frac{a}{h}\right)^2 = 1152 \rightarrow \lambda = 5{,}83$$

Ⓐ WALEC / CYLINDER

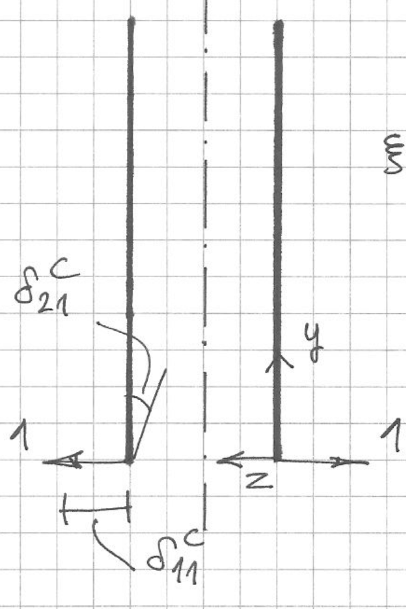
- Stan bezzgięciowy (Stan "0") / Non-bending case ("0"th)



$$\delta_{10}^c = \frac{pa^2}{Eh} = 6{,}67 \cdot 10^{-7} \, pa \; [m]$$

$$\delta_{20}^c = 0$$

- Stan $X_1 = 1$ / Case $X_1 = 1$



$$\xi = \frac{y}{a}$$

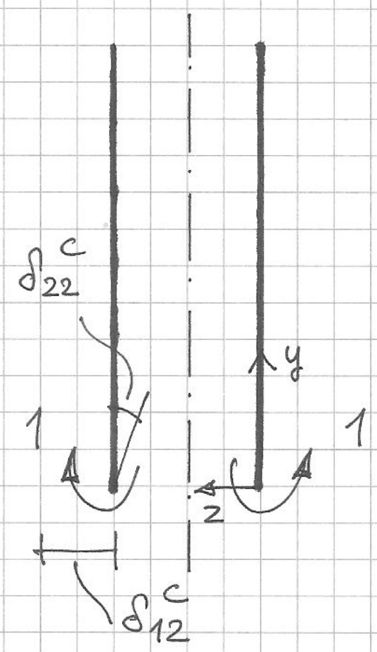
$$w(\xi) = e^{-\lambda\xi}\left[A_1 \cos(\lambda\xi) + A_2 \sin(\lambda\xi)\right]$$

warunki brzegowe / boundary conditions:

$$\left.\begin{array}{l} M_2(0) = 0 \\ Q_2(0) = 1 \end{array}\right| \rightarrow \begin{array}{l} A_1 = -0{,}777 \cdot 10^{-5} \\ A_2 = 0 \end{array}$$

$$\delta_{11}^{c} = -w(0) = 7{,}77 \cdot 10^{-6} \left[\frac{m^2}{kN}\right]$$
$$\delta_{21}^{c} = \chi_2(0) = \frac{1}{a} \cdot 4{,}52 \cdot 10^{-5} \left[\frac{m}{kN}\right]$$

- Stan $X_2 = 1$ / Case $X_2 = 1$



ξ, $w = w(\xi)$ jak wyżej / as above

warunki brzegowe / boundary conditions

$$\left.\begin{array}{l} M_2(0) = 1 \\ Q_2(0) = 0 \end{array}\right| \longrightarrow \begin{array}{l} A_1 = -\frac{1}{a} \cdot 4{,}52 \cdot 10^{-5} \\ A_2 = \frac{1}{a} \cdot 4{,}52 \cdot 10^{-5} \end{array}$$

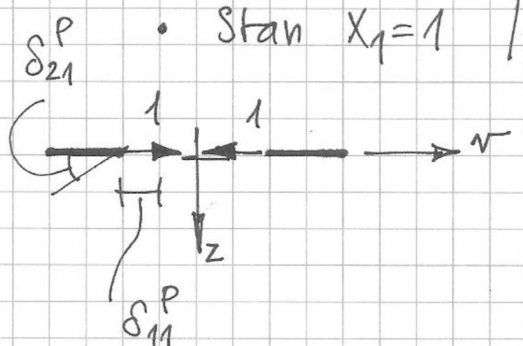
$$\delta_{12}^{c} = -w(0) = \frac{1}{a} \cdot 4{,}52 \cdot 10^{-5} \left[\frac{m}{kN}\right] = \delta_{21}^{c}$$
$$\delta_{22}^{c} = \chi_2(0) = \frac{1}{a^2} \cdot 5{,}27 \cdot 10^{-4} \left[\frac{1}{kN}\right]$$

(B) PŁYTA / PLATE

- Stan "0" / 0-th case

$$\delta_{10}^{P} = \delta_{20}^{P} = 0$$

- Stan $X_1 = 1$ / $X_1 = 1$ case



$$\rho = \frac{r}{a+b} = \frac{r}{2a} \qquad \alpha = \frac{a}{a+b} = \frac{1}{2}$$

