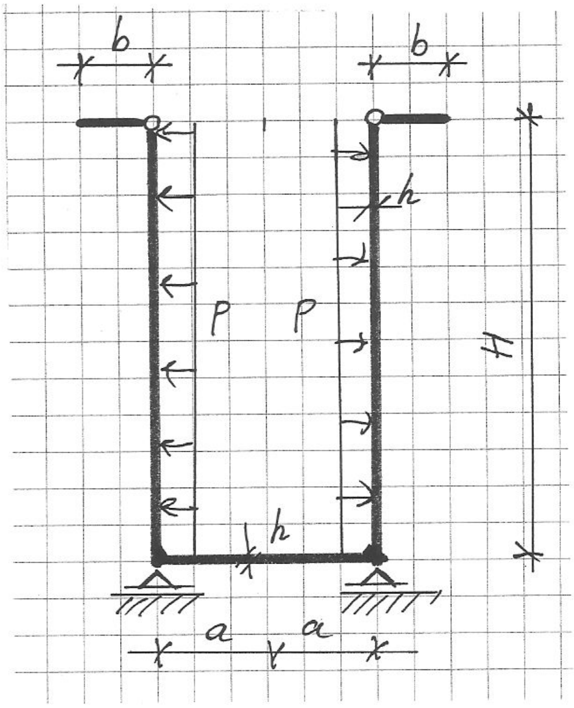
EGZAMIN MK (MK3) KB studia mgr stacj. dnia 3 II 2016 r.

NAZWISKO Imię			
Nr albumu			Ocena z ćwiczeń projektowych
ocena zadania 1	ocena zadania 2	Ocena z egzaminu po ustnym	
		Ocena łączna, data, podpis	

Zadanie 1.

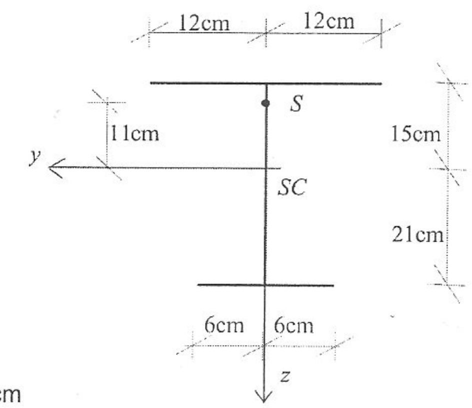
Dany jest zbiornik walcowy.
Znaleźć wykresy
momentów zginających M_1 , M_2
w powłoce i płycie dennej.
Obliczenia podatności
wykonać przy założeniu ,
że zbiornik jest długi.



Zadanie 2.

Znaleźć siłę krytyczną P
(przyłożoną w środku ciężkości SC przekroju)
wyboczenia giętno-skrętnego pręta cienkościennego
o danym przekroju, podpartego widełkowo,
o długości $l = 3.0$ m. Dane dotyczące przekroju:

$A=72cm^2$
$Jy=14904cm^4$
$Jz=1296cm^4$
$J\omega=165888cm^6$
$Js=24cm^4$



grubość g=1cm

Pozostałe charakterystyki wyznaczyć samodzielnie.

Exam on the Mechanics of Structures
3.02.2016

PROBLEM #1
Consider a cylindrical shell with top annular and bottom circular plates and loaded by a uniform loading p applied to a cylinder wall.
Assume that the shell is long.

Calculate the bending moments M_1, M_2 in a cylinder.

See front page for shell dimensions.

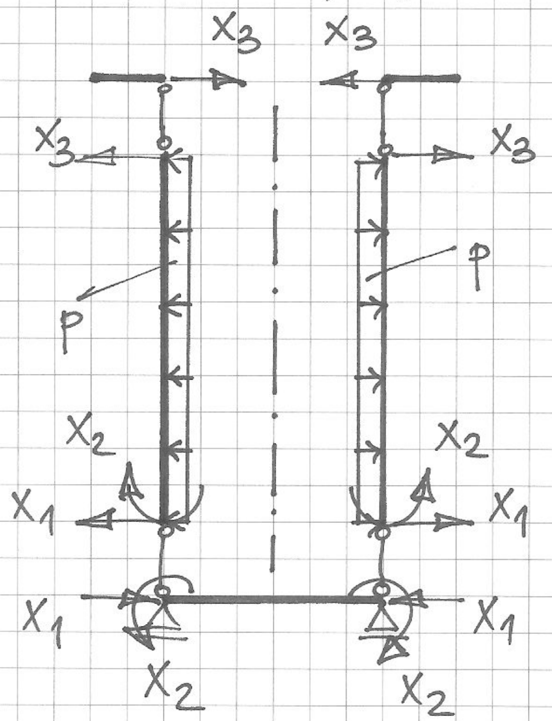
PROBLEM #2
Consider a thin-walled beam of length $l = 3.0\,m$, fork-supported and loaded by a force P applied at cross-section's centroid SC.

