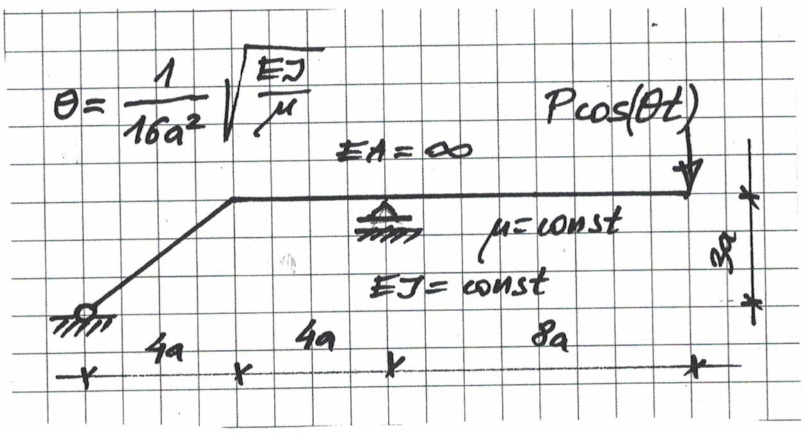
Egzamin z Mechaniki Konstrukcji II, 12 II 2020 r.
Wydział Inżynierii Lądowej, studia stacjonarne

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

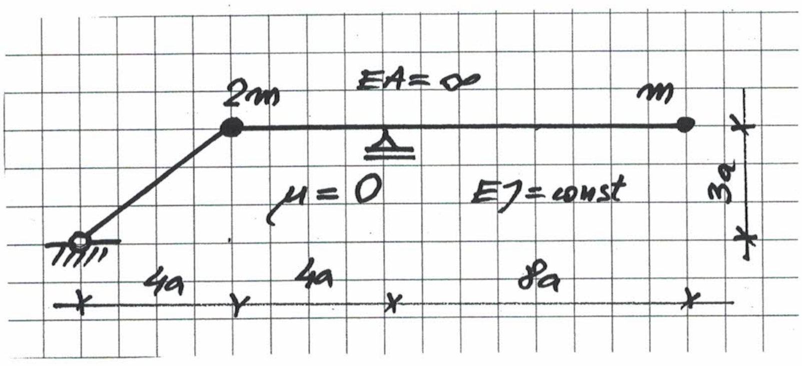
Zadanie 1

Dana jest rama jak na rysunku, pod obciążeniem harmonicznie zmiennym w czasie. Zapisać równania rozwiązujące to zagadnienie drgań.
(The given frame is subject to the harmonic load, cf. the figure. Write the down the governing equations of the problem)



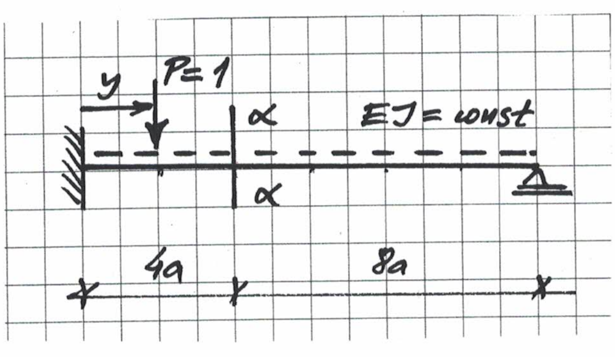
Zadanie 2

Dana jest nieważka rama (z masami skupionymi). Zapisać równania rozwiązujące zagadnienie drgań własnych. *(There is given a weightless frame with concentrated masses. Write down the equations solving the problem of eigenvibrations)*

