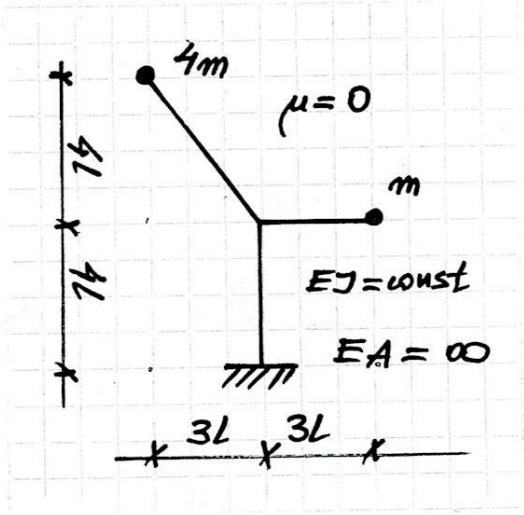
Egzamin pisemny z Mechaniki Konstrukcji II, 27 VI 2022 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1

Dana jest rama płaska jak na rysunku;
Zapisać równania określające
częstości drgań własnych

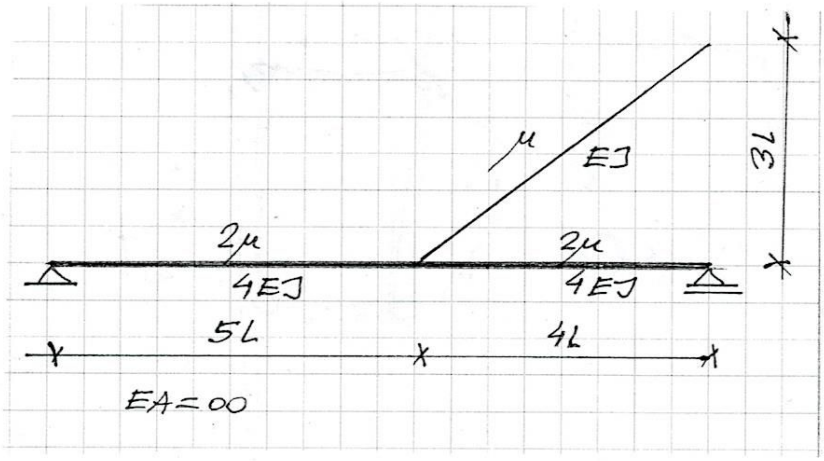
(Given is the plane frame as in the figure;
write down the equations
which determine
the eigenfrequencies)



Zadanie 2

Dana jest rama o odcinkowo
zmiennej sztywności
i masie, por. rysunek.
Zapisać równania określające
pierwszą częstość drgań własnych.

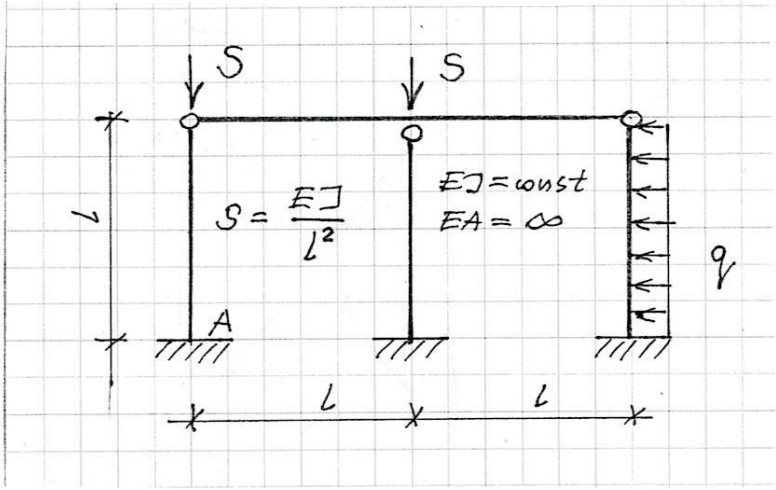
(*Given is the beam*
of bar-wise varying stiffness
and the mass density, see the figure.
Write down the equations
which determine
the first circular eigenfrequency).



