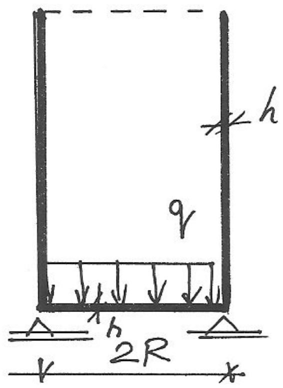
EGZAMIN MK (MK3) KB studia mgr stacj. dnia 29 VI 2015 r.

NAZWISKO Imię		
Nr albumu		Ocena z ćwiczeń projektowych
ocena zadania 1	ocena zadania 2	Ocena z egzaminu po ustnym
		Ocena łączna, data, podpis

Zadanie 1.

Dany jest zbiornik walcowy z płytą denną obciążona równomiernie. Przyjąć, że zbiornik jest długi. Wyznaczyć momenty zginające w płycie i powłoce.



$h = R/20$

$\nu = 0.3$

Zadanie 2. Znaleźć wartość momentu wywołującego zwichrzenie pręta cienkościennego o danym przekroju, podpartego widełkowo, o długości $l = 2.5$ m. Dane dotyczące przekroju: SC =S oraz

$J_{\omega} = 1.33333 \times 10^5 \cdot cm^6$

$J_s = 20 \cdot cm^4$

$A = 60 \cdot cm^2$

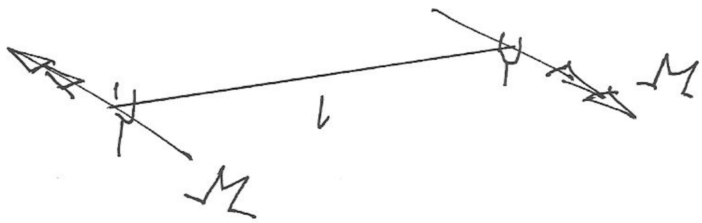
$J_y = 4333.33 cm^4$

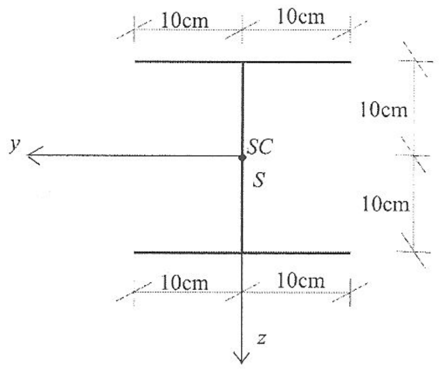
$J_z = 1333.33 cm^4$

$g = 1 cm$

$E = 205 GPa$

$\nu = 0.3$





Pret jest podparty widełkowo, obciążony dwoma momentami M_y=M na końcach

Exam on the Mechanics of Structures
29.06.2015

PROBLEM #1
Consider a cylindrical shell with bottom circular plate loaded by a uniform load q.
Assume that the shell is long.

Calculate the bending moments M_1, M_2 in a cylinder.

See front page for:
- value of the Poisson ratio ν;
- symbols used for dimensions and loading.

PROBLEM #2
Consider a thin-walled beam of length $l = 2.5m$, fork-supported at both ends and loaded by a moment M applied at the cross-section's centroid $SC = S$.

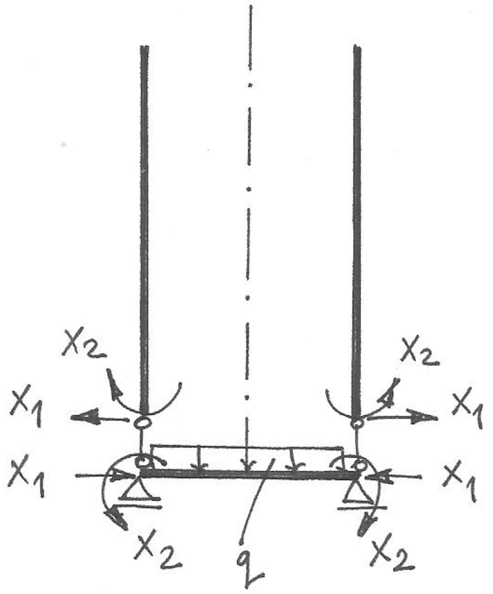
Calculate the value of the moment M critical for warping.

