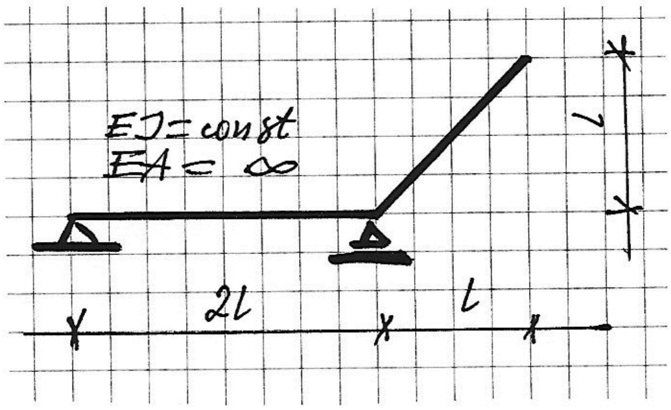
## Egzamin pisemny z Mechaniki Konstrukcji II, 10 IX 2018 r.

| NAZWISKO imię | | | | |
|---|---|---|---|---|
| Grupa | Data zaliczenia ćwiczeń | | Numer albumu | |
| Ocena zadania 1 | Ocena zadania 2 | Ocena zadania 3 | Ocena z egzaminu | Ocena łączna<br><br>Data |

**Zadanie 1**

Znaleźć pierwszą częstość drgań własnych danej ramy wykonanej z kształtownika I 180 o charakterystykach: $A=27{,}9\ cm^2$ , $J= 1450\ cm^4$; gęstość masy wynosi $7{,}88\ g/cm^3$. Przyjąć $E=205$ GPa oraz $l=1.2$ m.

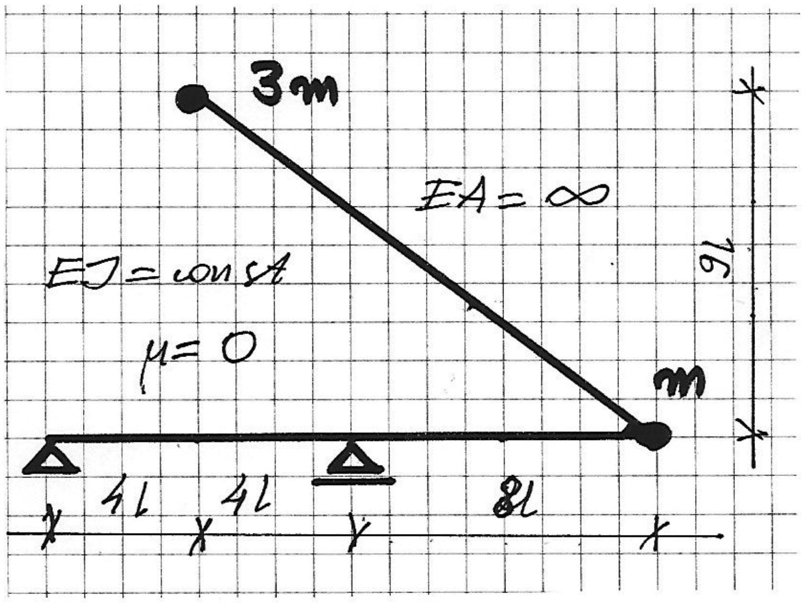
(*Find the first eigenfrequency of the given frame made from the profile I 180 of the characteristics $A=27{,}9\ cm^2$ , $J= 1450\ cm^4$; the mass density being $7{,}88\ g/cm^3$. Assume $E=205$ GPa* and $l=1.2$ m)




**Zadanie 2**

Znaleźć równania określające częstości drgań własnych danej ramy płaskiej o punktowym rozkładzie masy.

(*Find the equations which determine the eigenfrequencies of the given weightless frame with two point masses*)

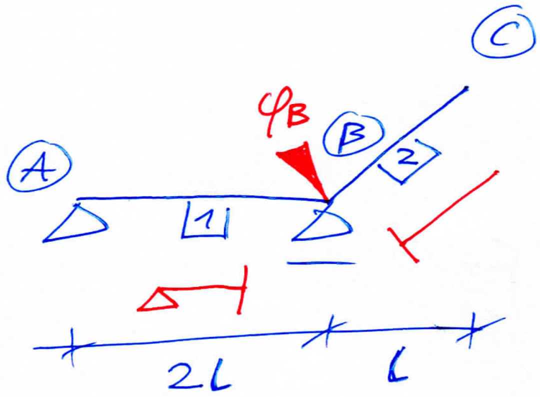



**Zadanie 3**

Wyprowadzić wzór Eulera na siłę krytyczną pryzmatycznego (EJ=const) wspornika.