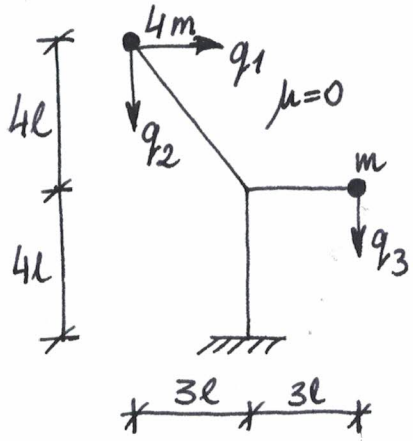
Zadanie 3

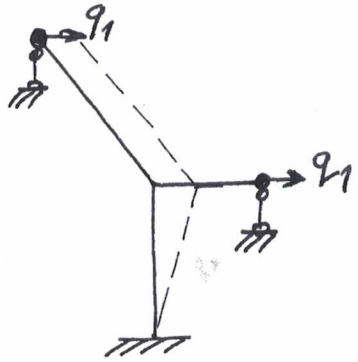
Dana jest rama poddana obciążeniu *q*
oraz dużym siłom osiowym S,
por. rysunek.
Obliczyć moment zginający
w utwierdzeniu A.

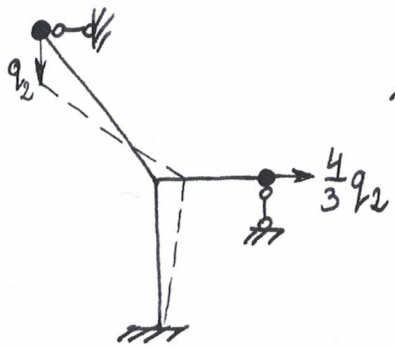
(Given is the frame
of inextensional bars, subject
to the big axial forces S
and –to the distributed
transverse load of intensity q,
see the figure.
Compute the bending moment
at the clamped node A.).

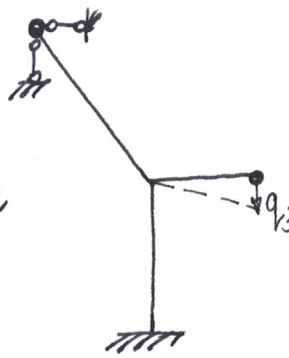


zadanie 1



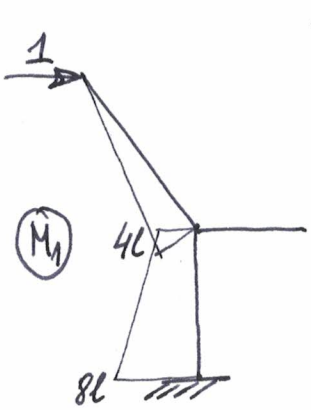


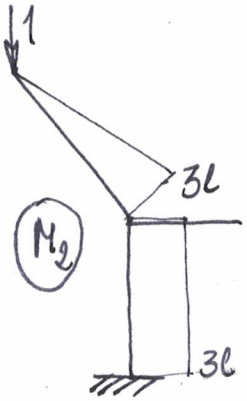


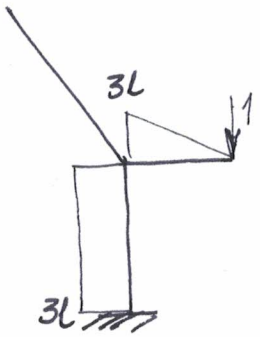


$$E_k = \frac{1}{2}\left(4m\cdot(\dot{q}_1^2+\dot{q}_2^2)+m\cdot\left((\dot{q}_1+\tfrac{4}{3}\dot{q}_2)^2+\dot{q}_3^2\right)\right)=\frac{1}{2}\cdot m\left[5\dot{q}_1^2+\tfrac{4}{3}\dot{q}_1\dot{q}_2+\tfrac{52}{9}\dot{q}_2^2+\dot{q}_3^2\right]$$

$$\mathbb{M}=\begin{bmatrix}5 & \frac{4}{3} & 0\\ \frac{4}{3} & \frac{52}{9} & 0\\ 0 & 0 & 1\end{bmatrix} \qquad q=\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix}$$







