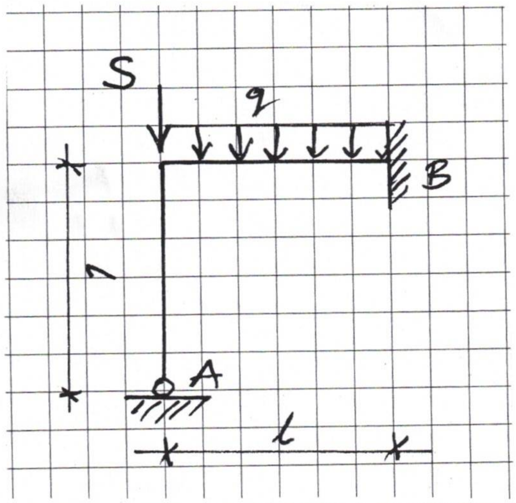
Egzamin pisemny z Mechaniki Konstrukcji II, 25 czerwca 2026 r.

Imię i NAZWISKO				
Prowadzący ćwiczenia, nr grupy				
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egz. pis.	Ocena ostateczna z egzaminu
				Ocena łączna

Zadanie 1

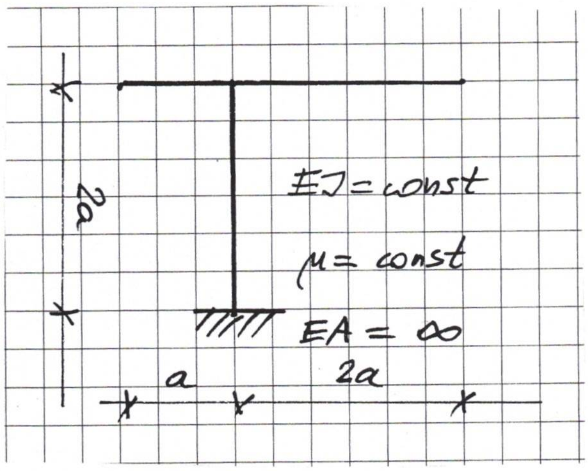
Dana jest rama wykonana
z dwuteownika
I 180 o charakterystykach:
J=1450cm^4 , E=210GPa,
l=2m
Obciążona dużą siłą osiową S=30,45kN
oraz q=1kN/m.
Znaleźć moment zginający
w utwierdzeniu B.

(*The frame is made from the bars of
I 180 profile:
J=1450cm^4 ,
E=210GPa,
l=2m
The big axial force equals S=30,45kN
and q=1kN/m.
Compute the bending moment
in the clamped edge B.*)



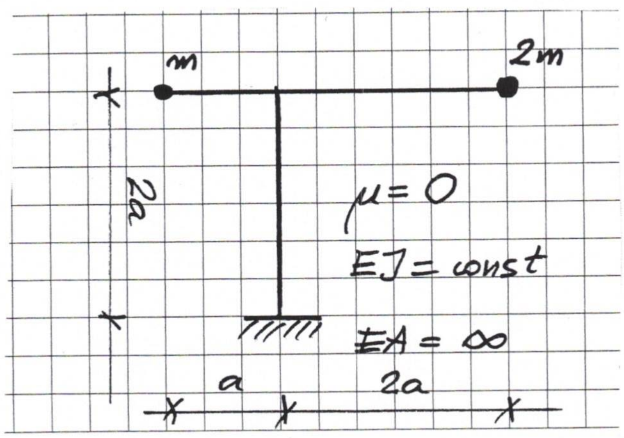
Zadanie 2

Zapisać równania
określające
częstości drgań własnych
danej ramy.
(Write down the equations
for computing
the natural frequencies
of the given frame.)



