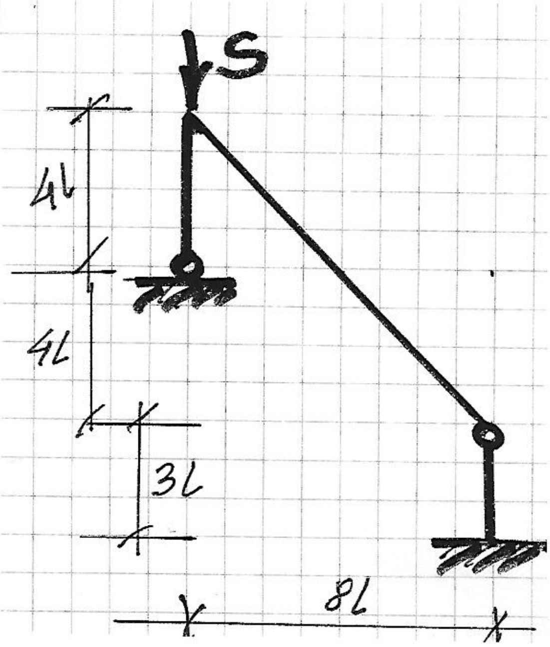
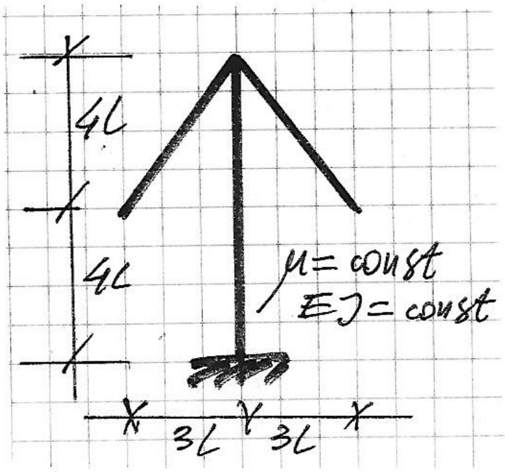
Egzamin z Mechaniki Konstrukcji II, 20 VI 2017 r.
Wydział Inżynierii Lądowej, studia stacjonarne

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

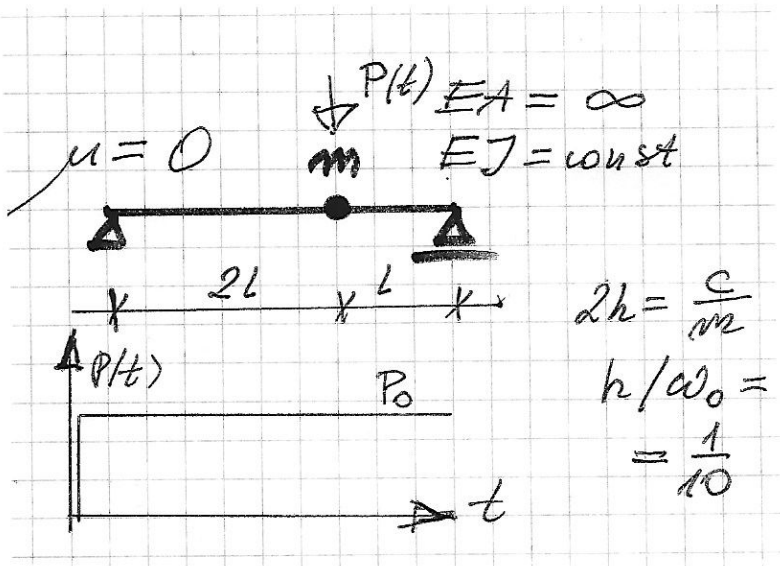


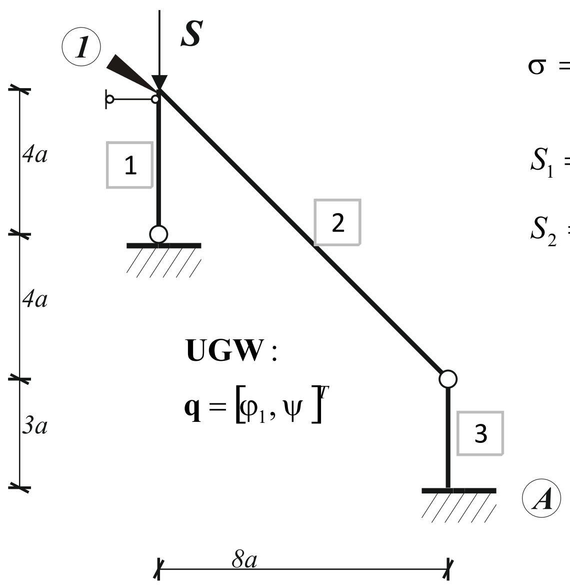
Zadanie 1
Zapisz równania określające wartość siły krytycznej. Sprawdzić ponadto wyboczenie lokalne.
(*Write down the equations determining the value of the critical force. Check the conditions of local buckling*)