Calculate the value of the force P critical for flexural-torsional buckling.

See front page for cross-section dimensions and geometrical characteristics $A, J_y, J_z, J_\omega, J_s$.
Assume the thickness of a section $g = 1cm$.
Calculate the remaining characteristics, if necessary.

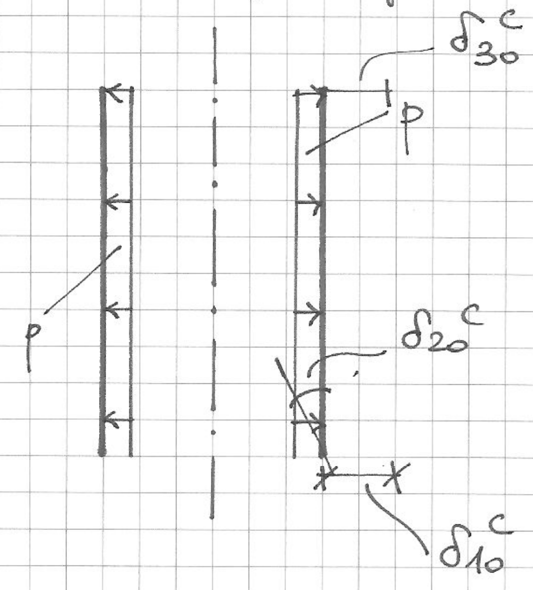
Zadanie 1 Problem #1

Schemat zastępczy / Primary structure



(A) WALEC / CYLINDER

- Stan bezzgięciowy (stan "0") / Non-bending state ("0"-th state)



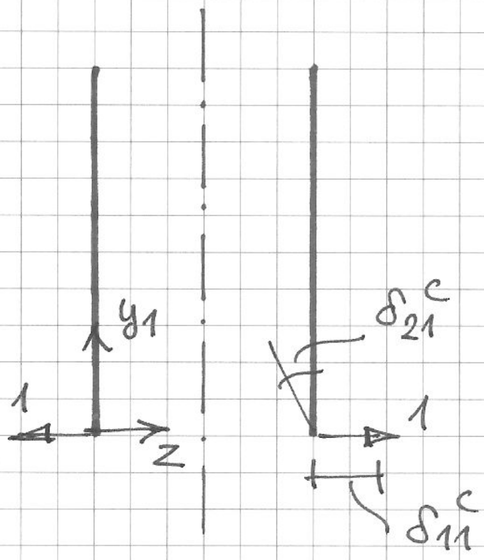
$$\delta_{10}^c = \delta_{20}^c = \frac{pa^2}{Eh}$$

$$\delta_{20}^c = 0$$

$$M_{10}^c = 0$$

$$M_{20}^c = 0$$

- Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$



$$\xi_1 = \frac{y_1}{a}$$

$$\lambda^4 = 3(1-\nu^2)\left(\frac{a}{h}\right)^2$$

$$C = \frac{Eh}{1-\nu^2} \qquad D = \frac{Eh^3}{12(1-\nu^2)}$$

$$w(\xi_1) = e^{-\lambda\xi_1}\left[A_1 \cos(\lambda\xi_1) + A_2 \sin(\lambda\xi_1)\right]$$

warunki brzegowe / boundary conditions

$$\left.\begin{array}{l} M_2(0) = 0 \\ Q_2(0) = 1 \end{array}\right| \rightarrow A_1, A_2$$