See front page for:
- the cross-section dimensions;
- value of the Young modulus E and Poisson ratio ν;
- values of the geometrical characteristics $A, J_y, J_z, J_\omega, J_s$.

Calculate the remaining characteristics, if necessary.

Zadanie 1
Problem #1
29.6.2015

Układ zastępczy / Primary system



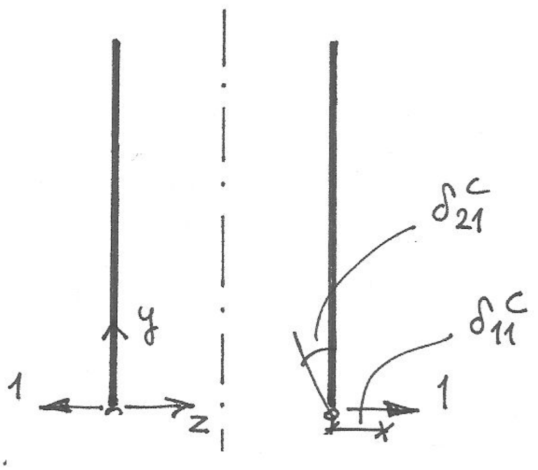
$$\lambda^4 = 3(1-\nu^2)\left(\frac{R}{h}\right)^2$$

$$C = \frac{Eh}{1-\nu^2} \qquad D = \frac{Eh^3}{12(1-\nu^2)}$$

Ⓐ WALEC / CYLINDER

- Nie ma obciążeń w stanie "0" — bezzgięciowym.
  No loading in the "0"-th state — non-bending state.

- Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$



$$\xi = \frac{y}{R}$$

$$w(\xi) = e^{-\lambda\xi}\left[A_1\cos(\lambda\xi) + A_2\sin(\lambda\xi)\right]$$

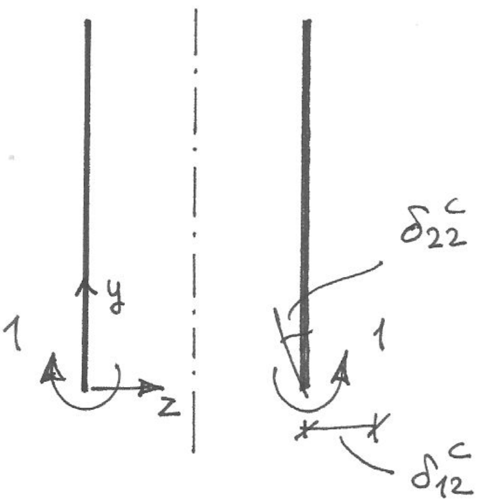
war. brzegowe / boundary conditions

$$\left.\begin{aligned} Q_2(0) &= 1 \\ M_2(0) &= 0 \end{aligned}\right| \rightarrow A_1, A_2$$

$$\delta_{11}^c = -w(0) = \frac{2R\lambda}{Eh}$$

$$\delta_{21}^c = \chi_2(0) = \frac{2\lambda^2}{Eh}$$

• Zaburzenie $X_2=1$ / Perturbation $X_2=1$



$w\,(\lambda\xi)$ jak wyżej / as above

war. brzegowe / boundary conditions:

$\varphi_2(0)=0$

$M_2(0)=1$ $\longrightarrow A_1, A_2$

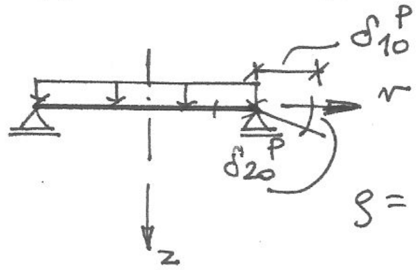
$$\delta_{12}^c = -w(0) = \frac{2\lambda^2}{Eh}$$

$$\delta_{22}^c = \chi_2(0) = \frac{4\lambda^3}{ERh}$$

(B) PŁYTA / PLATE

• Stan "0" – obciążenie zewnętrzne

The "0"-th state of loading



$\rho = \frac{r}{R}$

$$w(\rho) = A_1 + A_2\rho^2 + \frac{qR^4}{64D}\rho^4$$

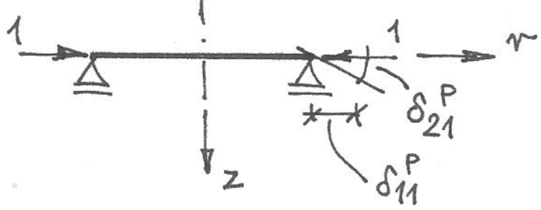
wb. / bc.