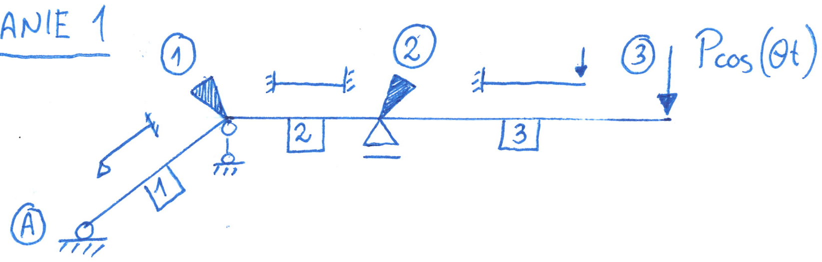


Zadanie 3

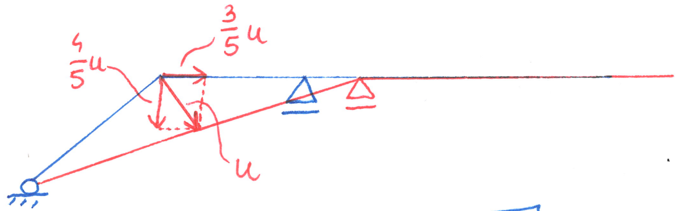
Sporządzić linię wpływu siły poprzecznej w zaznaczonym przekroju.
(Construct the influence line of the transverse force at the given cross-section.)



ZADANIE 1



$$q = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \frac{u}{a} \end{bmatrix}$$



$$\Theta = \frac{1}{16a^2}\sqrt{\frac{EJ}{\mu}} \qquad \lambda = a\sqrt[4]{\frac{\mu\Theta^2}{EJ}} = \frac{1}{4}$$

	w_i	w_k	u
1	0	u	0
2	$\frac{4}{5}u$	0	$\frac{3}{5}u$
3	0	$-$	$\frac{3}{5}u$

$$\lambda_1 = 5\lambda = \frac{5}{4}$$

$$\lambda_2 = 4\lambda = 1$$

$$\lambda_3 = 8\lambda = 2$$

$$B_{11}^1 = \mu \cdot 5a \cdot \Theta^2 \cdot u^1 = 0$$

$$B_{11}^2 = \mu \cdot 4a \cdot \Theta^2 \cdot u^2 = \frac{3}{320}\frac{EJ}{a^2}\frac{u}{a}$$

$$B_{11}^3 = \mu \cdot 8l \cdot \Theta^2 \cdot u^3 = \frac{3}{160}\frac{EJ}{a^2}\frac{u}{a}$$

1) $\Phi_1^1 + \Phi_1^2 = 0$

2) $\Phi_2^2 + \Phi_2^3 = 0$

3) $W_1^1 \cdot \bar{u} + W_1^2 \cdot \frac{4}{5}\bar{u} - B_{11}^2 \cdot \frac{3}{5}\bar{u} - B_{11}^3 \cdot \frac{3}{5}\bar{u} = 0$

$$\Phi_1^1 = \frac{EJ}{5a}\left[\alpha'\left(\frac{5}{4}\right)\varphi_1 - \vartheta'\left(\frac{5}{4}\right)\frac{u}{5a}\right] = \frac{EJ}{a}\left[0{,}590\,\varphi_1 - 0{,}112\,\frac{u}{a}\right]$$

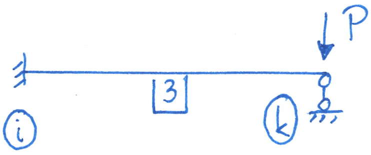
$$\Phi_1^2 = \frac{EJ}{4a}\left[\alpha(1)\varphi_1 + \beta(1)\varphi_2 + \vartheta(1)\frac{\frac{4}{5}u}{4a}\right] = \frac{EJ}{a}\left[0{,}998\,\varphi_1 + 0{,}502\,\varphi_2 + 0{,}297\,\frac{u}{a}\right]$$

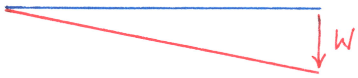
$$\Phi_2^2 = \frac{EJ}{4a}\left[\beta(1)\varphi_1 + \alpha(1)\varphi_2 + \delta(1)\frac{\frac{4}{5}u}{4a}\right] = \frac{EJ}{a}\left[0{,}502\,\varphi_1 + 0{,}998\,\varphi_2 + 0{,}302\,\frac{u}{a}\right]$$

$$\Phi_2^3 = \frac{EJ}{8a}\left[\alpha''(2)\varphi_2\right] + \Phi_2^{03} = \frac{EJ}{a}\left[2{,}179\,\varphi_2\right] + 32{,}070\,Pa$$

