Egzamin pisemny z Mechaniki Konstrukcji II, 16 VI 2025 r.

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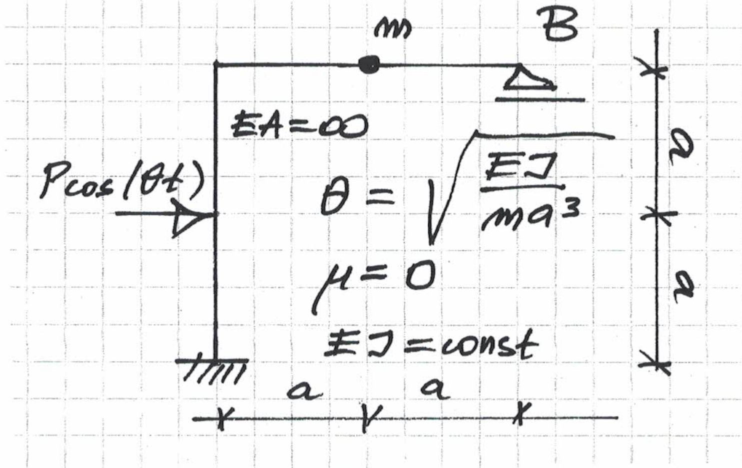
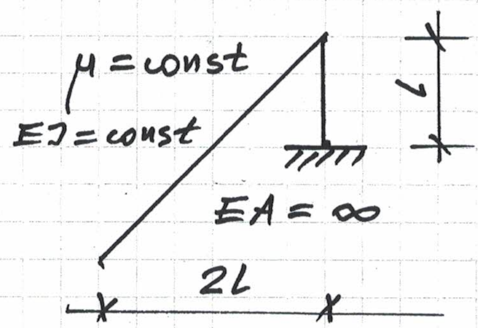
Zadanie 1

Dana jest rama z prętów nieważkich z masą skupioną, obciążona jak na rysunku. Znaleźć amplitudę reakcji pionowej w B
(Given is a frame of weightless bars with a concentrated mass, loaded as shown in the figure. Compute the amplitude of the vertical reaction at B.)



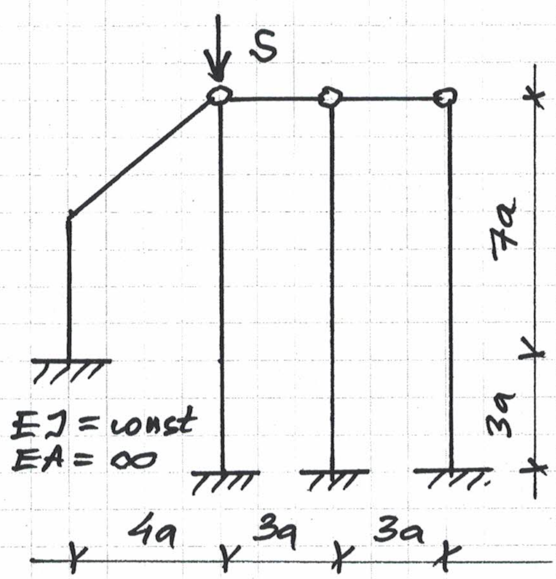
Zadanie 2

Zapisać równania określające pierwszą częstość drgań własnych danej ramy.
(For given frame write down the equations which make it possible to compute the first eigenfrequency)



Zadanie 3

Dana jest rama płaska obciążona dużą siłą osiową, jak na rys.
Zapisać równania utraty stateczności całej ramy.
(Given is a plane frame subjected to a big axial force; write down equations of the loss of stability of the frame)