$$d_{11}=176\frac{l^3}{EJ}$$
$$d_{12}=-92\frac{l^3}{EJ}$$
$$d_{13}=72\frac{l^3}{EJ}$$
$$d_{22}=51\frac{l^3}{EJ}$$
$$d_{23}=-36\frac{l^3}{EJ}$$
$$d_{33}=45\frac{l^3}{EJ}$$

$$\mathbb{D}=\frac{l^3}{EJ}\begin{bmatrix}176 & -92 & 72\\ -92 & 51 & -36\\ 72 & -36 & 45\end{bmatrix}$$

$$(\mathbb{I}-\omega^2\mathbb{D}\mathbb{M})q=0 \qquad \lambda=\frac{\omega^2 m l^3}{EJ}$$

$$\det\begin{bmatrix}1-\frac{2272}{9}\lambda & \frac{2672}{9}\lambda & -72\lambda\\ 392\lambda & 1-172\lambda & 36\lambda\\ -312\lambda & 112\lambda & 1-45\lambda\end{bmatrix}=0 \rightarrow \lambda^{(1)},\lambda^{(2)},\lambda^{(3)}$$

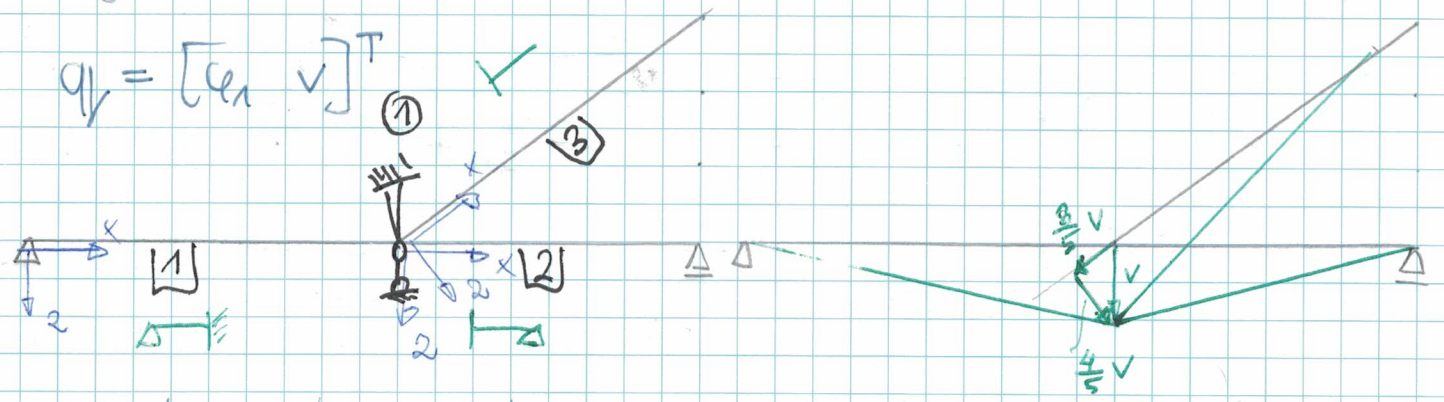
opracował Jan Perczyński

ZADANIE 2

UgW

$q_y = [\varphi_1 \; v]^T$

plan przemieszczeń



Pręt	w_i	w_u	u	λ
1	0	v	0	$\frac{5}{\sqrt{2}}\lambda$
2	v	0	0	$\frac{4}{\sqrt{2}}\lambda$
3	$\frac{4}{5}v$	—	$-\frac{3}{5}v$	5λ

$$\lambda = l\sqrt[4]{\frac{\mu\omega^2}{EJ}}$$

r. w. MP

$$\underline{\Phi}_1^{(1)} + \underline{\Phi}_1^{(2)} + \underline{\Phi}_1^{(3)} = 0$$

$$-\left(\underline{W}_1^{(1)}\bar{v} + \underline{W}_1^{(2)}\bar{v} + \underline{W}_1^{(3)}\frac{4}{5}\bar{v}\right) + \omega^2 5l\mu\left(-\frac{3}{5}v\right)\left(-\frac{3}{5}\bar{v}\right) = 0$$