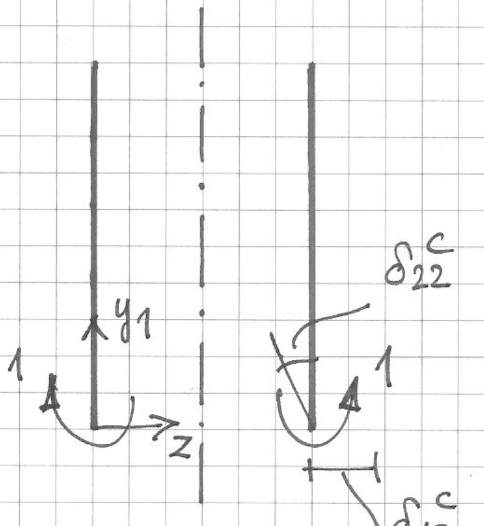
$$\delta_{11}^c = -w(0) = \frac{1}{2}\frac{a^3}{\lambda^3 D}$$

$$\delta_{21}^c = \chi_2(0) = \frac{1}{2}\frac{a^2}{\lambda^2 D}$$

$$M_{11}^c(\xi_1) = \nu M_{21}^c(\xi_1)$$

$$M_{21}^c(\xi_1) = \frac{a}{\lambda} e^{-\lambda\xi_1} \sin(\lambda\xi_1)$$

- Zaburzenie $X_2 = 1$ / Perturbation $X_2 = 1$



$w(\xi_1) =$ jak wyżej / as above

warunki brzegowe / boundary conditions

$$\left.\begin{array}{l} M_2(0) = 1 \\ Q_2(0) = 0 \end{array}\right| \rightarrow A_1, A_2$$

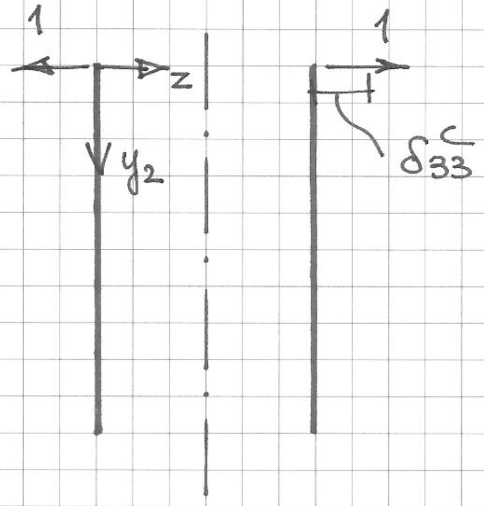
$$\delta_{12}^c = -w(0) = \frac{1}{2}\frac{a^2}{\lambda^2 D}$$

$$\delta_{22}^c = \chi_2(0) = \frac{a}{\lambda D}$$

$$M_{12}^c(\xi_1) = \nu M_{22}^c(\xi_1)$$

$$M_{22}^c(\xi_1) = e^{-\lambda\xi_1}\left[\cos(\lambda\xi_1) + \sin(\lambda\xi_1)\right]$$

• Zaburzenie $X_3=1$ / Perturbation $X_3=1$



$$\xi_2 = \frac{y_2}{a}$$

$$w(\xi_2) = e^{-\lambda\xi_2}\left[A_1\cos(\lambda\xi_2) + A_2\sin(\lambda\xi_2)\right]$$

warunki brzegowe / boundary conditions

$$\left.\begin{matrix} M_2(0)=0 \\ Q_2(0)=0 \end{matrix}\right| \longrightarrow A_1, A_2$$

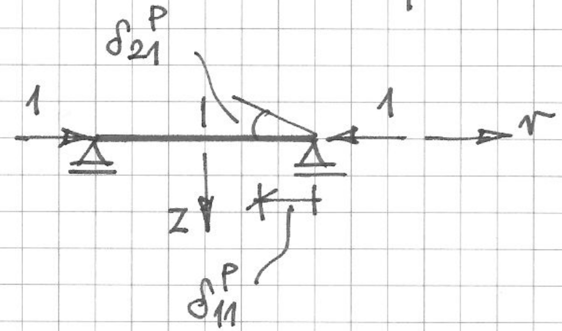
$$\delta_{33}^{c} = -w(0) = \frac{1}{2}\frac{a^3}{\lambda^3 D}$$