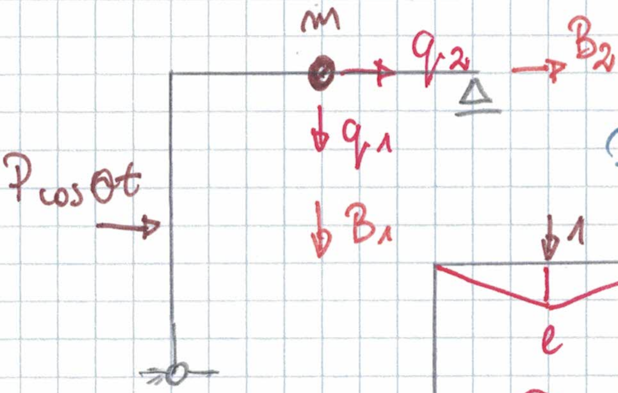


ZADANIE 1

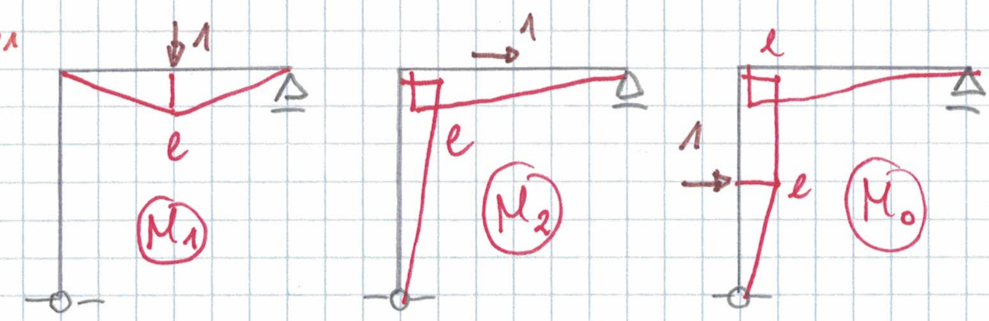
2 dyn. st. sw.

Macierz mas

$$\mathbb{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} m$$



Podatności



Macierz podatności

$$\mathbb{D} = \frac{l^3}{EJ} \begin{bmatrix} \frac{1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{16}{3} \end{bmatrix} \qquad \mathbb{D}_0 = \begin{bmatrix} \frac{1}{4} \\ \frac{19}{6} \end{bmatrix} \frac{l^3}{EJ}$$

Równanie ruchu

$$(\mathbb{I} - \theta^2 \mathbb{D}\mathbb{M})\, q = \mathbb{D}_0 P \Rightarrow q = \begin{bmatrix} -0{,}129 \\ -0{,}716 \end{bmatrix} \frac{P l^3}{EJ}$$

Amplitudy sił bezwładności

$$B_1 = \theta^2 q_1 m \qquad B_2 = \theta^2 q_2 m$$

Amplituda reakcji pionowej

$$V_B = \frac{P}{2} + \frac{B_1}{2} + B_2 = -0{,}281\, P$$

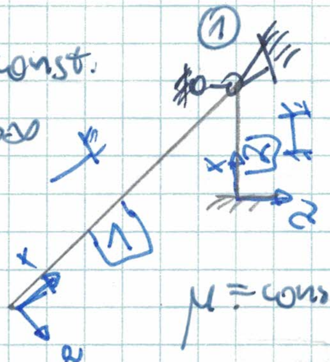
Oprac. S. Spodzieja

ZADANIE 2

UgW

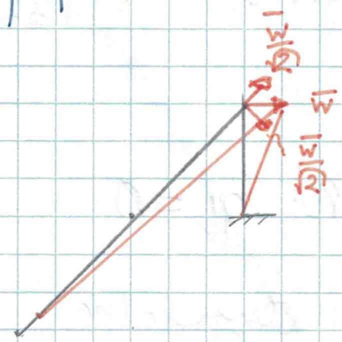
EJ = const.