WT

$$\underline{\Phi}_1^{(1)} = \frac{EJ}{l}\left[\frac{4}{5}\alpha'(\lambda_1)\varphi_1 - \frac{4}{25}\Theta'(\lambda_1)\frac{v}{l}\right]$$

$$\underline{\Phi}_1^{(2)} = \frac{EJ}{l}\left[\alpha'(\lambda_2)\varphi_1 + \frac{1}{4}\Theta'(\lambda_2)\frac{v}{l}\right]$$

$$\underline{\Phi}_1^{(3)} = \frac{EJ}{l}\left[\frac{1}{5}\alpha''(\lambda_3)\varphi_1 + \frac{4}{125}\Theta''(\lambda_3)\frac{v}{l}\right]$$

$$\underline{W}_1^{(1)} = \frac{EJ}{l^2}\left[-\frac{4}{25}\Theta'(\lambda_1)\varphi_1 + \frac{4}{125}\gamma'(\lambda_1)\frac{v}{l}\right]$$

$$\underline{W}_1^{(2)} = \frac{EJ}{l^2}\left[\frac{1}{4}\Theta'(\lambda_2)\varphi_1 + \frac{1}{10}\gamma'(\lambda_2)\frac{v}{l}\right]$$

$$\underline{W}_1^{(3)} = \frac{EJ}{l^2}\left[\frac{1}{25}\Theta''(\lambda_3)\varphi_1 + \frac{4}{625}\gamma''(\lambda_3)\frac{v}{l}\right]$$

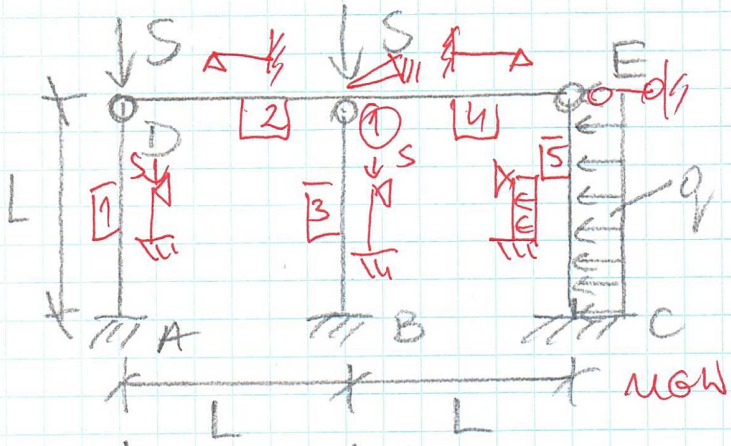
$$\frac{Eg}{e}\left|\begin{array}{cc} \frac{4}{5}\alpha'(x_1)+\alpha'(x_2)+\frac{1}{5}\alpha''(x_3) & -\frac{4}{25}\theta'(x_1)+\frac{1}{4}\theta'(x_2)+\frac{4}{125}\theta''(x_3) \\ -\frac{4}{25}\theta'(x_1)+\frac{1}{4}\theta'(x_2)+\frac{4}{125}\theta''(x_3) & \frac{4}{125}\gamma'(x_1)+\frac{1}{20}\gamma'(x_2)+\frac{16}{3\cdot125}\gamma''(x_3) \end{array}\right|\left[\begin{array}{c} \varphi_1 \\ \frac{V}{e} \end{array}\right]=\left[\begin{array}{c} 0 \\ 0 \end{array}\right]$$

$$-\frac{9}{5}x^4$$

$$|K(x)|\,q_1=0$$

$$\det|K(x)|=0 \Rightarrow x_i=\ldots \Rightarrow \omega_i=\ldots$$

ZAD. 3. EGZAMIN MK2 27.06.2022



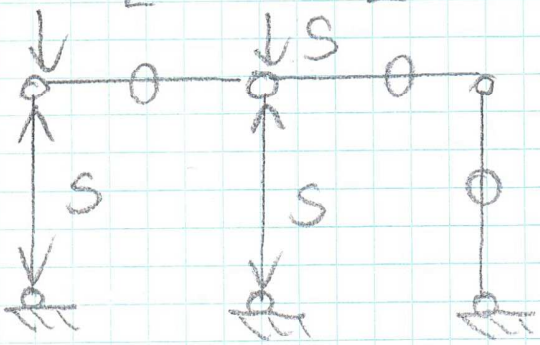
$S = \frac{EJ}{L^2}$ $\quad \delta = L\sqrt{\frac{S}{EJ}} = 1$