(*Derive the Euler formula for the critical force compressing a prismatic (EJ=const) cantilever*).

Egzamin MK $\underline{II}$ 10.09.2018 Zadanie 1




$$q = [\varphi_B]$$

$$\boxed{\lambda := l\sqrt[4]{\frac{\mu\omega^2}{EJ}}}$$

$$\lambda^{(1)} = l^{(1)}\sqrt[4]{\frac{\mu\omega^2}{EJ}} = 2\lambda$$

$$\lambda^{(2)} = \sqrt{2}\lambda$$

RRMP:

1) $\Phi_B^{(1)} + \Phi_B^{(2)} = 0$

Wzory transf.:

$$\Phi_B^{(1)} = \frac{EJ}{(2l)}\left[\alpha'(2\lambda)\cdot\varphi_B\right]$$

$$\Phi_B^{(2)} = \frac{EJ}{(\sqrt{2}l)}\left[\alpha''(\sqrt{2}\lambda)\cdot\varphi_B\right]$$

RRMB:

$$\frac{EJ}{l}\left[\frac{1}{2}\alpha'(2\lambda) + \frac{1}{\sqrt{2}}\alpha''(\sqrt{2}\lambda)\right]\cdot\varphi_B = 0$$

Niezerowe drgania $\equiv$ $\varphi_B \neq 0$, a więc

$$\underbrace{\frac{1}{2}\alpha'(2\lambda) + \frac{1}{\sqrt{2}}\alpha''(\sqrt{2}\lambda)}_{\overset{!!}{=} A(\lambda)} = 0$$

| $\lambda$ | $A(\lambda)$ |
|---|---|
| 0.1 | $\approx 1.499$ |
| 0.2 | $\approx 1.433$ |
| $\vdots$ | |
| $\boxed{1.0}$ | $\boxed{\approx 0.036}$ |
| 1.1 | $\approx -0.170$ |

Przyjmuję $\lambda = 1.0$

$$\lambda = l\sqrt[4]{\frac{\mu\omega^2}{EJ}} \Rightarrow \omega = \frac{\lambda^2}{l^2}\sqrt{\frac{EJ}{\mu}}, \text{ gdzie } \boxed{\mu = A\rho}$$

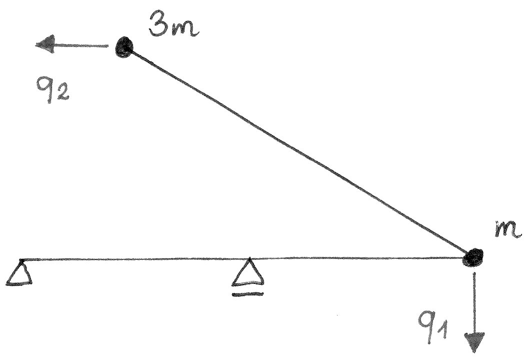
czyli:

$$\omega = \frac{1.0^2}{(1.2\,m)^2}\sqrt{\frac{(205\cdot 10^9\,N/m^2)(1450\cdot 10^{-8}\,m^4)}{(27.9\cdot 10^{-4}\,m^2)(7.88\cdot 10^{-3}\,kg/(10^{-6}\,m^3))}} =$$

