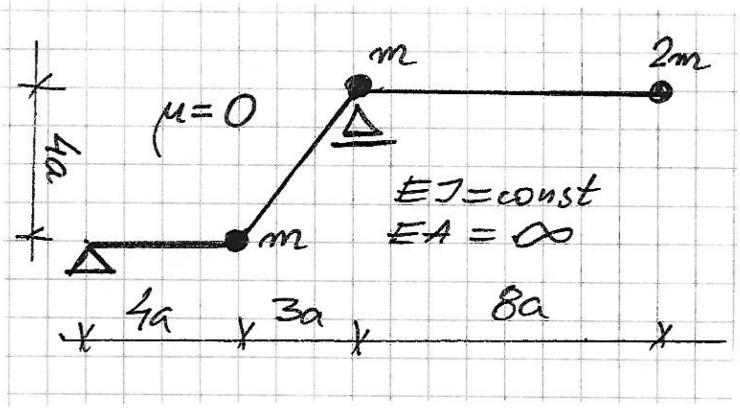
Egzamin pisemny z Mechaniki Konstrukcji II, 7 II 2018 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1

Zapisać równania
określające częstości drgań własnych .

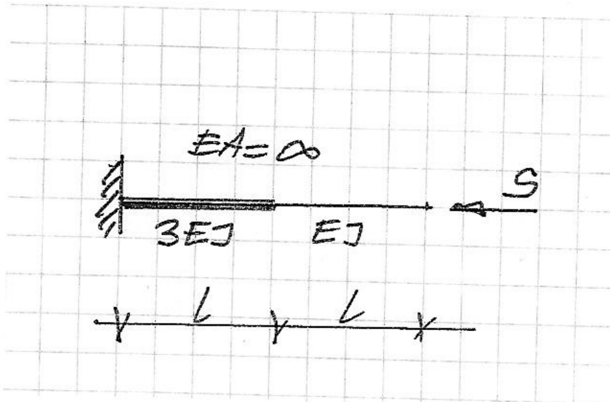
(Write down the equations
which determine the eigenfrequencies)



Zadanie 2

Znaleźć siłę krytyczną
danego pręta o zmiennej sztywności

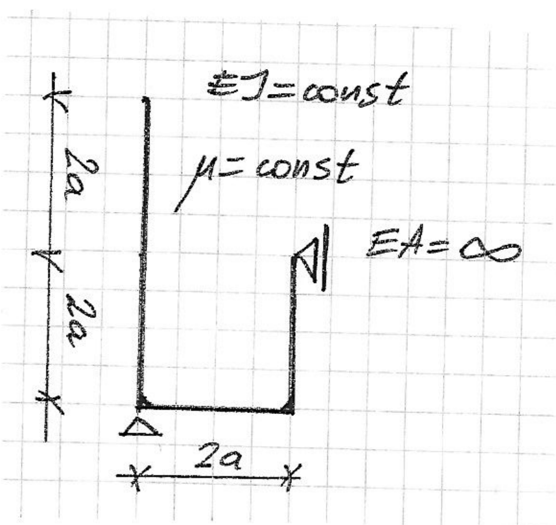
(Compute the critical force
of the bar of varying stiffness)

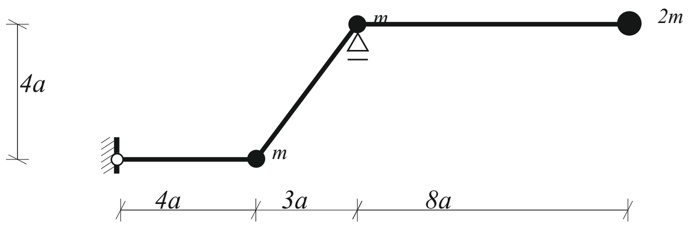


Zadanie 3

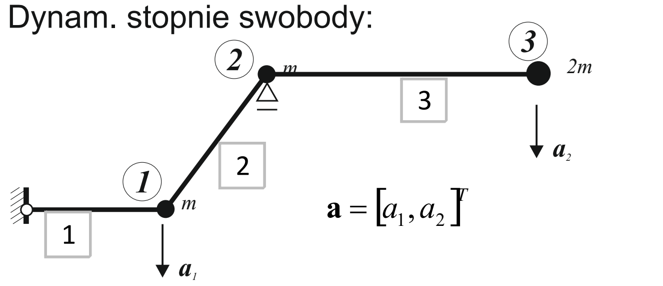
Zapisać równania
określające częstości drgań własnych .

