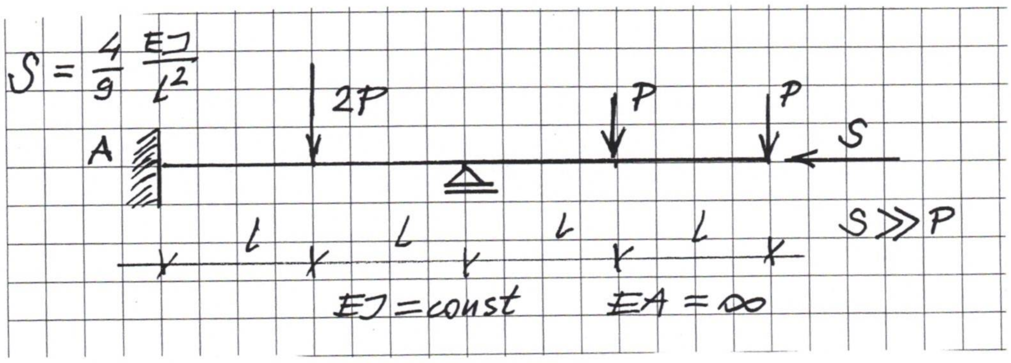
## Egzamin pisemny z Mechaniki Konstrukcji II, 18 czerwca 2026 r.

| Imię i NAZWISKO | | | | |
|---|---|---|---|---|
| Prowadzący ćwiczenia, nr grupy | | | | |
| ocena zadania 1 | ocena zadania 2 | ocena zadania 3 | ocena egz. pis. | Ocena ostateczna z egzaminu |
| | | | | Ocena łączna |

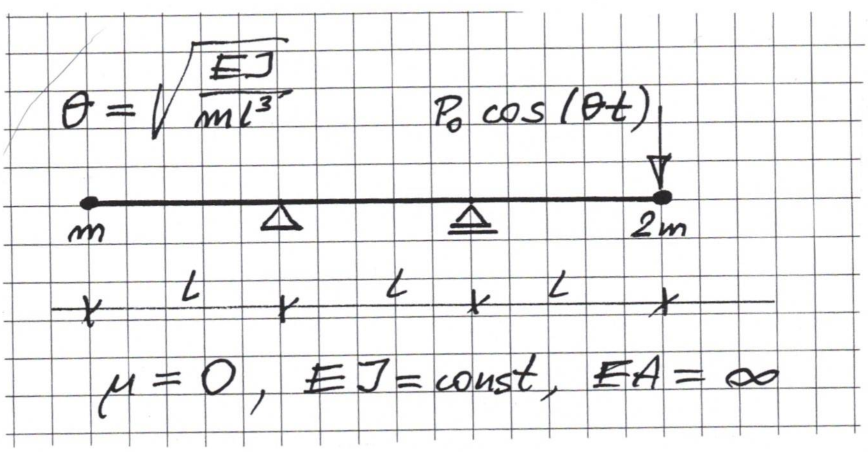
**Zadanie 1**

Zapisać formuły na podstawie których można obliczyć moment zginający w utwierdzeniu A. Skorzystać z twierdzenia Bettiego.
(*Write down the formulae which make it possible to compute the bending moment at the clamped edge A. Make use of Betti's theorem*).



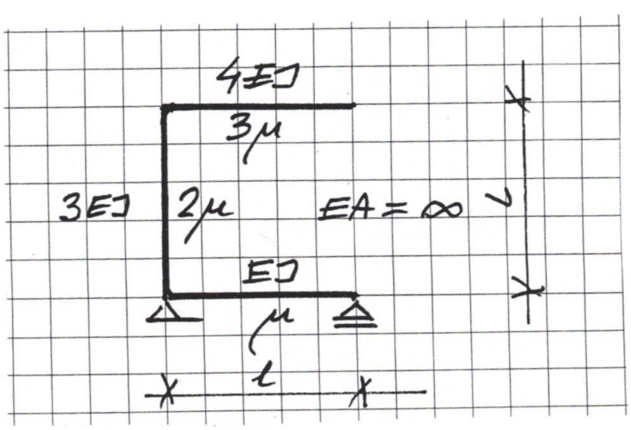

**Zadanie 2**

Obliczyć
a) Częstości drgań własnych danego pręta; b) Amplitudę przemieszczenia masy „m"
(*Compute*
*a) The natural frequencies of the given bar; b) The amplitude of the displacement of the mass "m")*