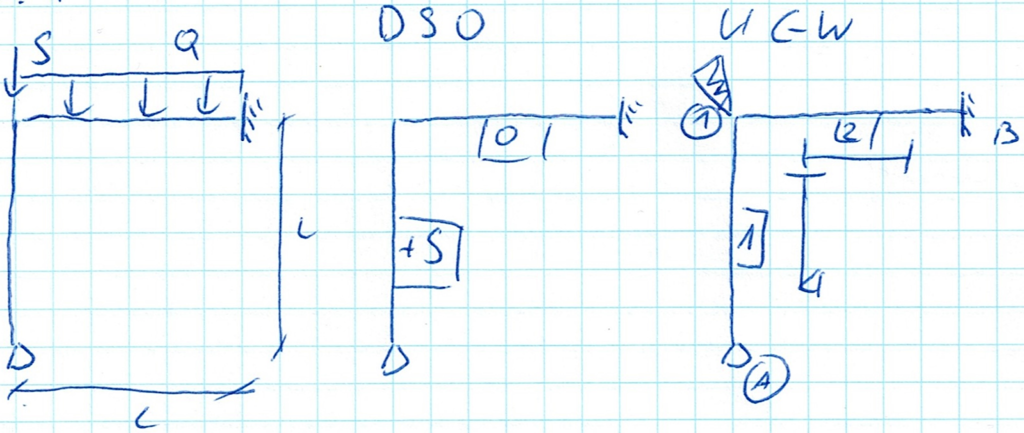
Zadanie 3

Zapisać równania
określające
częstości drgań własnych
danej ramy z masami skupionymi.
(Write down the equations
for computing
the natural frequencies
of the given frame
with concentrated masses)



Egzamin MK2 25.06.2026

Zad. 1



$$\lambda_1 = l\sqrt{\frac{S}{EJ}} = 200\,cm\sqrt{\frac{30{,}45\,kN}{21000\frac{kN}{cm^2}\cdot 1450\,cm^4}} = 0{,}2$$

$$\lambda_2 = 0$$

RR:

$$\Phi_1^1 + \Phi_1^2 = 0$$

WT:

$$\Phi_1^1 = \frac{EJ}{l}\left[\alpha'(0{,}2)\varphi_1\right] = \frac{EJ}{l}\left[2{,}992\,\varphi_1\right]$$

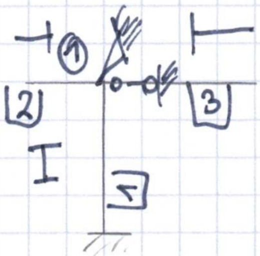
$$\Phi_1^2 = \frac{2EJ}{l}\left[2\varphi_1\right] + \Phi_1^{02} = \frac{EJ}{l}\left[4\varphi_1\right] - \frac{1}{12}ql^2$$

$$\varphi_1 = \frac{1}{12(\alpha'(0{,}2)+4)}\,\frac{ql^3}{EJ}$$

$$M_B = \Phi_B^2 = \frac{2EJ}{l}\left[\varphi_1\right] + \Phi_B^{02} = \frac{EJ}{l}\left[\frac{2}{12(\alpha'(0{,}2)+4)}\right]\frac{ql^3}{EJ} + \frac{1}{12}ql^2 = \frac{281}{2622}ql^2 =$$

$$= \frac{562}{1311}\,kNm \approx 0{,}42868\,kNm$$

ZADANIE 2



$$q = \begin{bmatrix} \varphi_1 \\ \frac{w}{l} \end{bmatrix}$$

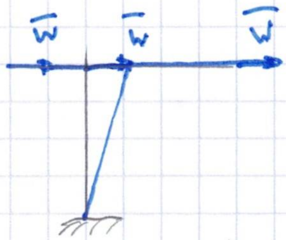
$$\lambda^{(1)} = 2l\sqrt[4]{\frac{\mu\omega^2}{EJ}} = 2\lambda$$

$$\lambda^{(2)} = l\sqrt[4]{\frac{\mu\omega^2}{EJ}} = \lambda$$

$$\lambda^{(3)} = 2\lambda$$

$$\omega^2 = \frac{EJ}{\mu l^4}\lambda^4$$

p.p.



r. r. MP

$$\Phi_1^{(1)} + \Phi_1^{(2)} + \Phi_1^{(3)} = 0 \quad (1)$$

$$\bar{W}_1^{(1)} - B_{11}^{(2)} - B_{11}^{(3)} = 0 \quad (2)$$

$$B_{11}^{(2)} = l \cdot \mu \cdot \omega^2 w = \frac{EJ}{l^3}\lambda^4 w$$

$$B_{11}^{(3)} = 2l\mu\omega^2 w = 2\frac{EJ}{l^3}\lambda^4 w$$

WT

1) I

$$\Phi_1^{(1)} = \frac{EJ}{2l}\left[\alpha(2\lambda)\varphi_1 - \theta(2\lambda)\frac{w}{2l}\right] = \frac{EJ}{l}\left[\frac{1}{2}\alpha(2\lambda)\varphi_1 - \frac{1}{4}\theta(2\lambda)\frac{w}{l}\right]$$

$$\bar{W}_1^{(1)} = -\frac{EJ}{4l^2}\left[\theta(2\lambda)\varphi_1 - \gamma(2\lambda)\frac{w}{2l}\right] = \frac{EJ}{l^2}\left[-\frac{1}{4}\theta(2\lambda)\varphi_1 + \frac{1}{8}\gamma(2\lambda)\frac{w}{l}\right]$$