$$M_{13}^{c}(\xi_2) = \nu M_{23}^{c}(\xi_2)$$

$$M_{23}^{c}(\xi_2) = \frac{a}{\lambda}e^{-\lambda\xi_2}\sin(\lambda\xi_2)$$

(B1) PŁYTA DOLNA / BOTTOM PLATE

• Zaburzenie $X_1=1$ / Perturbation $X_1=1$



$$\rho = \frac{r}{a} \qquad C = \frac{Eh}{1-\nu^2}$$

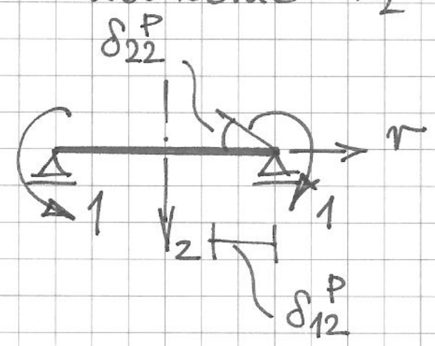
$$u(\rho) = A_1\rho$$

warunki brzegowe / boundary conditions

$$N_2(1) = -1 \longrightarrow A_1$$

$$M_{11}^{P} = M_{21}^{P} = 0 \qquad \delta_{11}^{P} = \frac{a}{C(1+\nu)} \qquad \delta_{21}^{P} = 0$$

• Zaburzenie $X_2=1$ / Perturbation $X_2=1$



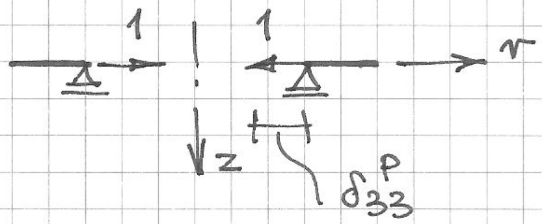
$$w(\rho) = A_1 + A_2\rho^2 \qquad D = \frac{Eh^3}{12(1-\nu^2)}$$

warunki brzegowe / boundary conditions

$$\left.\begin{matrix} w(1)=0 \\ M_2(1)=-1 \end{matrix}\right| \longrightarrow A_1, A_2$$

$$M_{12}^{P}(\rho) = M_{22}^{P}(\rho) = -1 \qquad \delta_{12}^{P} = 0 \qquad \delta_{22}^{P} = -\chi_2(1) = \frac{a}{D(1+\nu)}$$

(B₂) PŁYTA GÓRNA / TOP PLATE



$$\varrho = \frac{r}{a+b}$$

$$u(\varrho) = A_1 \varrho + \frac{A_2}{\varrho}$$

warunki brzegowe / boundary conditions

$$\left.\begin{aligned} N_2\left(\frac{a}{a+b}\right) &= 1 \\ N_2(1) &= 0 \end{aligned}\right| \longrightarrow A_1, A_2$$

$$M_{13}^P = M_{23}^P = 0 \qquad \delta_{33}^P = u\left(\frac{a}{a+b}\right) = \frac{a+b}{C(1-\nu^2)} \frac{\alpha\left[\alpha^2(1-\nu)+(1+\nu)\right]}{1-\alpha^2}$$

$$\alpha = \frac{a}{a+b}$$

(C) OBLICZENIE X_1, X_2, X_3 / CALCULATION OF X_1, X_2, X_3

$$\left.\begin{aligned} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{10} &= 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{20} &= 0 \end{aligned}\right\} \quad X_1, X_2$$

$$\delta_{33} X_3 + \delta_{30} = 0 \longrightarrow X_3$$

$$\delta_{ij} = \delta_{ij}^c + \delta_{ij}^P \qquad \delta_{i0} = \delta_{i0}^c + \delta_{i0}^P$$

$$\delta_{11} = \frac{a}{C(1+\nu)} + \frac{1}{2}\frac{a^3}{\lambda^3 D} \qquad \delta_{10} = \frac{pa^2}{Eh}$$

$$\delta_{12} = \delta_{21} = \frac{1}{2}\frac{a^2}{\lambda^2 D} \qquad \delta_{20} = 0$$

$$\delta_{22} = \frac{a}{D(1+\nu)} + \frac{a}{\lambda D} \qquad \delta_{30} = \frac{pa^2}{Eh}$$

(D) OBLICZENIE M_1, M_2 / CALCULATION OF M_1, M_2

$\cancel{M_1^C(\xi_1) = M_{11}^C X_1 +}$

WALEC / CYLINDER

$$\left\{\begin{aligned} &M_1^C(\xi_1) = M_{11}^C(\xi_1) X_1 + M_{12}^C(\xi_1) X_2 + M_{13}^C\left(\frac{H}{a} - \xi_1\right) X_3 \\ &M_2^C(\xi_1) \text{ — podobnie / similarly} \end{aligned}\right.$$

$M_1 = M_2 = 0$ w płycie górnej.