Zadanie 2
Zapisz równania określające częstości drgań własnych antysymetrycznych.
(*Write down the equations determining the eigenfrequencies of antisymmetric vibration*)

Zadanie 3
a. Znaleźć częstość drgań własnych ω_0 danej belki nieważkiej z masą skupioną
b. Rozważyć drgania tłumione wywołane nagle przyłożonym obciążeniem P_0; przyjąć poziom tłumienia $h/\omega_0 = 1/10$ oraz jednorodne warunki początkowe. Znaleźć przemieszczenie masy w dowolnej chwili czasu
(*a. Compute the eigenfrequency* ω_0 of the given weightless beam with a lumped mass
b. Consider the damped vibrations caused by a suddenly applied load P_0; assume the damping according to $h/\omega_0 = 1/10$ *as well as the homogeneous initial conditions. Find the displacement of the mass at an arbitrary time instant*)





UGW :

$$\mathbf{q} = [\varphi_1, \psi]^T$$

DSO :

$$\sigma = a\sqrt{\frac{S}{EJ}} \Rightarrow S = \frac{EJ}{a^2}\sigma^2$$

$$S_1 = S, \sigma_1 = 4a\sqrt{\frac{S}{EJ}} = 4\sigma,$$

$$S_2 = S_3 = 0, \sigma_2 = \sigma_3 = 0.$$

RR :

$$1.\ \Phi_1^1 + \Phi_1^2 = 0,$$

$$2.\ |\overline{\psi} = -1|\quad \Phi_1^1(-1) + \Phi_A^3\left(-\frac{4}{3}\right) - 4\frac{EJ}{a}\sigma^2\psi = 0.$$

Wzory transformacyjne :

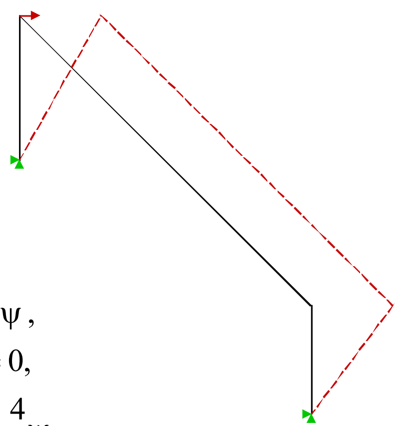
$$\Phi_1^1 = \frac{EJ}{4a}[\alpha'(4\sigma)(\varphi_1 - \psi)] = \frac{EJ}{a}\left[\frac{\alpha'(4\sigma)}{4}\varphi_1 - \frac{\alpha'(4\sigma)}{4}\psi\right]$$

$$\Phi_1^2 = \frac{3EJ}{8a\sqrt{2}}[\varphi_1] = \frac{EJ}{a}\left[\frac{3}{8\sqrt{2}}\varphi_1\right]$$

$$\Phi_A^3 = \frac{3EJ}{3a}\left[-\frac{4}{3}\psi\right] = \frac{EJ}{a}\left[-\frac{4}{3}\psi\right]$$

Układ równań:

$$\mathbf{K}(\sigma)\mathbf{q} = \mathbf{0}$$



PP :

$$\psi_1 = \psi,$$

$$\psi_2 = 0,$$

$$\psi_3 = \frac{4}{3}\psi,$$

$$\overline{L}_S = S4a\psi\overline{\psi} = 4Sa\psi\overline{\psi} = 4\frac{EJ}{a}\sigma^2\psi\overline{\psi}.$$

$$K(\sigma) = \frac{EJ}{a}\left[\begin{array}{c|c} +\frac{1}{4}\alpha'(4\sigma) + \frac{3}{8\sqrt{2}} & -\frac{1}{4}\alpha'(4\sigma) \\ -\frac{1}{4}\alpha'(4\sigma) & +\frac{1}{4}\alpha'(4\sigma) + \frac{16}{9} - 4\cdot\sigma^2 \end{array}\right]$$

$$\det K(\sigma) = 0 \quad \Rightarrow \sigma_{kr} \Rightarrow S_{kr} = \frac{EJ}{a^2}(\sigma_{kr})^2$$

