EGZAMIN MK (MK3) KB studia mgr stacj. dnia 22 VI 2015 r.

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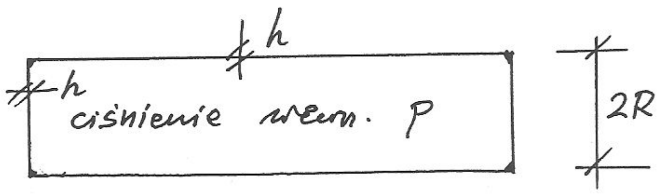
Zadanie 1.

Dany jest zbiornik walcowy zamknięty z obu stron płytami kołowymi. Przyjąć, że zbiornik jest długi.



$E = 30\ GPa$ $\nu = 0.2$

Wyznaczyć momenty zginające M_1, M_2 w powłoce i płytach dennych.

Zadanie 2. Znaleźć siłę krytyczną P (przyłożoną w środku ciężkości SC przekroju) wyboczenia giętnoskrętnego pręta cienkościennego o danym przekroju, podpartego widełkowo, o długości $l = 3.0$ m. Dane dotyczące przekroju:

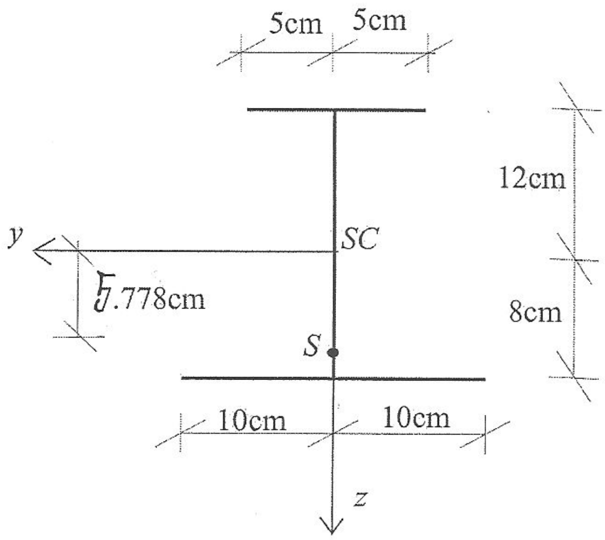
$A = 50 \cdot cm^2$

$J_y = 3573.333 cm^4$

$J_z = 750 \cdot cm^4$

$J_\omega = 29629.63 cm^6$

$J_s = 16.667 cm^4$



Pozostałe charakterystyki wyznaczyć samodzielnie.

Exam on the Mechanics of Structures
22.06.2015

PROBLEM #1
Consider a cylindrical shell closed by circular plates and loaded by an internal pressure *p*. Assume that the shell is long.

Calculate the bending moments M_1, M_2 in a cylinder and plates.

See front page for:
- values of Young's modulus E and Poisson ratio ν;
- symbols used for dimensions and loading.

PROBLEM #2
Consider a thin-walled beam of length $l = 3.0\,m$, fork-supported at both ends and loaded by a force P applied at the cross-section's centroid SC.

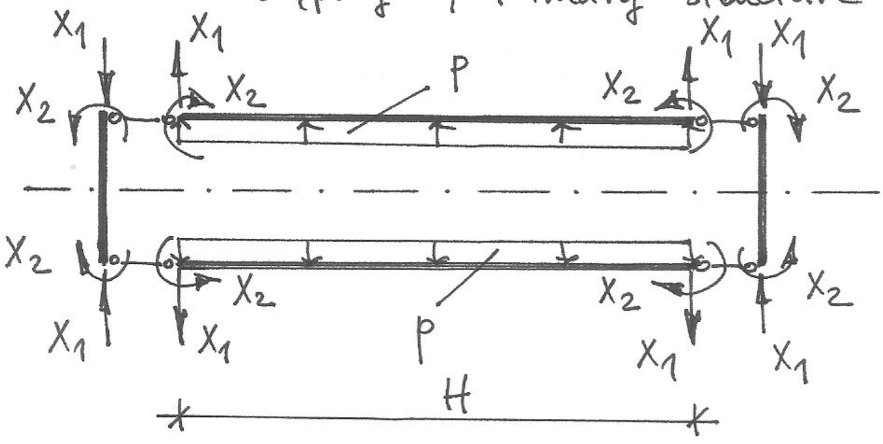
Calculate the value of a force P critical for flexural-torsional buckling.