**Zadanie 3**

Zapisać równania określające pierwszą częstość drgań własnych danej ramy
(*Write down the formulae which make it possible to compute the first natural frequency of the given frame)*



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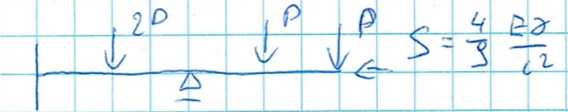
Tw Bettiego

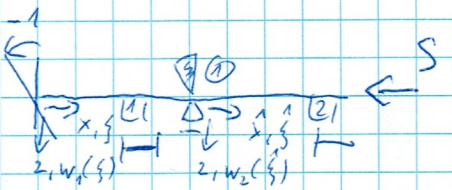
Zad. 1




$$M_A(-1) + 2P w_1\left(\tfrac{1}{2}\right) + P w_2\left(\tfrac{1}{2}\right) + P w_2(1) = 0$$

$$M_A = 2P w_1\left(\tfrac{1}{2}\right) + P w_2\left(\tfrac{1}{2}\right) + P w_2(1)$$

$M_A = ?$




$$\sigma_1 = \sigma_2 = \frac{4}{3} \approx 1{,}3$$

$$w_1(\xi) = \ominus 1 \cdot 2l\, \omega_1(\xi) \ominus \varphi_1 \cdot 2l\, \omega_1(1-\xi)$$

$$w_2(\hat{\xi}) = \varphi_1\, 2l\, \omega_5(\hat{\xi})$$

$$\Phi_1^1 + \Phi_1^2 = 0$$

$$\Phi_1^1 = \frac{EJ}{l}\left[\frac{\alpha\left(\frac{4}{3}\right)}{2}\varphi_1 \ominus \frac{\beta\left(\frac{4}{3}\right)}{2}\right]$$

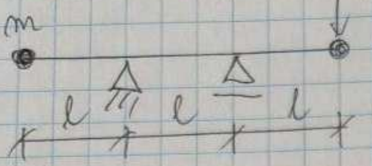
$$\Phi_1^2 = \frac{EJ}{l}\left[\frac{\alpha'''\left(\frac{4}{3}\right)}{2}\varphi_1\right]$$

$$\varphi_1 = \frac{\beta\left(\frac{4}{3}\right)}{\alpha\left(\frac{4}{3}\right) + \alpha'''\left(\frac{4}{3}\right)} \approx \ominus 2{,}253$$

$$M_A \approx \ominus 16{,}48\, Pl$$

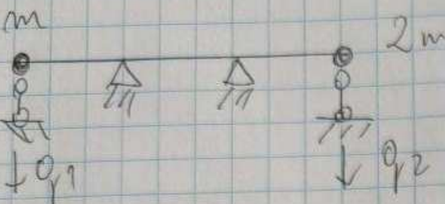
ZAD. 2. EGZAMIN MK2 18.06.2026r.

$P(t) = P_0 \sin \theta t$




$\theta = \sqrt{\frac{EJ}{ml^3}}$

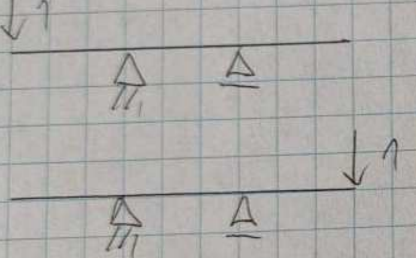
$EJ = const$ $\mu = 0$ $EA \to \infty$

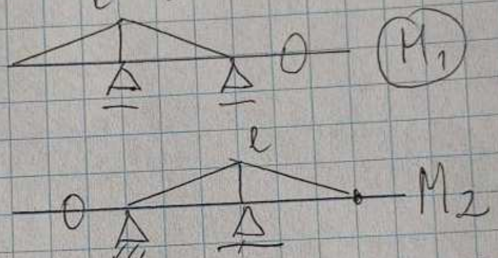



$\mathbb{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$

MACIERZ MAS $\mathbb{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}$

MACIERZ PODATNOŚCI $\mathbb{D} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$




$d_{11} = \int \frac{M_1^2}{EJ} ds = \frac{1}{EJ}\left[2 \cdot \frac{1}{2} \cdot l \cdot l \cdot \frac{2}{3} l\right] = \frac{2}{3}\frac{l^3}{EJ}$

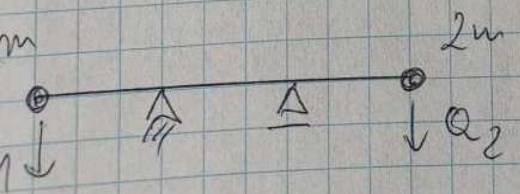
$d_{22} = \int \frac{M_2^2}{EJ} ds = \frac{2}{3}\frac{l^3}{EJ}$