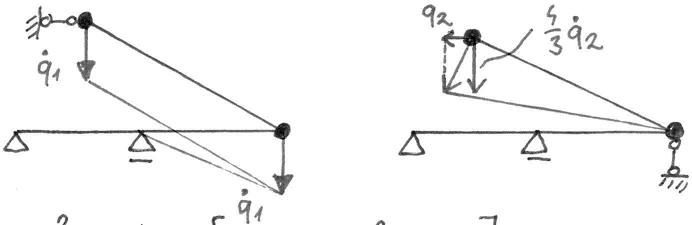
$$= 255\,\frac{rad}{s}$$

przygotował
Karol Bołbotowski

ZADANIE 2 / PROBLEM 2

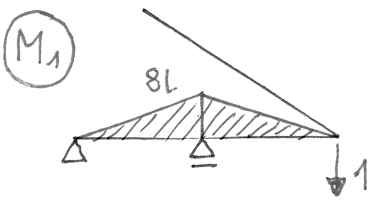


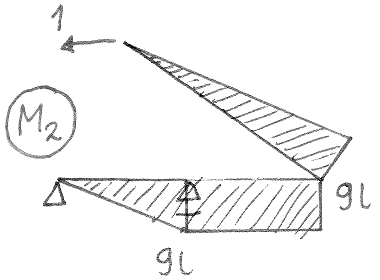

PLANY PRĘDKOŚCI / VELOCITY PLANS




$$E_K = \frac{1}{2} m \dot{q}_1^2 + \frac{1}{2} 3m \left[ \left( \dot{q}_1 + \frac{4}{3}\dot{q}_2 \right)^2 + \dot{q}_2^2 \right] = \dots$$

$$\mathbb{M} = \begin{bmatrix} 4 & 4 \\ 4 & \frac{25}{3} \end{bmatrix} m \approx \begin{bmatrix} 4 & 4 \\ 4 & 8{,}33 \end{bmatrix} m$$




$$d_{11} = \frac{1}{EJ}\left[ 2 \cdot \frac{1}{2} \cdot 8l \cdot 8l \cdot \frac{2}{3} \cdot 8l \right] = 341{,}33 \frac{l^3}{EJ}$$

$$d_{12} = \frac{1}{EJ}\left[ \frac{1}{2} \cdot 8l \cdot 8l \cdot \left(-\frac{2}{3} \cdot 9l\right) + \frac{1}{2} \cdot 8l \cdot 8l \cdot (-9l) \right] = -480 \frac{l^3}{EJ}$$

$$d_{22} = \frac{1}{EJ}\left[ \frac{1}{2} \cdot 8l \cdot 9l \cdot \frac{2}{3} \cdot 9l + 8l \cdot 9l \cdot 9l + \frac{1}{2} \cdot 15l \cdot 9l \cdot \frac{2}{3} \cdot 9l \right] = 1269 \frac{l^3}{EJ}$$

$$\mathbb{D} = \begin{bmatrix} 341{,}33 & -480 \\ -480 & 1269 \end{bmatrix} \frac{l^3}{EJ}$$

$$(\mathbb{I} - \omega^2 \mathbb{D}\mathbb{M})\,\mathbb{q} = \mathbb{0};$$ SZUKAMY TAKICH $\omega$, ŻE / WE'RE LOOKING FOR SUCH $\omega$, THAT $$\det(\mathbb{I} - \omega^2 \mathbb{D}\mathbb{M}) = 0$$

$$\det\left( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \underbrace{\frac{\omega^2 m l^3}{EJ}}_{= x} \begin{bmatrix} 341{,}33 & -480 \\ -480 & 1269 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ 4 & 8{,}33 \end{bmatrix} \right) = \det \begin{bmatrix} 1 + 554{,}67x & 2634{,}67x \\ -3156x & 1 - 8655x \end{bmatrix} = 0$$

$$(1 + 554{,}67x)(1 - 8655x) - 2634{,}67x \cdot (-3156x) = 0$$

$$x_1 = 0{,}000131 \Rightarrow \boxed{\omega_1 = 0{,}0114 \sqrt{\frac{EJ}{ml^3}}}$$

$$x_2 = 0{,}002174 \Rightarrow \boxed{\omega_2 = 0{,}0466 \sqrt{\frac{EJ}{ml^3}}}$$

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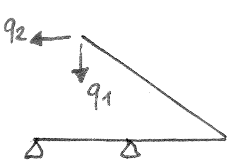
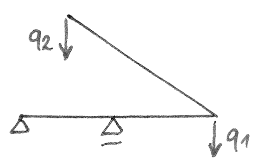
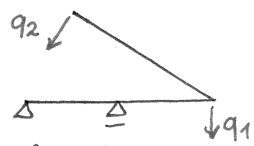
ROZWIĄZANIA ALTERNATYWNE / ALTERNATIVE SOLUTIONS




$$\mathbb{M} = \begin{bmatrix} 4 & -\frac{4}{3} \\ -\frac{4}{3} & \frac{43}{9} \end{bmatrix} m \quad \mathbb{D} = \begin{bmatrix} 1317{,}33 & 1212 \\ 1212 & 1269 \end{bmatrix} \frac{l^3}{EJ}$$




$$\mathbb{M} = \begin{bmatrix} \frac{43}{16} & -\frac{27}{16} \\ -\frac{27}{16} & \frac{75}{16} \end{bmatrix} m \quad \mathbb{D} = \begin{bmatrix} 341{,}33 & -298{,}67 \\ -298{,}67 & 1317{,}33 \end{bmatrix}$$