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2)

$$\Phi_1^{(2)} = \frac{EJ}{l}\left[\alpha''(\lambda)\varphi_1\right]$$

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3)

$$\Phi_1^{(3)} = \frac{EJ}{2l}\left[\alpha''(2\lambda)\varphi_1\right] = \frac{EJ}{l}\left[\frac{1}{2}\alpha''(2\lambda)\varphi_1\right]$$

$$\frac{EJ}{l}\begin{bmatrix} \frac{1}{2}\alpha(2\lambda) + \alpha''(\lambda) + \frac{1}{2}\alpha''(2\lambda) & -\frac{1}{4}\theta(2\lambda) \\ -\frac{1}{4}\theta(2\lambda) & \frac{1}{8}\gamma(2\lambda) - 3\lambda^4 \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \frac{w}{l} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

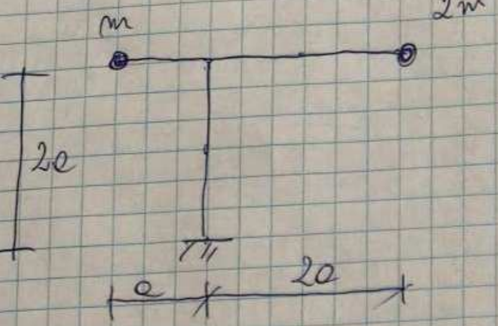
$$\mathbb{K}(\lambda)\, q = 0$$

$$\det \mathbb{K}(\lambda) = 0$$

$$\Rightarrow \lambda_i = \ldots$$

$$\Rightarrow \omega_i = \ldots$$

ZAD. 3. MK2 EGZAMIN

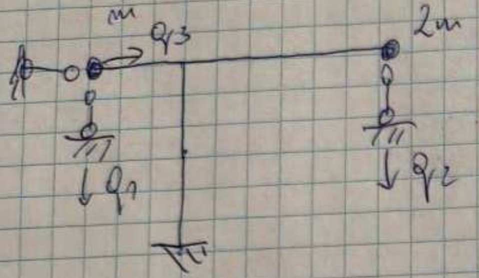


$\mu = 0$

$EJ = const$

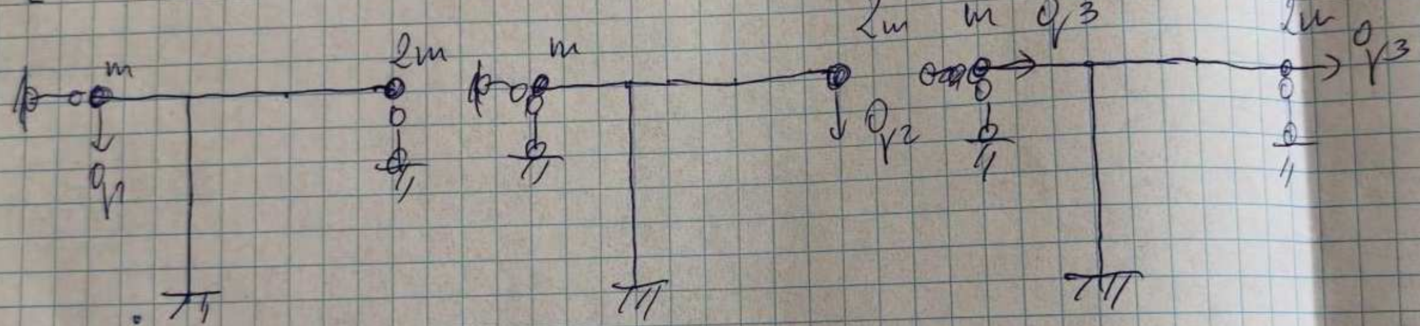
$EA \to \infty$

$\omega_i = ?$



$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \qquad (1 - \omega^2 \cdot D \cdot M)\, q = 0$$

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & 3m \end{bmatrix}$$



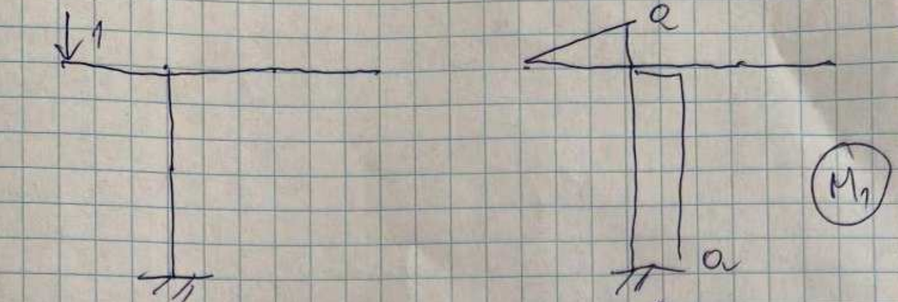
$$E_k = \frac{1}{2} m \left(\dot{q}_1^2 + \dot{q}_3^2\right) + \frac{1}{2} 2m \left(\dot{q}_2^2 + \dot{q}_3^2\right) =$$