$d_{12} = \int \frac{M_1 M_2}{EJ} ds = \frac{1}{EJ}\left[\frac{1}{2} l \cdot l \cdot \frac{1}{3} l\right] = \frac{1}{6}\frac{l^3}{EJ}$

$\mathbb{D} = \begin{bmatrix} \frac{2}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{3} \end{bmatrix} \frac{l^3}{EJ}$

a) częstość drgań własnych

$(\mathbb{1} - \omega^2 \cdot \mathbb{D} \cdot \mathbb{M})\,\mathbb{a} = 0$ $\mathbb{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$



c.d. ZAD. 2. EGZAMIN MK2 18.06.2026 r.

$$\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}-\underbrace{\omega^2 m\frac{l^3}{EJ}}_{x}\begin{bmatrix}\frac{2}{3}&\frac{1}{6}\\\frac{1}{6}&\frac{2}{3}\end{bmatrix}\begin{bmatrix}1&0\\0&2\end{bmatrix}\right)\begin{bmatrix}a_1\\a_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\det\begin{bmatrix}1-\frac{2}{3}x & -\frac{1}{3}x\\ -\frac{1}{6}x & 1-\frac{4}{3}x\end{bmatrix}=0$$

$$\frac{15}{18}x^2-2x+1=0 \Rightarrow \quad x_1=0{,}7101 \quad x_2=1{,}6899$$

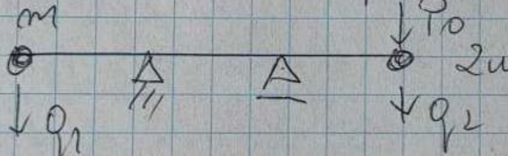
Odp.:

$$\omega_1=\sqrt{x_1}\cdot\sqrt{\frac{EJ}{ml^3}}=0{,}8427\sqrt{\frac{EJ}{ml^3}}$$

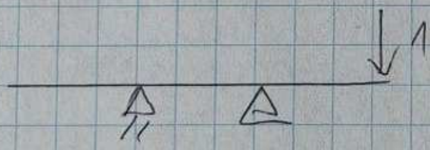
$$\omega_2=\sqrt{x_2}\sqrt{\frac{EJ}{ml^3}}=1{,}2999\sqrt{\frac{EJ}{ml^3}}$$

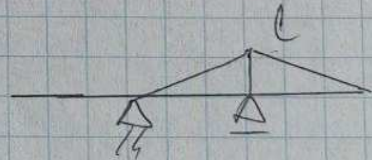
b) Amplituda przemieszczenia masy „m"

$$(\mathbb{1}-\theta^2\,\mathbb{D}\cdot\mathbb{M})\,\mathbb{q}=\mathbb{D}_0\cdot P_0$$




$$\mathbb{D}_0=\begin{bmatrix}d_{10}\\d_{20}\end{bmatrix}$$




$M_0=M_2$

$$d_{20}=\int\frac{M_2M_0}{EJ}ds=d_{22}=\frac{2}{3}\frac{l^3}{EJ}$$

$$d_{10}=\int\frac{M_1M_0}{EJ}ds=d_{12}=\frac{1}{6}\frac{l^3}{EJ}$$

$$\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}-\frac{EJ}{ml^3}\frac{l^3}{EJ}\cdot m\begin{bmatrix}\frac{2}{3}&\frac{1}{6}\\\frac{1}{6}&\frac{2}{3}\end{bmatrix}\begin{bmatrix}1&0\\0&2\end{bmatrix}\right)\begin{bmatrix}q_1\\q_2\end{bmatrix}=\begin{bmatrix}\frac{1}{6}\\\frac{2}{3}\end{bmatrix}\frac{P_0 l^3}{EJ}$$

$$\begin{bmatrix}\frac{1}{3}&-\frac{1}{3}\\-\frac{1}{6}&-\frac{1}{3}\end{bmatrix}\begin{bmatrix}q_1\\q_2\end{bmatrix}=\begin{bmatrix}\frac{1}{6}\\\frac{2}{3}\end{bmatrix}\frac{P_0l^3}{EJ} \qquad \mathbb{q}=\begin{bmatrix}-1\\-1{,}5\end{bmatrix}\frac{P_0l^3}{EJ}$$

odp. $q_1=-\frac{P_0l^3}{EJ}$