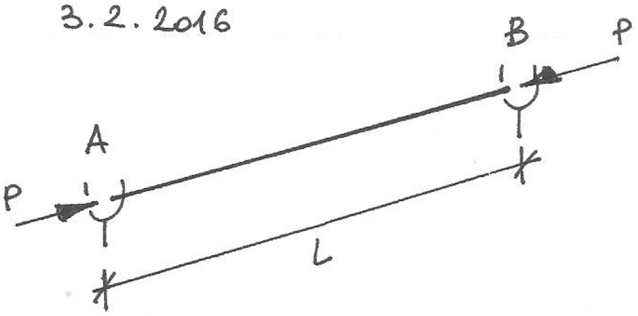
$M_1 = M_2 = 0$ in top plate.

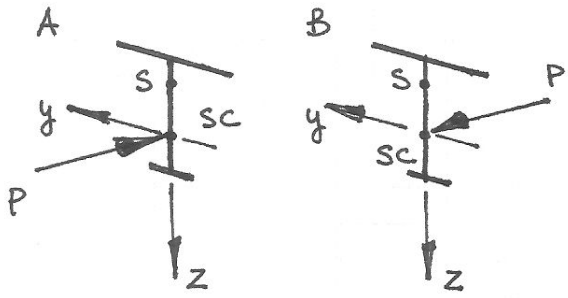
$$\left.\begin{aligned} M_1^P(\varrho) &= M_{12}^P(\varrho) X_2 \\ M_2^P(\varrho) &= M_{22}^P(\varrho) X_2 \end{aligned}\right\} \text{ PŁYTA DOLNA / BOTTOM PLATE}$$

Zadanie 2

Problem #2

3.2.2016





$z_s = -11\ cm$, $y_s = 0$

$$(r_0)^2 = \frac{J_y + J_z}{A} + (z_s)^2 + (y_s)^2 = 346\ cm^2$$

$$\beta_z = \left(\frac{z_s}{r_0}\right)^2 = 0{,}35$$

$$P_z = \frac{\pi^2 E_1 J_z}{L^2} = 0{,}142\ E_1$$

$$P_y = \frac{\pi^2 E_1 J_y}{L^2} = 1{,}634\ E_1$$

$$P_s = \left(\frac{1}{r_0}\right)^2 \left[G J_s + \frac{\pi^2 E_1 J_\omega}{L^2} \right] = 0{,}07\,G + 0{,}053\,E_1$$

Wartość siły krytycznej wyznacza się z warunku :

The value of a critical force is calculated from :

$$\det\left[A(P)\right] = 0 \rightarrow P = P_{kr}$$

W przypadku $y_s = 0$, $z_s \neq 0$ $A(P)$ ma postać :

In case of $y_s = 0$, $z_s \neq 0$ $A(P)$ takes the form :

$$A(P) = \begin{bmatrix} P - P_z & 0 & z_s P \\ 0 & P - P_y & 0 \\ z_s P & 0 & (r_0)^2 (P - P_s) \end{bmatrix}$$

$$\det\left[A(p)\right] = (r_0)^2 \cdot (p - p_y) \cdot W_2(p) = (r_0)^2 (p-p_y)(p-p_1)(p-p_2)(1-\beta_z)$$

$$W_2(p) = (1-\beta_z)p^2 - (p_s + p_z)p + p_s p_z$$

p_1, p_2 – pierwiastki wielomianu $W_2(p)$ / roots of the polynomial $W_2(p)$

$$p_1 = \frac{p_z + p_s - \sqrt{\Delta}}{2(1-\beta_z)}$$

$$\Delta = (p_z + p_s)^2 - 4(1-\beta_z)p_z p_s$$

$$p_2 = \frac{p_z + p_s + \sqrt{\Delta}}{2(1-\beta_z)}$$

$$p_{kr} = \min\{p_1, p_2, p_y\}$$