$w(1)=0$

$M_2(1)=0$ $\longrightarrow A_1, A_2$

$$\delta_{10}^p = 0 \qquad \delta_{20}^p = -\chi_2(1) = -\frac{1}{8}\frac{qR^3}{D(1+\nu)}$$

- Zaburzenie $X_1=1$ / Perturbation $X_1=1$



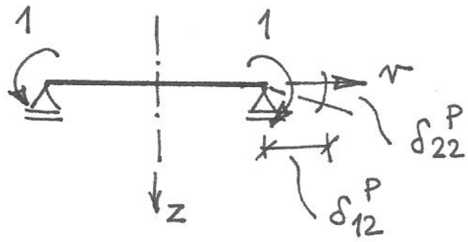
$$u(\varrho) = A_1 + A_2\varrho \quad , \quad A_1 = 0$$

w.b. / b.c. :

$$N_2(1) = -1 \longrightarrow A_2$$

$$\delta_{21}^P = 0 \qquad \delta_{11}^P = -u(1) = -\frac{R}{C(1+\nu)}$$

- Zaburzenie $X_2=1$ / Perturbation $X_2=1$



$$w(\varrho) = A_1 + A_2\varrho^2$$

w.b. / b.c.

$$\left.\begin{array}{l} w(1) = 0 \\ M_2(1) = -1 \end{array}\right| \longrightarrow A_1, A_2$$

$$\delta_{12}^P = 0 \qquad \delta_{22}^P = -\frac{R}{D(1+\nu)}$$

(C) obliczenie X_1, X_2 / Calculation of X_1, X_2

$$\delta_{ij} = \delta_{ij}^c + \delta_{ij}^P \quad , \quad \delta_{i0} = \delta_{i0}^c + \delta_{i0}^P \quad , \quad i,j = 1,2$$

$$\delta_{11} = \frac{2R\lambda}{Eh} - \frac{R}{C(1+\nu)} \; ; \; \delta_{12} = \delta_{21} = \frac{2\lambda^2}{Eh}$$

$$\delta_{22} = \frac{4\lambda^3}{ERh} - \frac{R^2}{D(1+\nu)} \; ; \; \delta_{10} = 0 \; ; \; \delta_{20} = -\frac{1}{8}\frac{qR^3}{D(1+\nu)}$$

$$X_1 = 0{,}856\, qR \quad ; \quad X_2 = -0{,}14\, qR^2$$

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{10} = 0 \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{20} = 0 \end{cases}$$

(D) obliczenie M_1, M_2

Calculation of M_1, M_2

walec / cylinder : $M_1^c(\xi) = \nu M_2^c(\xi) = X_1 \cdot M_{21}^c(\xi) + X_2 \cdot M_{22}^c(\xi)$

$$M_{21}^c(\xi) = \frac{R}{\lambda} e^{-\lambda\xi}\sin(\lambda\xi)$$

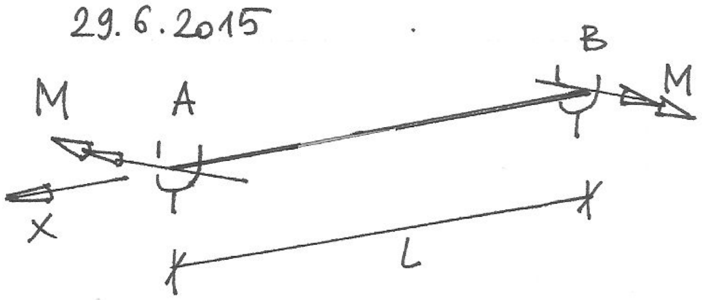
$$M_{22}^c(\xi) = e^{-\lambda\xi}\left[\cos(\lambda\xi) - \sin(\lambda\xi)\right]$$