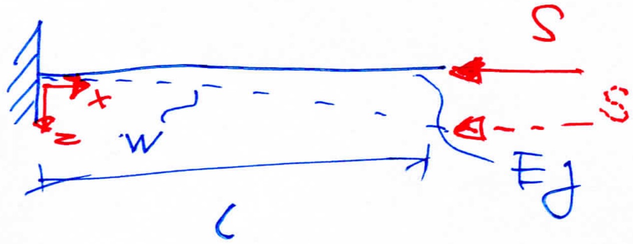



$$\mathbb{M} = \begin{bmatrix} \frac{52}{25} & 0 \\ 0 & 3 \end{bmatrix} m$$

$$\mathbb{D} = \begin{bmatrix} 341{,}33 & -526{,}93 \\ -526{,}93 & 2463{,}45 \end{bmatrix} \frac{l^3}{EJ}$$

# Egzamin MK II 10.09.2018 Zadanie 3

$\boxed{\text{założenie: } S>0}$




Przemieszczeniowe równanie równowagi:

$$EJ\cdot\frac{1}{l^4}\frac{d^4w}{d\xi^4}(\xi)+S\cdot\frac{1}{l^2}\frac{d^2w}{d\xi^2}(\xi)=0, \quad \xi=\frac{x}{l}$$

czyli

$$\frac{d^4w}{d\xi^4}(\xi)+\delta^2\frac{d^2w}{d\xi^2}(\xi)=0, \quad \delta=l\sqrt{\frac{S}{EJ}} \qquad \boxed{\delta>0}$$

Rozwiązanie:

$$w(\xi)=C_0+\delta C_1\xi+C_2\cos(\delta\xi)+C_3\sin(\delta\xi)$$

$$\varphi(\xi)=\frac{1}{l}\frac{dw}{d\xi}(\xi)=\frac{1}{l}\left[\delta C_1-\delta C_2\sin(\delta\xi)+\delta C_3\cos(\delta\xi)\right]$$

$$M(\xi)=-\frac{EJ}{l^2}\frac{d^2w}{d\xi^2}(\xi)=-\frac{EJ}{l^2}\left[-\delta^2C_2\cos(\delta\xi)-\delta^2C_3\sin(\delta\xi)\right]$$

$$\boxed{T(\xi)=\frac{1}{l}\frac{dM}{d\xi}(\xi)-\underbrace{S}_{\frac{\delta^2EJ}{l^2}}\cdot\frac{1}{l}\frac{dw}{d\xi}(\xi)}\overset{\text{po redukcji}}{=\!=}-\delta^3\frac{EJ}{l^3}\cdot C_1$$

warunki brzegowe:

$$\left.\begin{aligned}
w(0)=0 &\Rightarrow C_0+C_2=0\\
\varphi(0)=0 &\Rightarrow \delta C_1+\delta C_3=0\\
M(1)=0 &\Rightarrow \delta^2\cos(\delta)\cdot C_2+\delta^2\sin(\delta)C_3=0\\
T(1)=0 &\Rightarrow -\delta^3C_1=0
\end{aligned}\right\} A(\delta)\begin{bmatrix}C_0\\C_1\\C_2\\C_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$\delta$ odpowiada sile krytycznej wtedy i tylko wtedy gdy $\begin{bmatrix}C_0\\C_1\\C_2\\C_3\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$,

czyli $\det A(\delta)=0$ (z algebry)

ale można prościej (pamiętając, że $\boxed{\delta>0}$):

$$C_0=-C_2,\quad C_1=0 \Rightarrow C_3=0 \Rightarrow \cos(\delta)\cdot C_2=0$$

nie może być $C_2=0 \Rightarrow \cos(\delta)=0 \Rightarrow \boxed{\delta=\frac{\pi}{2}+n\cdot\pi}$, $n=0,1,2,\ldots$

Pierwsza siła krytyczna ($\delta_{KR}=\frac{\pi}{2}$):

$$S_{KR}=\frac{(\delta_{KR})^2EJ}{l^2}=\frac{\pi^2EJ}{(2l)^2}=\boxed{\frac{\pi^2EJ}{(\mu l)^2}}, \text{ gdzie } \boxed{\mu=2}$$

przygotował: Karol Bołbotowski