EA → ∞



$\mu = const.$

P.P.



$$q = \begin{bmatrix} \varphi_1 \\ \frac{w}{l} \end{bmatrix}$$

$$\lambda^{(1)} = 2\sqrt{2}\, l \sqrt[4]{\frac{\mu\omega^2}{EJ}} = 2\sqrt{2}\,\lambda$$

$$\lambda^{(2)} = l \sqrt[4]{\frac{\mu\omega^2}{EJ}} = \lambda$$

$$\omega^2 = \frac{\lambda^4}{l^4}\frac{EJ}{\mu}$$

Pręt	$l^{(u)}$	$\lambda^{(u)}$	w_i	w_u	ψ
1	$2\sqrt{2}\,l$	$2\sqrt{2}\,\lambda$	X	$\frac{w}{\sqrt{2}}$	$\frac{w}{\sqrt{2}}$
2	l	λ	0	w	0

r. r. MP

$$\Phi_1^{(1)} + \Phi_1^{(2)} = 0 \qquad (1)$$

$$-\left(\overline{W}_1^{(1)} \frac{\bar{w}}{\sqrt{2}} + \overline{W}_1^{(2)}\bar{w}\right) + \mu\, 2\sqrt{2}\, l \cdot \omega^2 \frac{w}{\sqrt{2}} \cdot \frac{\bar{w}}{\sqrt{2}} = 0 \qquad (2)$$

WT

$$\Phi_1^{(1)} = \frac{EJ}{2\sqrt{2}\,l}\left[\alpha''(2\sqrt{2}\lambda)\,\varphi_1 - \theta''(2\sqrt{2}\lambda)\frac{w}{\sqrt{2}\cdot 2\sqrt{2}\,l}\right]$$

$$= \frac{EJ}{l}\left[\frac{1}{2\sqrt{2}}\alpha''(2\sqrt{2}\lambda)\,\varphi_1 - \frac{1}{8\sqrt{2}}\theta''(2\sqrt{2}\lambda)\frac{w}{l}\right]$$

$$\overline{W}_1^{(1)} = -\frac{EJ}{(2\sqrt{2}\,l)^2}\left[\theta''(2\sqrt{2}\lambda)\,\varphi_1 - \gamma''(2\sqrt{2}\lambda)\frac{w}{4l}\right] =$$

$$= \frac{EJ}{l}\left[-\frac{1}{8}\theta''(2\sqrt{2}\lambda)\,\varphi_1 + \frac{1}{32}\gamma''(2\sqrt{2}\lambda)\frac{w}{l}\right] \quad \left(\cdot\frac{1}{\sqrt{2}}\right)$$

$$\Phi_1^{(2)} = \frac{EJ}{l}\left[\alpha(\lambda)\,\varphi_1 - \theta(\lambda)\frac{w}{l}\right]$$

$$\overline{W}_1^{(2)} = -\frac{EJ}{l^2}\left[-\theta(\lambda)\,\varphi_1 + \gamma(\lambda)\frac{w}{l}\right] \quad (\cdot -1)$$

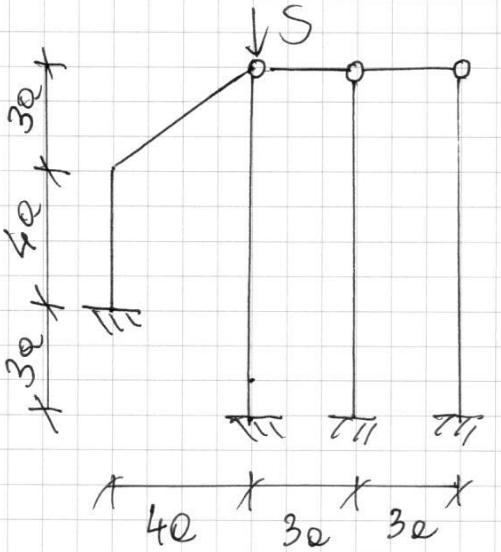
$$\frac{EJ}{\ell}\begin{bmatrix} \frac{1}{2\sqrt{\Omega}}\alpha''(2\sqrt{\Omega}\lambda)+\alpha(\lambda) & -\frac{1}{8\sqrt{\Omega}}\theta''(2\sqrt{\Omega}\lambda)-\theta(\lambda) \\ -\frac{1}{8\sqrt{\Omega}}\theta''(2\sqrt{\Omega}\lambda)-\theta(\lambda) & \frac{1}{32\sqrt{\Omega}}\gamma''(2\sqrt{\Omega}\lambda)+\gamma(\lambda)-\sqrt{\Omega}\,\lambda^4 \end{bmatrix}\begin{bmatrix}\varphi_1 \\ \frac{w}{\ell}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\mathbb{K}(\lambda)\, q = 0$$