See front page for:
- the cross-section dimensions;
- values of the geometrical characteristics $A, J_y, J_z, J_\omega, J_s$.

Calculate the remaining characteristics, if necessary.

Zadanie 1
Problem #1
22.6.2016

Układ zastępczy / Primary structure



Symetria / symmetry

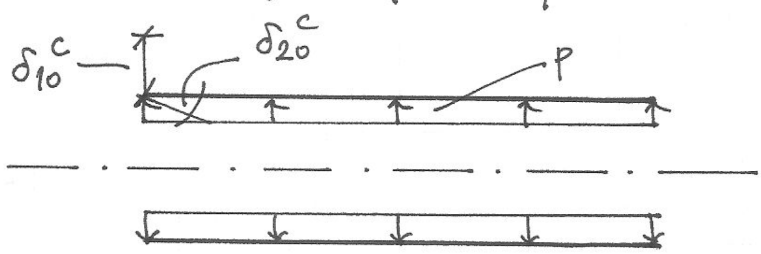
$$\lambda^4 = 3(1-\nu^2)\left(\frac{R}{h}\right)^2$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

$$C = \frac{Eh}{1-\nu^2}$$

Ⓐ POWŁOKA / CYLINDER

- Stan bezzgięciowy "0" / Non-bending state "0"

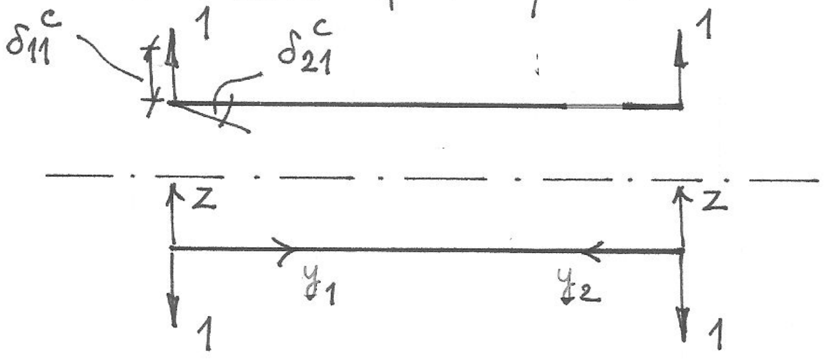


$$\delta_{10}^c = \frac{pR^2}{Eh}$$

$$\delta_{20}^c = 0$$

$$M_{10}^c = 0 \qquad M_{20}^c = 0$$

- Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$



$$\delta_{11}^c = \frac{1}{2D}\,\frac{R^3}{\lambda^3}$$

$$\delta_{21}^c = \frac{1}{2D}\,\frac{R^2}{\lambda^2}$$

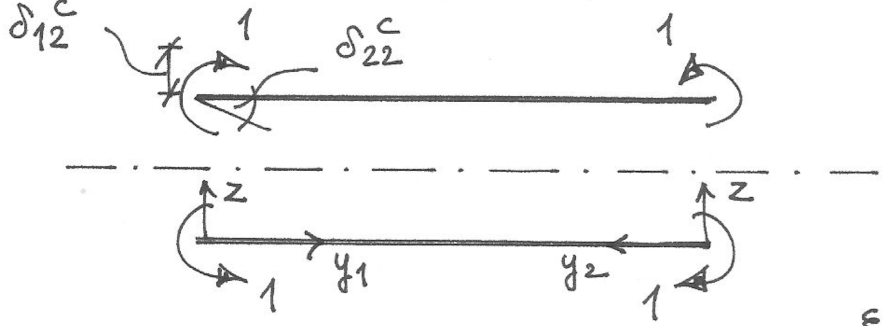
$$\xi_1 = \frac{y_1}{R}$$

$$\xi_2 = \frac{y_2}{R} \qquad \xi_2 = \frac{H-y_1}{R}$$

$$M_{11}^c(\xi_i) = \nu\, M_{21}^c(\xi_i)$$

$$M_{21}^c(\xi_i) = \frac{R}{\lambda}\, e^{-\lambda\xi_i} \sin(\lambda\xi_i) \qquad \xi_i = \xi_1 \text{ lub } \xi_2$$