(Write down the equations
which determine the eigenfrequencies)

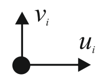




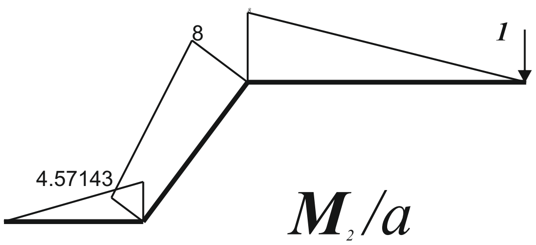
Dynam. stopnie swobody:



Składowe amplitud przemieszczeń mas:



$$u_1 = 0$$
$$v_1 = -a_1$$
$$v_2 = 0$$
$$0 = -a_1 - 3a\psi_2 \Rightarrow \psi_2 = -\frac{a_1}{3a}$$
$$u_2 = 0 + 4a\psi_2 = -\frac{4}{3}a_1$$
$$u_3 = u_2 = -\frac{4}{3}a_1$$
$$v_3 = -a_2$$
$$2E_k = m\left[(-\dot{a}_1)^2\right] + m\left[\left(-\frac{4}{3}\dot{a}_1\right)^2\right] + 2m\left[\left(-\frac{4}{3}\dot{a}_1\right)^2 + (-\dot{a}_2)^2\right] =$$
$$= (\dot{a}_1)^2\left(\frac{19}{3}m\right) + (\dot{a}_2)^2(2m)$$



Macierz mas M:

m	6 1/3	
		2

Macierz podatnosci D:

$$\left[\frac{a^3}{EJ}\right]\cdot$$

8.8163E+00	-3.4939E+01
-3.4939E+01	4.0098E+02

Odp:

$$\underbrace{\left(\mathbf{I} - \omega^2\mathbf{DM}\right)}_{\mathbf{A}(\omega)}\mathbf{a} = \mathbf{0}$$
$$\det \mathbf{A}(\omega) = 0 \Rightarrow \omega_i$$

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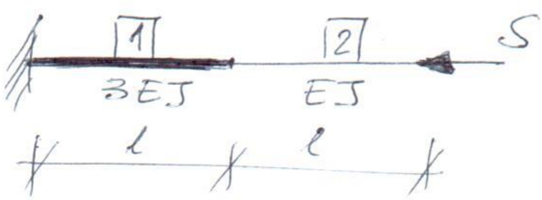
ZAD 2

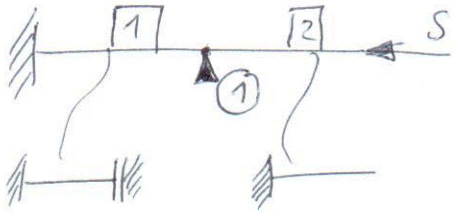
MKZ 07.02.2018.



$$\sigma_1 = \frac{\sqrt{3}}{3}\sigma$$

$$\sigma_2 = \sigma = \sqrt{\frac{Sl^2}{EJ}}$$

[1] zastosowanie kondensacji statycznej pręta [1]



$$\phi_1^1 = \frac{3EJ}{l}\left[\alpha''(\sigma_1)\varphi_1\right]$$

$$\phi_1^2 = \frac{EJ}{l}\left[\alpha'''(\sigma_2)\varphi_1\right]$$

1) $\phi_1^1 + \phi_1^2 = 0$

$$\frac{EJ}{l}\left[\alpha'''(\sigma)\varphi_1\right] + \frac{3EJ}{l}\left[\alpha''\left(\frac{\sqrt{3}}{3}\sigma\right)\varphi_1\right] = 0$$

$$\frac{EJ}{l}\left[\alpha'''(\sigma) + 3\alpha''\left(\frac{\sqrt{3}}{3}\sigma\right)\right]\varphi_1 = 0$$

$$\boxed{\alpha'''(\sigma) + 3\alpha''\left(\frac{\sqrt{3}}{3}\sigma\right) = 0}$$

$\rightarrow \boxed{\sigma_{kr}}$

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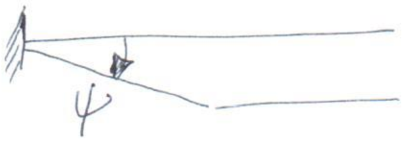
[2]



$$\phi_A^1 = \frac{3EJ}{l}\left[\beta(\sigma_1)\varphi_1 - \Theta(\sigma_1)\psi\right]$$

$$\phi_1^1 = \frac{3EJ}{l}\left[\alpha(\sigma_1)\varphi_1 - \Theta(\sigma_1)\psi\right]$$

$$\phi_1^2 = \frac{EJ}{l}\left[\alpha'''(\sigma_2)\varphi_1\right]$$

1) $\phi_1^1 + \phi_1^2 = 0$

2) $\phi_A^1\,\overline{\psi} + \phi_1^1\,\overline{\psi} + \overline{L}_s = 0$

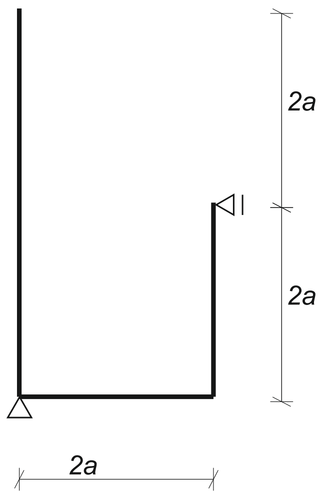
$$\overline{L}_s = (S\cdot\psi\cdot l)\overline{\psi}$$