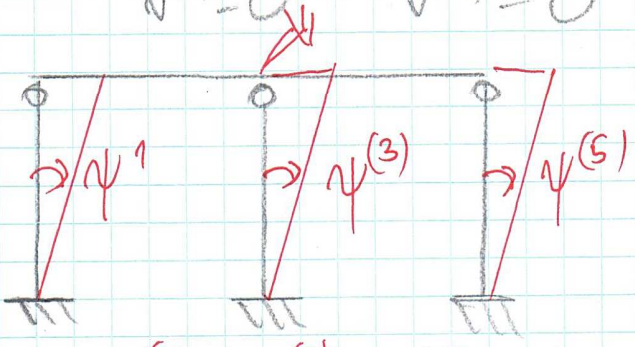
$EJ = const$ $\quad q\| = \begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix}$

$EA \to \infty$

$\delta^{(1)} = 1 \quad \delta^{(3)} = 1 \quad \delta^{(5)} = 1$

$\delta^{(2)} = 0 \quad \delta^{(4)} = 0$





$\psi^{(1)} = \psi^{(3)} = \psi^{(5)} = \psi$

RÓWNANIA RÓWNOWAGI

1) $\Sigma M_1 = 0 \quad \phi_1^{(2)} + \phi_1^{(4)} = 0$

2) $\phi_A^{(1)} \bar{\psi} + \phi_B^{(3)} \bar{\psi} + \phi_C^{(5)} \cdot \bar{\psi} + S \cdot L \cdot \psi \cdot \bar{\psi} \cdot 2 + L_2 = 0$

$L_2 = -q \cdot L \cdot \frac{L}{2} \cdot \psi$

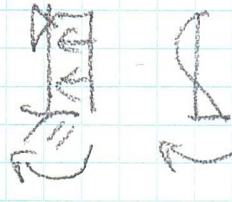
$\bar{\psi} = -1 \Rightarrow -\phi_A^{(1)} - \phi_B^{(3)} - \phi_C^{(5)} - 2 \cdot \frac{EJ}{L^2} \cdot L \cdot \psi + \frac{qL^2}{2} = 0$

WZORY TRANSFORMACYJNE

$\phi_A^{(1)} = \frac{EJ}{L}[\alpha'(1)(-\psi)] = \frac{EJ}{L}[-2{,}784\,\psi]$

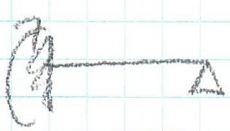
$\phi_B^{(3)} = \frac{EJ}{L}[\alpha'(1)(-\psi)] = \frac{EJ}{L}[-2{,}784\,\psi]$

c.d. ZAD. 3 EGZAMIN MK2 27.06.2022



$$\phi_C^{(5)} = \frac{3EJ}{L}[-\psi] + \frac{ql^2}{8}$$

$$\phi_1^{(2)} = \frac{3EJ}{L}[\varphi_1]$$



$$\phi_1^{(4)} = \frac{3EJ}{L}[\varphi_1]$$

$$\frac{EJ}{L}\begin{bmatrix} 3+3 & 0 \\ 0 & 2\cdot 2{,}784+3 \quad -2 \end{bmatrix}\begin{bmatrix}\varphi_1 \\ \psi\end{bmatrix} = \begin{bmatrix} 0 \\ +\frac{1}{8} \quad -\frac{1}{2}\end{bmatrix} ql^2$$

$$\frac{EJ}{L}\begin{bmatrix} 6 & 0 \\ 0 & 6{,}588\end{bmatrix}\begin{bmatrix}\varphi_1 \\ \psi\end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{3}{8}\end{bmatrix} ql^2$$

$$\begin{bmatrix}\varphi_1 \\ \psi\end{bmatrix} = \begin{bmatrix} 0 \\ -0{,}0569\end{bmatrix}\frac{ql^3}{EJ}$$

$$M_A = \phi_A^{(1)} = \frac{EJ}{L}\left[-2{,}784\cdot\left(-0{,}0569\,\frac{ql^3}{EJ}\right)\right] = 0{,}159\,ql^2$$