$$W_1^1 = -\frac{EJ}{(5a)^2}\left[\vartheta'\left(\frac{5}{4}\right)\varphi_1 - \gamma'\left(\frac{5}{4}\right)\frac{u}{5a}\right] = \frac{EJ}{a^2}\left[-0{,}112\,\frac{u}{a} + 0{,}014\,\frac{u}{a}\right]$$

$$W_1^2 = \frac{EJ}{(4a)^2}\left[\vartheta(1)\varphi_1 + \delta(1)\varphi_2 + \gamma(1)\frac{\frac{4}{5}u}{4a}\right] = \frac{EJ}{a^2}\left[0{,}372\,\varphi_1 + 0{,}377\,\varphi_2 + 0{,}145\,\frac{u}{a}\right]$$





$$W_k^3 \cdot \bar{w} - P \cdot \bar{w} = 0$$

$$W_k^3 = -\frac{EJ}{(8a)^2}\left[-\varkappa'(2)\frac{w}{8a}\right] = P$$

$$\bar{\Phi}_2^{03} = \Phi_i^3 = \frac{EJ}{8a}\left[-\delta'(2)\frac{w}{8a}\right] = -\frac{\overset{3,680}{\delta'(2)}}{\underset{-0,918}{\varkappa'(2)}} P \cdot 8a$$

$$\bar{\Phi}_2^{03} = 32{,}070\ Pa$$

$$\mathbb{K} = \frac{EJ}{a}\begin{bmatrix} \frac{1}{5}\alpha'\left(\frac{5}{4}\right) + \frac{1}{4}\alpha(1) & \frac{1}{4}\beta(1) & -\frac{1}{25}\vartheta'\left(\frac{5}{4}\right) + \frac{1}{20}\vartheta(1) \\ \frac{1}{4}\beta(1) & \frac{1}{4}\alpha(1) + \frac{1}{8}\alpha''(2) & \frac{1}{20}\delta(1) \\ -\frac{1}{25}\vartheta'\left(\frac{5}{4}\right) + \frac{1}{20}\vartheta(1) & \frac{1}{20}\delta(1) & \frac{1}{125}\gamma'\left(\frac{5}{4}\right) + \frac{1}{100}\gamma(1) - \frac{27}{1600} \end{bmatrix}$$

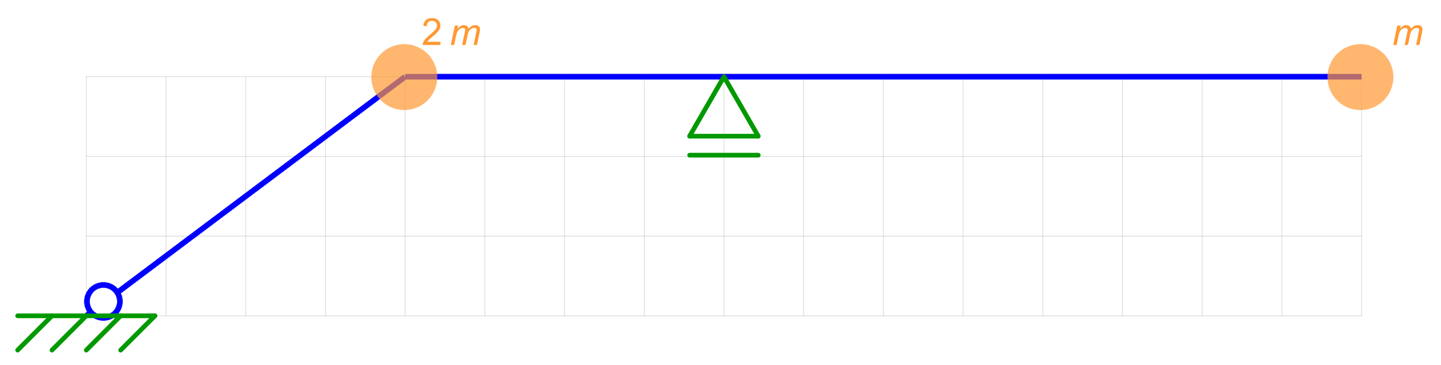
$$\mathbb{K} \approx \frac{EJ}{a}\begin{bmatrix} 1{,}588 & 0{,}502 & 0{,}186 \\ 0{,}502 & 3{,}177 & 0{,}302 \\ 0{,}186 & 0{,}302 & 0{,}114 \end{bmatrix} \qquad \mathbb{q} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \frac{u}{a} \end{bmatrix} \qquad \mathbb{Q} = \begin{bmatrix} 0 \\ 32{,}070 \\ 0 \end{bmatrix} Pa$$