- Zaburzenie $X_2 = 1$ / Perturbation $X_2 = 1$



$$\delta_{12}^{c} = \frac{1}{2D}\,\frac{R^2}{\lambda^2}$$

$$\delta_{22}^{c} = \frac{1}{D}\,\frac{R}{\lambda}$$

ξ_1, ξ_2 – jak wyżej / as above

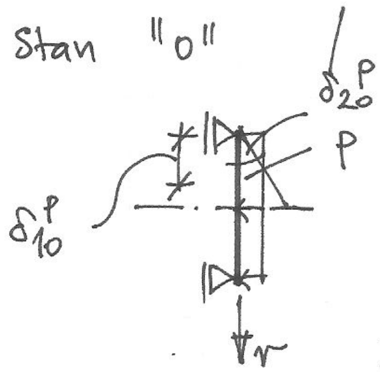
$$M_{12}^{c}(\xi_i) = \nu M_{22}^{c}(\xi_i)$$

$$M_{22}^{c}(\xi_i) = e^{-\lambda \xi_i}\left[\sin(\lambda \xi_i) + \cos(\lambda \xi_i)\right]$$

$i = 1, 2$

(B) PŁYTA / PLATE

- Stan "0" / The "0"-th state



$$\delta_{10}^{p} = 0$$

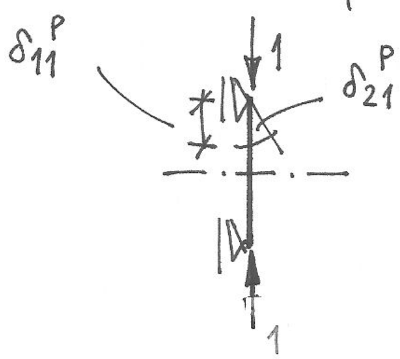
$$\delta_{20}^{p} = -\frac{pR^3}{8D(1+\nu)}$$

$$\rho = \frac{r}{R}$$

$$M_{10}^{p}(\rho) = \frac{pR^2}{16}\left[(3+\nu)(1-\rho^2) + (1-\nu)\cdot 2\rho^2\right]$$

$$M_{20}^{p}(\rho) = \frac{pR^2}{16}(3+\nu)(1-\rho^2)$$

- Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$

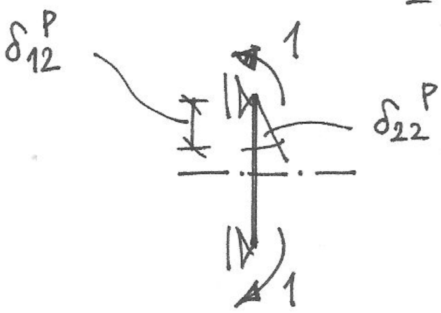


$$\delta_{11}^{p} = \frac{R}{C(1+\nu)}$$

$$\delta_{21}^{p} = 0$$

$$M_{11}^{p}(\rho) = 0 \qquad M_{21}^{p}(\rho) = 0$$

- Zaburzenie $X_2 = 1$ / Perturbation $X_2 = 1$



$$\delta_{12}^{P} = 0$$

$$\delta_{22}^{P} = \frac{R}{D(1+\nu)}$$

$$M_{12}^{P}(\rho) = -1 \qquad M_{22}^{P}(\rho) = -1$$

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c) Obliczenia X_1, X_2 / Calculation of X_1, X_2

$$\delta_{ij} = \delta_{ij}^{c} + \delta_{ij}^{P}$$

$$\delta_{i0} = \delta_{i0}^{c} + \delta_{i0}^{P}$$

$$\delta_{11} = \frac{1}{2D}\frac{R^3}{\lambda^3} + \frac{R}{C(1+\nu)}$$

$$\delta_{12} = \delta_{21} = \frac{1}{2D}\frac{R^2}{\lambda^2}$$

$$\delta_{22} = \frac{1}{D}\frac{R}{\lambda} + \frac{R}{D(1+\nu)}$$

$$\delta_{10} = \frac{pR^2}{Eh}$$

$$\delta_{20} = -\frac{pR^3}{8D(1+\nu)}$$

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{10} = 0 \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{20} = 0 \end{cases} \longrightarrow X_1, X_2$$