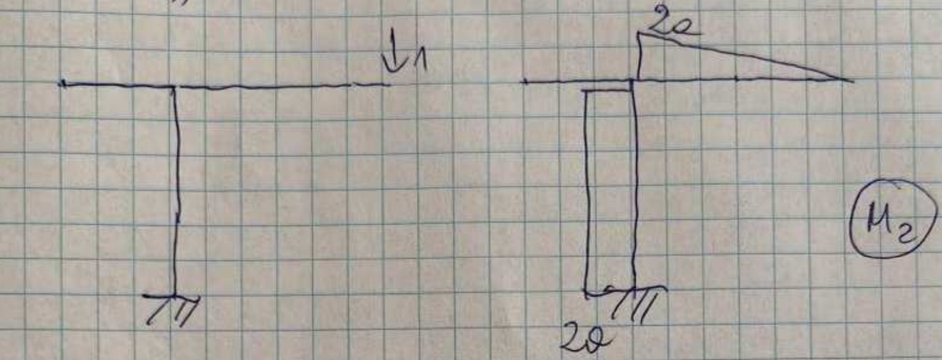
$$= \frac{1}{2} \left\{ m \dot{q}_1^2 + 2m \dot{q}_2^2 + 3m \dot{q}_3^2 \right\}$$

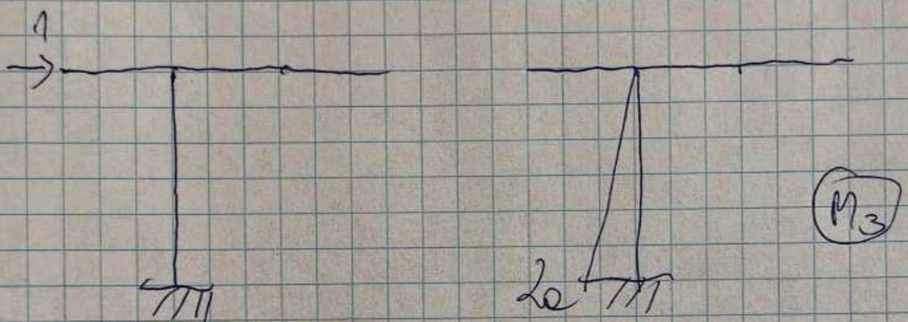
$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} = \begin{bmatrix} \frac{7}{3} & -4 & -2 \\ -4 & \frac{32}{3} & 4 \\ -2 & 4 & \frac{8}{3} \end{bmatrix} \frac{a^3}{EJ}$$

$$d_{ij} = \int \frac{M_i M_j}{EJ} ds$$

ZAD. 3. c.d MK2 EGZAMIN 28.06.2026r.







$$d_{11} = \frac{1}{EJ}\left[\frac{1}{2}a\cdot a\cdot\frac{2}{3}a + a\cdot 2a\cdot a\right] = \frac{7}{3}\frac{a^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}\left[\frac{1}{2}2a\cdot 2a\cdot\frac{2}{3}2a + 2a\cdot 2a\cdot 2a\right] = \frac{32}{3}\frac{a^3}{EJ}$$

$$d_{33} = \frac{1}{EJ}\left[\frac{1}{2}2a\cdot 2a\cdot\frac{2}{3}2a\right] = \frac{8}{3}\frac{a^3}{EJ}$$

$$d_{12} = \frac{1}{EJ}\left[-a\cdot 2a\cdot 2a\right] = -4\frac{a^3}{EJ}$$

$$d_{13} = \frac{1}{EJ}\left[-a\cdot 2a\cdot\frac{1}{2}2a\right] = -2\frac{a^3}{EJ}$$

$$d_{23} = \frac{1}{EJ}\left[2a\cdot 2a\cdot\frac{1}{2}2a\right] = 4\frac{a^3}{EJ}$$

$$x = \omega^2\frac{ml^3}{EJ}$$

$$\underbrace{\left(\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - \omega^2\cdot\frac{ml^3}{EJ}\begin{bmatrix}\frac{7}{3}&-4&-2\\-4&\frac{32}{3}&4\\-2&4&\frac{8}{3}\end{bmatrix}\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}\right)}_{A}\begin{bmatrix}q_1\\q_2\\q_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\det A = 0 \Rightarrow x_i \Rightarrow \omega_i$$