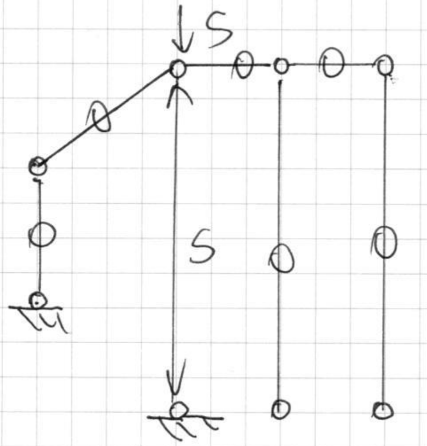
rozw. nietrywialne $\det \mathbb{K}(\lambda) = 0$

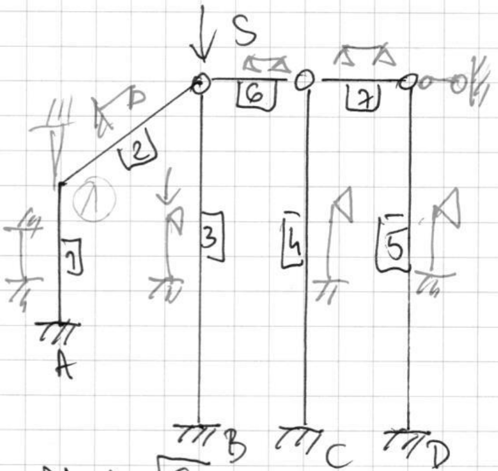
$\Rightarrow \lambda = \dots \Rightarrow \omega = \dots \frac{\lambda^2}{\ell^2}\sqrt{\frac{EJ}{\mu}}$

ZAD: 3. EGZAMIN MK2 ST 16. VI 2025



ZAPISAĆ RÓWNANIA UTRATY STATECZNOŚCI CAŁEJ RAMY



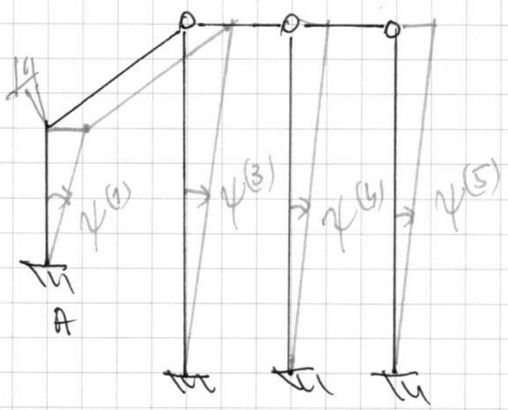


MGW

$$q = \begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix}$$

$\sigma = a\sqrt{\frac{S}{EJ}}$

PLAN PRZEM.



$l^{(1)} = 4a$		$S^{(1)} = 0$	$\sigma^{(1)} = 0$	$\psi^{(1)} = \psi$
$l^{(2)} = 5a$		$S^{(2)} = 0$	$\sigma^{(2)} = 0$	$\psi^{(2)} = 0$
$l^{(3)} = 10a$		$S^{(3)} = S$	$\sigma^{(3)} = 10\sigma$	$\psi^{(3)} = \psi$
$l^{(4)} = 10a$	EJ	$S^{(4)} = 0$	$\sigma^{(4)} = 0$	$\psi^{(4)} = \psi$
$l^{(5)} = 10a$		$S^{(5)} = 0$	$\sigma^{(5)} = 0$	$\psi^{(5)} = \psi$
$l^{(6)} = 3a$		$S^{(6)} = 0$	$\sigma^{(6)} = 0$	$\psi^{(6)} = 0$
$l^{(7)} = 3a$		$S^{(7)} = 0$	$\sigma^{(7)} = 0$	$\psi^{(7)} = 0$