- Obliczenia M_1, M_2 / Calculation of M_1, M_2

powłoka / cylinder:

$$M_1^c(\xi_1) = \nu M_2^c(\xi_1)$$

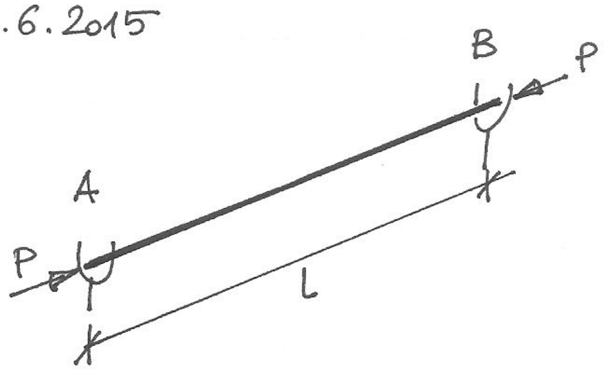
$$M_2^c(\xi_1) = \left[M_{21}^c(\xi_1) + M_{21}^c\left(\frac{H}{R} - \xi_1\right)\right] \cdot X_1 + \left[M_{22}^c(\xi_1) + M_{22}^c\left(\frac{H}{R} - \xi_1\right)\right] \cdot X_2$$

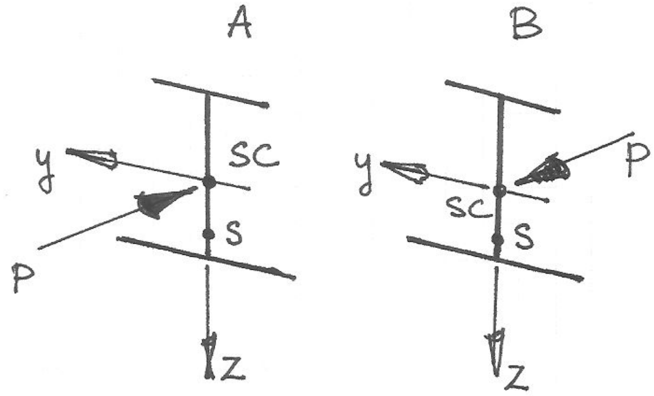
płyta / plate:

$$M_1^p(\rho) = -1 \cdot X_2 + M_{10}^p(\rho)$$

$$M_2^p(\rho) = -1 \cdot X_2 + M_{20}^p(\rho)$$

Zadanie 2
Problem #2
22.6.2015





$$E_1 = \frac{E}{1-\nu^2} \quad , \quad G = \frac{E}{2(1+\nu)}$$

$$y_s = 0 \quad , \quad z_s = 5{,}778 \text{ cm} \quad , \quad l = 300 \text{ cm}$$

$$r_0 = \sqrt{\frac{J_y + J_z}{A} + (z_s)^2} = 10{,}94 \text{ cm}$$

$$P_z = \frac{\pi^2 E_1 J_z}{l^2} = 0{,}0822\, E_1$$

$$P_y = \frac{\pi^2 E_1 J_y}{l^2} = 0{,}392\, E_1$$

$$P_s = \frac{1}{(r_0)^2}\left(G J_s + \frac{\pi^2 E_1 J_\omega}{l^2} \right) = 0{,}139\, G + 0{,}027\, E_1$$

$$A = \begin{bmatrix} P - P_z & 0 & z_s P \\ 0 & P - P_y & 0 \\ z_s P & 0 & (r_0)^2 (P - P_s) \end{bmatrix}$$

$$\det A = (r_0)^2 \cdot (P - P_y) \cdot W_2(P) \quad , \quad W_2(P) = (1-\beta_z) P^2 - (P_z + P_s) P + P_z P_s$$

$$= (r_0)^2 (1-\beta_z)(P-P_y)(P-P_1)(P-P_2) \qquad \beta_z = \left(\frac{z_s}{r_0}\right)^2 = 0{,}278$$

$$W_2(P) = (1-\beta_z)(P - P_1)(P - P_2)$$

$$\underline{\underline{P_{kr} = \min\{P_y, P_1, P_2\}}}$$

$$P_{1,2} = \frac{(P_z + P_s) \pm \sqrt{\Delta}}{2(1-\beta_z)}$$

$$\Delta = (P_z + P_s)^2 - 4(1-\beta_z) P_z P_s$$