$$Aq = 0 \qquad q = \begin{bmatrix}\varphi_1\\ \psi\end{bmatrix}$$

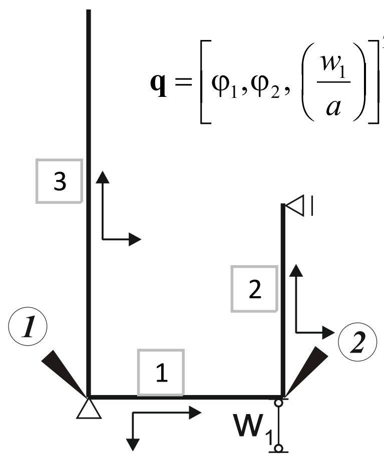
$$A = \begin{bmatrix} 3\alpha\left(\frac{\sqrt{3}}{3}\sigma\right) + \alpha'''(\sigma) & -3\Theta\left(\frac{\sqrt{3}}{3}\sigma\right) \\ -3\left[\alpha\left(\sigma\frac{\sqrt{3}}{3}\right) + \beta\left(\sigma\frac{\sqrt{3}}{3}\right)\right] & 6\Theta\left(\frac{\sqrt{3}}{3}\sigma\right) - \sigma^2 \end{bmatrix}$$

$$\alpha\left(\sigma\frac{\sqrt{3}}{3}\right) + \beta\left(\sigma\frac{\sqrt{3}}{3}\right) = \Theta\left(\frac{\sqrt{3}}{3}\sigma\right)$$

Policzyć det A i przyrównać do 0. Miejsca zerowe wyznaczają σ_{kr}.



UGW:



$$\mathbf{q} = \left[\varphi_1, \varphi_2, \left(\frac{w_1}{a}\right)\right]^T$$

Opis el.	Wzory transformacyjne
Element 1 $\lambda_1 = 2\lambda$	
$w_i^1 = 0$ $w_k^1 = +1w_1$ $u_1 = 0$ $B_{\|\|}^1 = 0$	$W_k^1 = \frac{EJ}{a^2}[-\frac{1}{4}\delta(\lambda_1)\varphi_1 - \frac{1}{4}\vartheta(\lambda_1)\varphi_2 + \frac{1}{8}\gamma(\lambda_1)\frac{w_1}{a}]$ $\Phi_1^1 = \frac{EJ}{a}[+\frac{1}{2}\alpha(\lambda_1)\varphi_1 + \frac{1}{2}\beta(\lambda_1)\varphi_2 - \frac{1}{4}\delta(\lambda_1)\frac{w_1}{a}]$ $\Phi_2^1 = \frac{EJ}{a}[+\frac{1}{2}\beta(\lambda_1)\varphi_1 + \frac{1}{2}\alpha(\lambda_1)\varphi_2 - \frac{1}{4}\vartheta(\lambda_1)\frac{w_1}{a}]$
Element 2 $\lambda_2 = 2\lambda$	
$w_i^2 = 0$ $w_k^2 = 0$ $u_2 = -1w_1$ $B_{\|\|}^2 = (-2w_1) \cdot \frac{EJ}{a^3}\lambda^4$	$\Phi_2^2 = \frac{EJ}{a}[+\frac{1}{2}\alpha'(\lambda_2)\varphi_2]$
Element 3 $\lambda_3 = 4\lambda$	
$w_i^3 = 0$ $w_k^3 = ?$ $u_3 = 0$ $B_{\|\|}^3 = 0$	$\Phi_1^3 = \frac{EJ}{a}[+\frac{1}{4}\alpha''(\lambda_3)\varphi_1]$

$\lambda = a\sqrt[4]{\frac{\mu\omega^2}{EJ}}$, $L_B = B_{||}^2(-a)$

RR:

1. $\Phi_1^1 + \Phi_1^3 = 0$
2. $\Phi_2^1 + \Phi_2^2 = 0$
3. $aW_k^1 - L_B = 0$

Macierz układu równań:

$$K(\lambda) = \frac{EJ}{a}\begin{bmatrix} +\frac{1}{2}\alpha(\lambda_1) + \frac{1}{4}\alpha''(\lambda_3) & +\frac{1}{2}\beta(\lambda_1) & -\frac{1}{4}\delta(\lambda_1) \\ +\frac{1}{2}\beta(\lambda_1) & +\frac{1}{2}\alpha(\lambda_1) + \frac{1}{2}\alpha'(\lambda_2) & -\frac{1}{4}\vartheta(\lambda_1) \\ -\frac{1}{4}\delta(\lambda_1) & -\frac{1}{4}\vartheta(\lambda_1) & +\frac{1}{8}\gamma(\lambda_1) - 2\cdot\lambda^4 \end{bmatrix}$$

Odp:

$$\mathbf{K}(\lambda)\mathbf{q} = \mathbf{0}$$
$$\det \mathbf{K}(\lambda) = 0 \Rightarrow \lambda_i \Rightarrow \omega_i$$

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