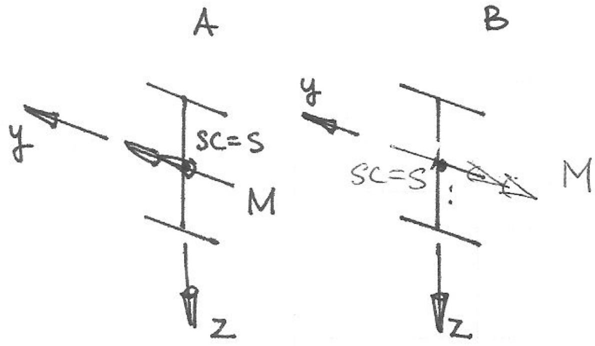
płyta / plate:

$$M_1^P(\rho) = X_2 \cdot M_{12}^P(\rho) + M_{10}^P(\rho)$$
$$M_2^P(\rho) = X_2 \cdot M_{22}^P(\rho) + M_{20}^P(\rho)$$

$$M_{12}^P(\rho) = -1 \qquad M_{22}^P(\rho) = -1$$
$$M_{10}^P(\rho) = -\frac{1}{16} qR^2 (1+3\nu)\rho^2$$
$$M_{20}^P(\rho) = -\frac{1}{16} qR^2 (3+\nu)\rho^2$$

Zadanie 2
Problem #2
29.6.2015





$$\xi = \frac{x}{L} \qquad A: \xi = 1$$
$$B: \xi = 0$$

Równania / equations:

1) $GJ_s\,\theta' - E_1 J_\omega\,\theta''' = Mv'$

2) $-E_1 J_y\, w'' = M$

3) $E_1 J_z\, v'' = -\theta M$

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1) + 3) $$GJ_s\theta'' - E_1 J_\omega \theta^{IV} = -\frac{\theta M}{E_1 J_z}$$

$$\theta^{IV} - \theta''\cdot 2\alpha_1 - \theta\cdot\alpha_2 = 0$$

$$2\alpha_1 = \frac{GJ_s L^2}{E_1 J_\omega} \qquad \alpha_2 = \frac{M^2 L^4}{(E_1)^2 J_z J_\omega}$$

Niech / let:

$$\beta_1 = \sqrt{-\alpha_1 + \sqrt{\alpha_1^2 + \alpha_2}}$$
$$\beta_2 = \sqrt{\alpha_1 + \sqrt{\alpha_1^2 + \alpha_2}}$$

Rozwiązanie / solution:

$$\theta(\xi) = C_1 \sin(\beta_1 \xi) + C_2 \cos(\beta_1 \xi) + C_3 \sinh(\beta_2 \xi) + C_4 \cosh(\beta_2 \xi)$$

Warunki brzegowe / boundary conditions:

$\theta(0) = 0$		$\theta(0) = 0$		
$B(0) = 0$	$\Longrightarrow$	$\theta''(0) = 0$	$\Longrightarrow$	$A\,C = 0$
$\theta(1) = 0$		$\theta(1) = 0$		$\det A = 0 \rightarrow M_{kr}$
$B(1) = 0$		$\theta''(1) = 0$		

$$\det \mathbb{A} = -4(\alpha_1^2 + \alpha_2) \sin\beta_1 \sinh\beta_2$$

$$\sin\beta_1 = 0 \iff \sqrt{-\alpha_1 + (\alpha_1^2 + \alpha_2)} = k\pi \quad , \quad k = 1, 2, \ldots$$

$$\alpha_2 = k^4\pi^4 + 2\alpha_1 k^2 \pi^2$$

Niech / Let $\quad k = 1$

Wtedy / Then :

$$M_{kr} = \frac{\pi}{L} \sqrt{(GJ_s)(E_1 J_z)\left(1 + \frac{\pi^2}{L^2} \frac{E_1 J_\omega}{GJ_s}\right)}$$

$$M_{kr} = 54{,}75 \quad kNm$$