$$\mathbb{K}\mathbb{q} + \mathbb{Q} = \mathbb{O}$$

Egzamin MKII/MoSII 12.02.2020, zadanie 2.

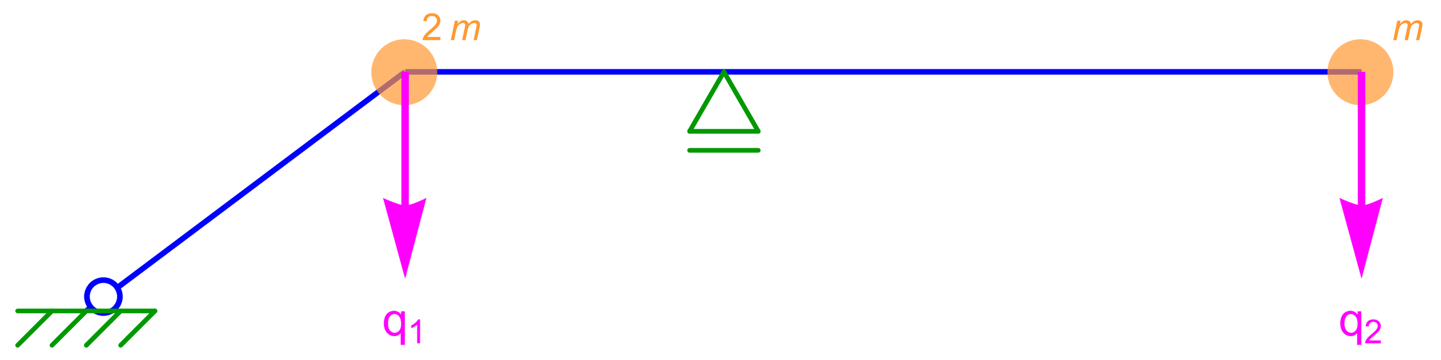
Ułożyć równania określające drgania własne ramy:

Geometria oraz obciążenia konstrukcji (wymiar oczka siatki – l):

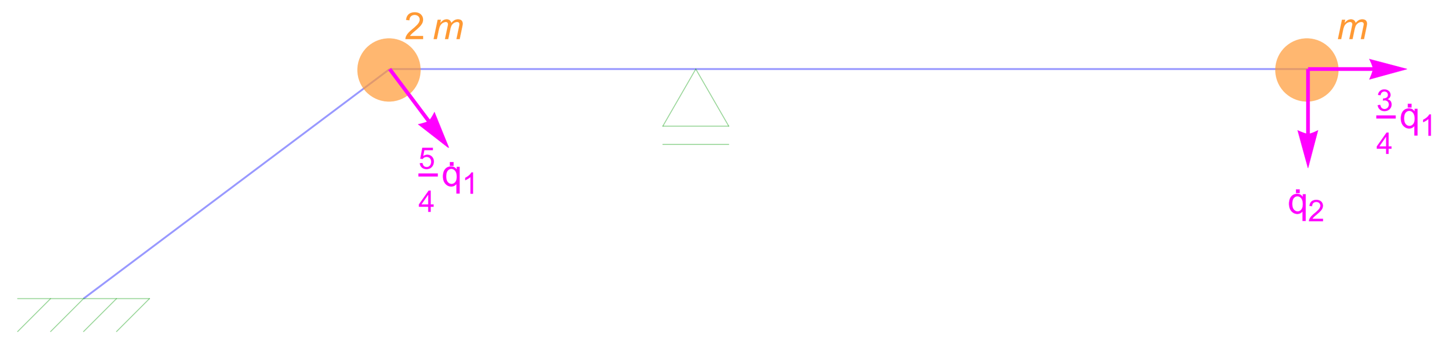


Zadanie statyki konstrukcji jest statycznie wyznaczalne.