$\psi^{(1)} = \psi$

$\psi^{(3)} = \psi^{(4)} = \psi^{(5)} = \frac{2}{5}\psi$

RÓWNANIA RÓWNOWAGI:

1) $\sum M_1 = 0 \Rightarrow \phi_1^{(1)} + \phi_1^{(2)} = 0$

ŻAD. 3. EG2. MK2 ST 16. VI 2025

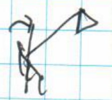
2) $$\left(\phi_A^{(1)}+\phi_1^{(1)}\right)\cdot\bar{\psi}+\phi_B^{(3)}\cdot\frac{2}{5}\bar{\psi}+\phi_C^{(4)}\cdot\frac{2}{5}\bar{\psi}+\phi_D^{(5)}\cdot\frac{2}{5}\bar{\psi}+$$
$$+S\cdot 10a\cdot\frac{2}{5}\psi\cdot\frac{2}{5}\bar{\psi}=0$$

$$\bar{\psi}=-1 \Rightarrow -\phi_A^{1}-\phi_1^{1}-\frac{2}{5}\phi_B^{(3)}-\frac{2}{5}\phi_C^{4}-\frac{2}{5}\phi_D^{(5)}+$$
$$-\frac{8}{5}S\cdot a\cdot\psi=0$$

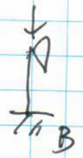
$$S=\frac{\sigma^2 EJ}{a^2}$$

WZORY TRANSFORMACYJNE:

$$\phi_1^{(1)}=\frac{2EJ}{4a}\left(2\varphi_1-3\psi\right)=\frac{EJ}{a}\left(\varphi_1-\frac{3}{2}\psi\right)$$
$$\phi_A^{(1)}=\frac{2EJ}{4a}\left(\varphi_1-3\psi\right)=\frac{EJ}{a}\left(\frac{1}{2}\varphi_1-\frac{3}{2}\psi\right)$$

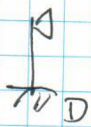


$$\phi_1^{(2)}=\frac{3EJ}{5a}\varphi_1=\frac{EJ}{a}\frac{3}{5}\varphi_1$$



$$\phi_B^{(3)}=\frac{EJ}{10a}\left[\alpha'(10\sigma)\cdot\left(-\frac{2}{5}\psi\right)\right]=\frac{EJ}{a}\left[-\frac{\alpha'(10\sigma)}{25}\psi\right]$$

$$\phi_C^{(4)}=\frac{3EJ}{10a}\left[-\frac{2}{5}\psi\right]=\frac{EJ}{a}\left[-\frac{3}{25}\psi\right]$$



$$\phi_D^{(5)}=\frac{EJ}{a}\left[-\frac{3}{25}\psi\right]$$

$$\frac{EJ}{a}\begin{bmatrix}1+\frac{3}{5} & -\frac{3}{2}\\ -\frac{3}{2} & 3+\frac{2}{5}\cdot\frac{\alpha'(10\sigma)}{25}+\frac{2}{5}\cdot\frac{3}{25}+\frac{2}{5}\cdot\frac{3}{25}-\frac{8}{5}\sigma^2\end{bmatrix}\begin{bmatrix}\varphi_1\\ \psi\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\det\begin{bmatrix}\frac{8}{5} & -\frac{3}{2}\\ -\frac{3}{5} & \frac{387}{125}+\frac{2}{125}\alpha'(10\sigma)-\frac{8}{5}\sigma^2\end{bmatrix}=0$$