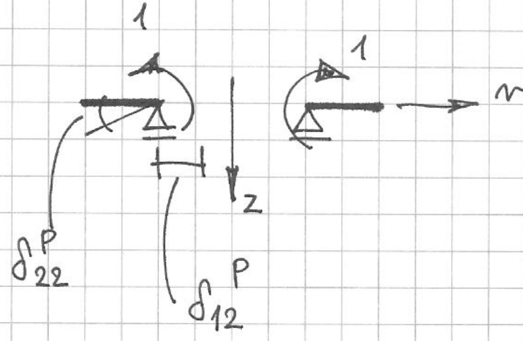
$$u(\rho) = A_1 \rho + \frac{A_2}{\rho}$$

warunki brzegowe / boundary conditions

$$\left.\begin{array}{l} N_2(\alpha) = 1 \\ N_2(1) = 0 \end{array}\right| \longrightarrow \begin{array}{l} A_1 = -3{,}55 \cdot 10^{-7} \\ A_2 = -5{,}33 \cdot 10^{-7} \end{array}$$

$$\delta_{11}^{P} = -u(\alpha) = 1{,}24 \cdot 10^{-6} \left[\frac{m^2}{kN}\right]$$
$$\delta_{21}^{P} = 0 \left[\frac{m}{kN}\right]$$

• Stan $X_2 = 1$ | $X_2 = 1$ case



$$\varrho = \frac{r}{2a}$$

$$w(\varrho) = A_1 + A_2\varrho^2 + A_3 \ln\varrho + A_4 \varrho^2 \ln\varrho$$

warunki brzegowe | boundary conditions

$w(\alpha) = 0$ $\quad M_2(1) = 0$

$M_2(\alpha) = 1$ $\quad Q_2(1) = 0$

$\delta_{12}^P = 0 \quad \left[\frac{m}{kN}\right]$

$\delta_{22}^P = 2{,}99 \cdot 10^{-3} \cdot \frac{1}{a^2} \left[\frac{1}{kN}\right]$

$A_1 = \frac{1}{a} \cdot 7{,}8 \cdot 10^{-4}$

$A_2 = \frac{1}{a} \cdot 4{,}27 \cdot 10^{-4}$

$A_3 = \frac{1}{a} \cdot 1{,}28 \cdot 10^{-3}$

$A_4 = 0$

(C) OBLICZENIE X_1, X_2 | CALCULATION OF X_1, X_2

$\delta_{ij} = \delta_{ij}^c + \delta_{ij}^P \qquad \delta_{i0} = \delta_{i0}^c + \delta_{i0}^P \qquad i,j = 1,2$

$\delta_{11} = 9{,}01 \cdot 10^{-6}$ $\qquad \delta_{10} = 6{,}67 \cdot 10^{-7}\, pa$

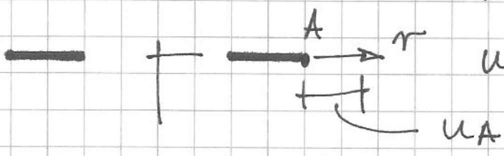
$\delta_{12} = \delta_{21} = 4{,}52 \cdot 10^{-5} \cdot \frac{1}{a}$ $\qquad \delta_{20} = 0$

$\delta_{22} = 3{,}51 \cdot 10^{-3} \cdot \frac{1}{a^2}$

$$\left.\begin{aligned} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{10} = 0 \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{20} = 0 \end{aligned}\right| \rightarrow \begin{aligned} X_1 &= -7{,}91 \cdot 10^{-2}\, pa \left[\frac{kN}{m}\right] \\ X_2 &= 1{,}02 \cdot 10^{-3}\, pa^2 \left[\frac{kNm}{m}\right] \end{aligned}$$

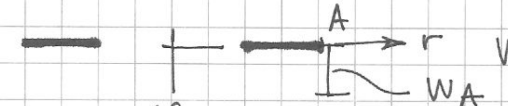
(D) OBLICZENIE PRZEMIESZCZEŃ | CALCULATION OF DISPLACEMENTS

• Horizontal



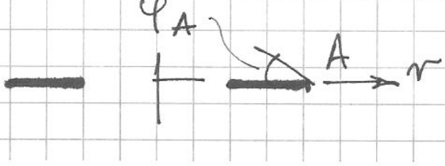
$u_A = X_1 \cdot u(1) = 7{,}03 \cdot 10^{-8}\, pa$ [m]