Współrzędne Lagrange'a:



Plan predkosci:



Energia kinetyczna jako forma kwadratowa wektora $\dot{\mathbf{q}}$:

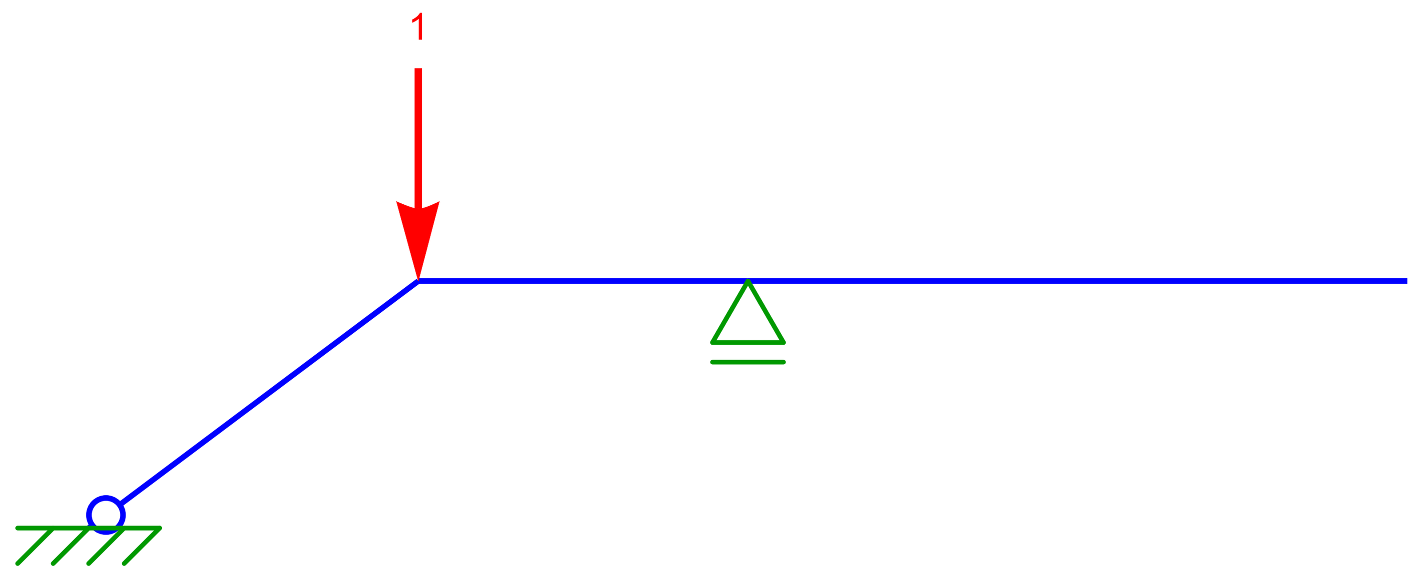
$$2\,E_k\,(\dot{\mathbf{q}}) = 2\,m\,\left(\frac{5}{4}\,\dot{q}_1\right)^2 + m\left[\dot{q}_2^{\,2} + \left(\frac{3}{4}\,\dot{q}_1\right)^2\right] = \frac{59}{16}\,m\,\dot{q}_1^{\,2} + m\,\dot{q}_2^{\,2} = \dot{\mathbf{q}}^T\,\mathbf{M}\,\dot{\mathbf{q}}$$

Macierz mas:

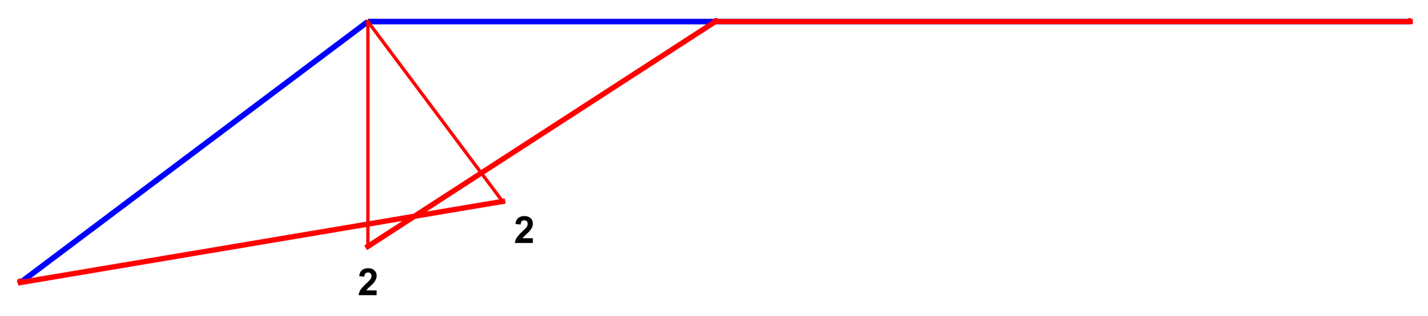
$$\mathbf{M} = m\begin{pmatrix} \frac{59}{16} & 0 \\ 0 & 1 \end{pmatrix}$$

Wykresy momentów zginających od jednostkowych sił bezwładności:

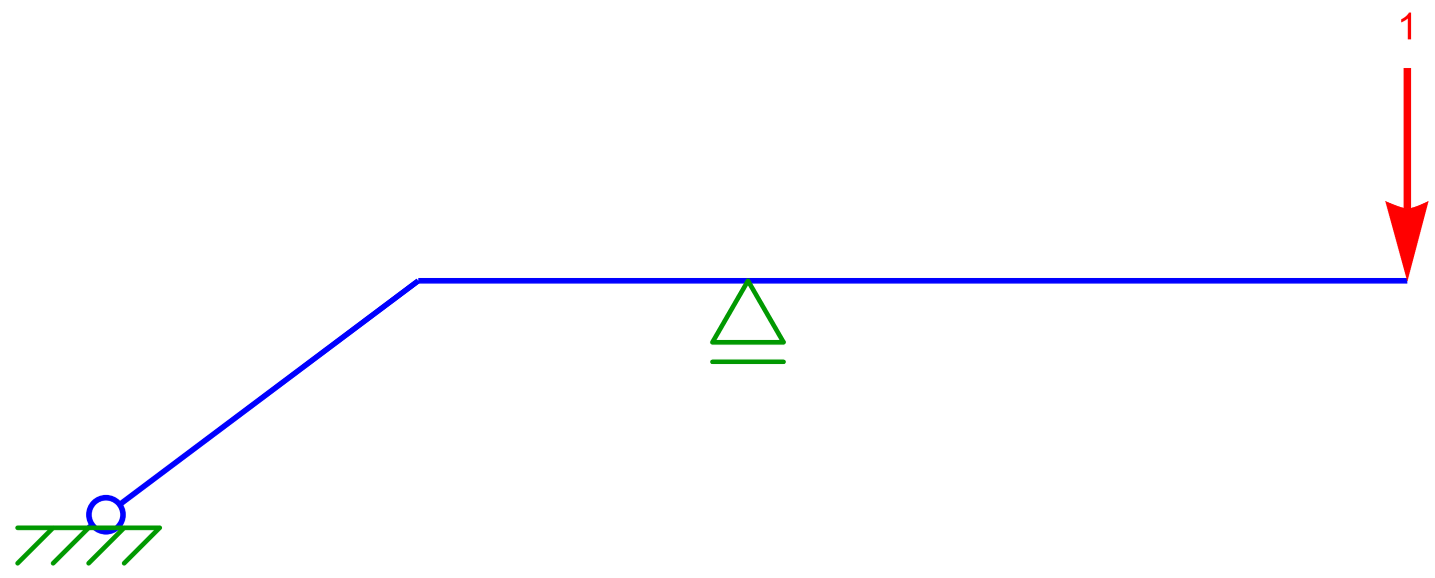
– od q_1:



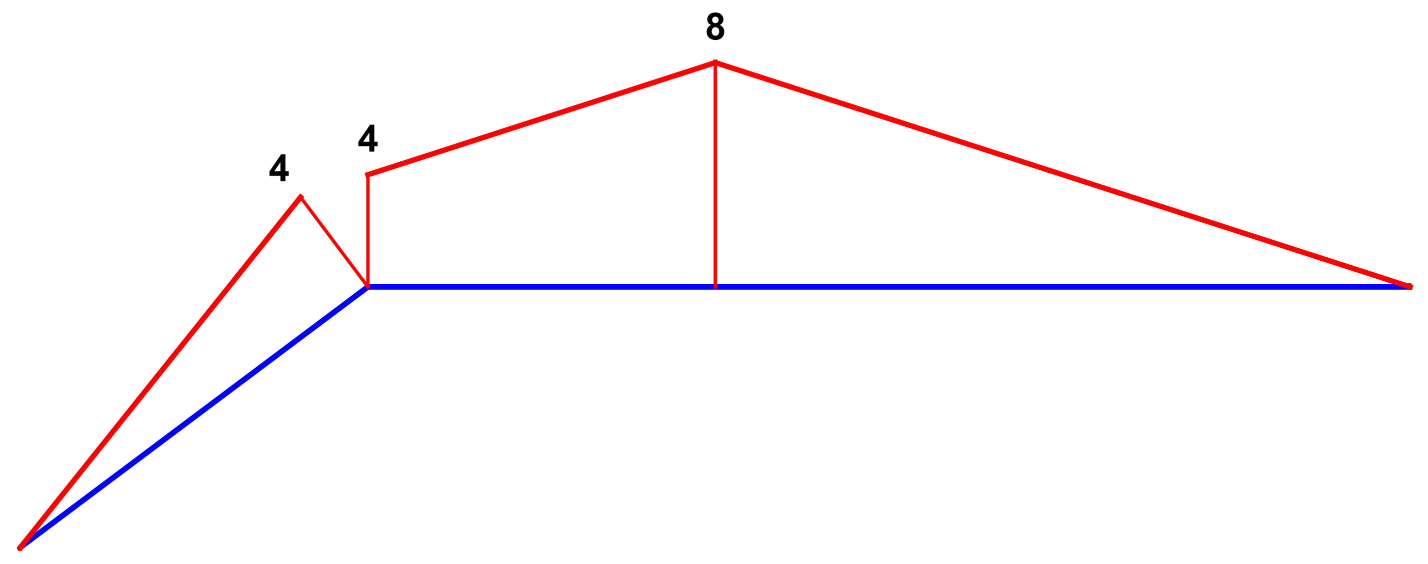
$M_1\,[l]$:



– od q_2:



$M_2\,[l]$:



Macierz podatności:

$$d_{11} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot 2l\cdot 5\,l\right)\left(\frac{2}{3}\cdot 2\,l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot 2l\cdot 4\,l\right)\left(\frac{2}{3}\cdot 2l\right)\right]_2 = 12\,\frac{l^3}{EJ}$$

$$d_{12} = d_{21} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot 2l\cdot 5\,l\right)\left(\frac{2}{3}\cdot(-4l)\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot 2l\cdot 4\,l\right)\left(\frac{2}{3}\cdot(-4l)+\frac{1}{3}\cdot(-8l)\right)\right]_2 = -\frac{104}{3}\,\frac{l^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot 4l\cdot 5\,l\right)\left(\frac{2}{3}\cdot 4l\right)\right]_1 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot 4l\cdot 4\,l\right)\left(\frac{2}{3}\cdot 4l+\frac{1}{3}\cdot 8l\right)+\left(\frac{1}{2}\cdot 8l\cdot 4\,l\right)\left(\frac{1}{3}\cdot 4l+\frac{2}{3}\cdot 8l\right)\right]_2 + \frac{1}{EJ}\left[\left(\frac{1}{2}\cdot 8l\cdot 8\,l\right)\left(\frac{2}{3}\cdot 8l\right)\right]_3 = \frac{1040}{3}\,\frac{l^3}{EJ}$$

$$\mathbf{D} = \frac{l^3}{EJ}\begin{pmatrix} 12 & -\frac{104}{3} \\ -\frac{104}{3} & \frac{1040}{3} \end{pmatrix}$$

ZADANIE DRGAŃ WŁASNYCH:

– poszukiwanie funkcji przemieszczeń postaci:

$$\mathbf{q}\,(t) = \mathbf{a}\,\sin(\omega t)$$

– zadanie własne:

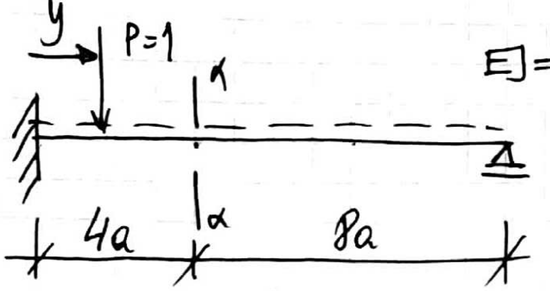
$$(\,\mathbf{I} - \omega^2\,\mathbf{D}\,\mathbf{M}\,)\,\mathbf{a} = \mathbf{0}$$

– równanie charakterystyczne $\left(\lambda = \frac{\omega^2\,l^3\,m}{EJ}\right)$:

$$\det(\,\mathbf{I} - \omega^2\,\mathbf{D}\,\mathbf{M}\,) = 0$$

$$\det\left(\begin{pmatrix} 1-\frac{177\,\lambda}{4} & \frac{104\,\lambda}{3} \\ \frac{767\,\lambda}{6} & 1-\frac{1040\,\lambda}{3} \end{pmatrix}\right) = 1 - \frac{4691\,\lambda}{12} + \frac{98\,176\,\lambda^2}{9} = 0$$

Zadanie przygotował Karol Bołbotowski.



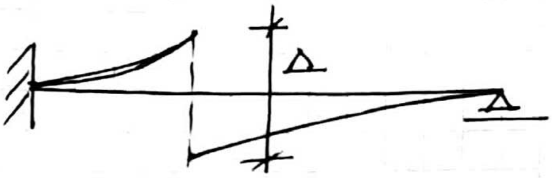
$EJ = const$

z twierdzenia Bettiego

$$P \cdot W(y) - T_\alpha(y) \cdot \Delta = 0$$

$$\frac{1}{\Delta} W(y) = T_\alpha(y) = \ln T_\alpha$$

$$\xi = \frac{x}{12a}$$



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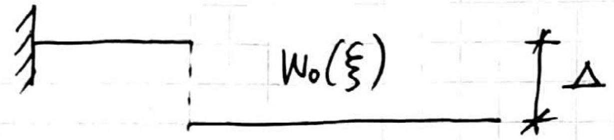
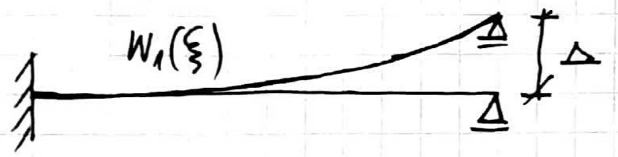


$$W_0(\xi) = \begin{cases} 0 & \text{dla } \xi \leqslant \frac{1}{3} \\ 1 \cdot \Delta & \text{dla } \xi \geqslant \frac{1}{3} \end{cases}$$

+



$$w_1(\xi) = C_0 + C_1\xi + C_2\xi^2 + C_3\xi^3$$

$$\left.\begin{array}{l} W_1(0) = 0 \rightarrow C_0 = 0 \\ W_1(1) = -\Delta \\ \varphi_1(0) = 0 \rightarrow \frac{1}{12a} W'(0) = 0 \rightarrow C_1 = 0 \\ M_1(1) = 0 \rightarrow -\frac{EJ}{(12a)^2} w''(1) = 0 \end{array}\right\} \rightarrow W(\xi) = -\frac{3}{2}\Delta\xi^2 + \frac{1}{2}\Delta\xi^3$$

$$W(\xi) = W_1(\xi) + W_0(\xi) = -\frac{3}{2}\Delta\xi^2 + \frac{1}{2}\Delta\xi^3 + \begin{cases} 0 & \text{dla } \xi \leqslant \frac{1}{3} \\ 1 \cdot \Delta & \text{dla } \xi > \frac{1}{3} \end{cases}$$

$$\ln T_\alpha = \frac{1}{\Delta} W(\eta), \quad \eta = \frac{y}{12a}$$