• Vertical



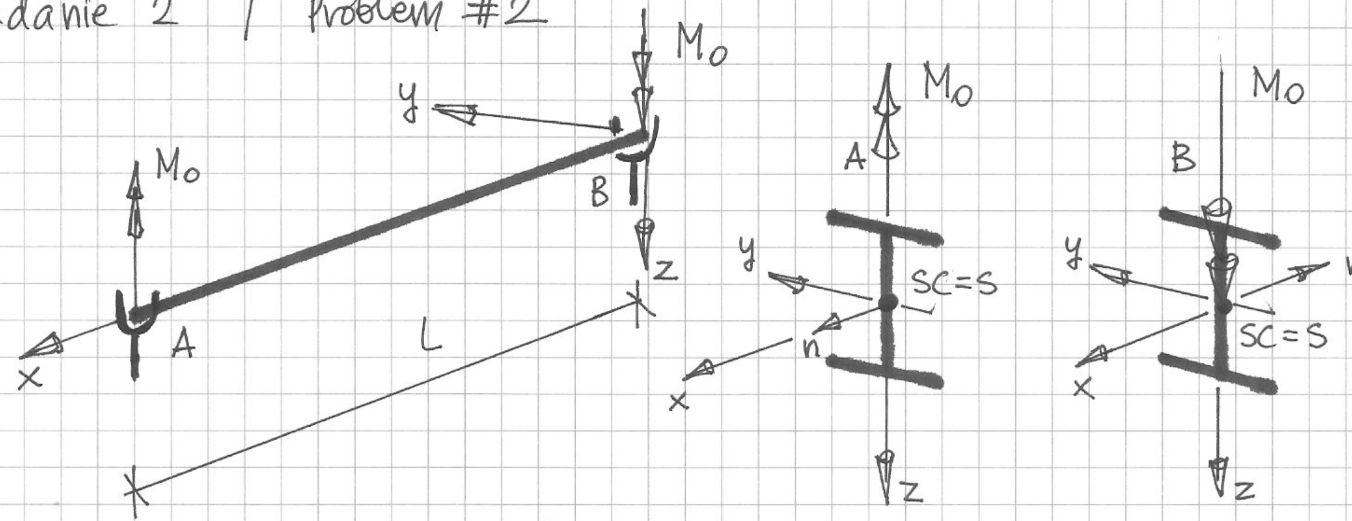
$w_A = X_2 \cdot w(1) = 1{,}23 \cdot 10^{-6}\, pa$ [m]

• Rotation



$\varphi_A = X_2 \cdot \varphi(1) = 2{,}17 \cdot 10^{-6}\, P$ [rad]

Zadanie 2 / Problem #2



M_0 działa w SC=S / M_0 is applied at SC=S

$$\hat{M}_x = 0 \qquad \hat{M}_y = 0 \qquad \hat{M}_z = -M_0$$

Równania / equations:

1) $$GJ_s \frac{d\theta}{dx} - E_1 J_\omega \frac{d^3\theta}{dx^3} = -M_0 \frac{dw}{dx}$$

2) $$-E_1 J_y \frac{d^2 w}{dx^2} = -M_0 \cdot \theta$$

3) $$E_1 J_z \frac{d^2 v}{dx^2} = -M_0$$

1)+2) prowadzi do / 1)+2) gives:

$$GJ_s \frac{d^2\theta}{dx^2} - E_1 J_\omega \frac{d^4\theta}{dx^4} = -M_0 \cdot \frac{M_0 \cdot \theta}{E_1 J_y}$$

Niech / Let:

$$\xi = \frac{x}{L} \qquad \frac{d}{dx} = \frac{1}{L}\frac{d}{d\xi} \qquad f' \equiv \frac{df}{d\xi}$$

Stąd / Hence:

$$\theta^{IV} - 2\alpha_1 \theta'' - \alpha_2 \theta = 0$$

gdzie / where:

$$2\alpha_1 = \frac{GJ_s L^2}{E_1 J_\omega} \qquad \alpha_2 = \frac{M_0^2 L^4}{(E_1 J_y)(E_1 J_\omega)}$$

Jeżeli / If:

$$\beta_1 = \sqrt{-\alpha_1 + \sqrt{\alpha_1^2 + \alpha_2}}$$

$$\beta_2 = \sqrt{\alpha_1 + \sqrt{\alpha_1^2 + \alpha_2}}$$

to / then:

$$\theta(\xi) = A_1 \cos(\beta_1 \xi) + A_2 \sin(\beta_1 \xi) + A_3 \operatorname{ch}(\beta_2 \xi) + A_4 \operatorname{sh}(\beta_2 \xi)$$

Warunki brzegowe / boundary conditions:

$$\begin{array}{l} \theta(0)=0 \\ B(0)=0 \\ \theta(1)=0 \\ B(1)=0 \end{array} \longrightarrow \begin{array}{l} \theta(0)=0 \\ \theta''(0)=0 \\ \theta(1)=0 \\ \theta''(1)=0 \end{array} \longrightarrow \begin{array}{l} \mathbb{C} A = 0 \\ \det \mathbb{C} = 0 \rightarrow M_{kr} \end{array}$$

$$\det \mathbb{C} = -4(\alpha_1^2 + \alpha_2)\sin\beta_1 \operatorname{sh}\beta_2$$

$$\sin\beta_1 = 0 \iff \sqrt{-\alpha_1 + \sqrt{\alpha_1^2 + \alpha_2}} = k\pi$$

Niech $k=1$ / Let $k=1$

Wtedy / then:

$$M_{kr} = \frac{\pi}{L}\sqrt{(GJs)(E_1 J_y)\left[1 + \left(\frac{\pi^2}{L^2}\right)\frac{E_1 J\omega}{GJs}\right]}$$

$$= 1101{,}27 \text{ kNm}$$