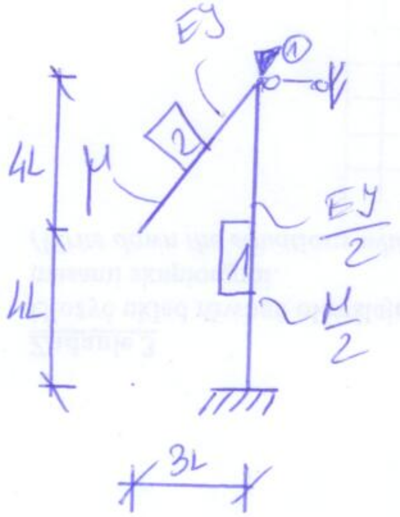
Wyboczenie lokalne:

$$S_1^{kr} = \pi^2\frac{EJ}{(\mu 4a)^2}, \mu \approx 0.7 \Rightarrow S_1^{kr} \approx 1.26\frac{EJ}{a^2}$$

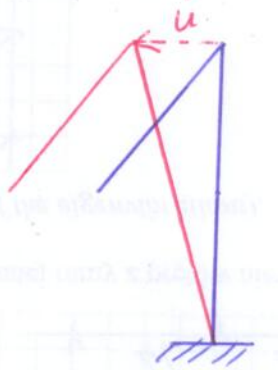
oprac. K. Hetmański

Zadanie 2.

Schemat połówkowy



plan przemieszczeń



pręt	W_i	W_k	u
1	0	$-u$	0
2	—	$-\frac{4}{5}$	$\frac{3}{5}u$

$$q = \begin{bmatrix} \varphi_1 \\ \frac{u}{L} \end{bmatrix}$$

$$\lambda_1 = 8L\sqrt[4]{\frac{\frac{\mu}{2}\omega^2}{\frac{EJ}{2}}} = 8\lambda \quad , \lambda = L\sqrt[4]{\frac{\mu\omega^2}{EJ}}$$

$$\lambda_2 = 5L\sqrt[4]{\frac{\mu\omega^2}{EJ}} = 5\lambda$$

Równania równowagi

1) $\bar{\Phi}_1^1 + \bar{\Phi}_1^2 = 0$

2) $-W_1^1(-u) - W_1^2\cdot\left(-\frac{4}{5}u\right) + B^{(2)}\cdot\frac{3}{5}u = 0$

$$\Phi_1^1 = \frac{\frac{EJ}{2}}{8L}\left[\alpha(8\lambda)\varphi_1 - \delta(8\lambda)\frac{-u}{8L}\right] = \frac{EJ}{L}\left[\frac{1}{16}\alpha(8\lambda)\varphi_1 + \frac{1}{128}\delta(8\lambda)\frac{u}{L}\right]$$

$$\Phi_1^2 = \frac{EJ}{5L}\left[\alpha''(5\lambda)\varphi_1 - \Theta''(5\lambda)\frac{-\frac{4}{5}u}{5L}\right] = \frac{EJ}{L}\left[\frac{1}{5}\alpha''(5\lambda)\varphi_1 + \frac{4}{125}\Theta''(5\lambda)\frac{u}{L}\right]$$

$$W_1^1 = \frac{-\frac{EJ}{2}}{(8L)^2}\left[\Theta(8\lambda)\varphi_1 - \gamma(8\lambda)\frac{-u}{8L}\right] = \frac{EJ}{L^2}\left[-\frac{1}{128}\Theta(8\lambda)\varphi_1 - \frac{1}{1024}\gamma(8\lambda)\frac{u}{L}\right]$$

$$W_1^2 = -\frac{EJ}{(5L)^2}\left[\Theta''(5\lambda)\varphi_1 - \gamma''(5\lambda)\frac{-\frac{4}{5}u}{5L}\right] = \frac{EJ}{L^2}\left[-\frac{1}{25}\Theta''(5\lambda)\varphi_1 - \frac{4}{625}\gamma''(5\lambda)\frac{u}{L}\right]$$

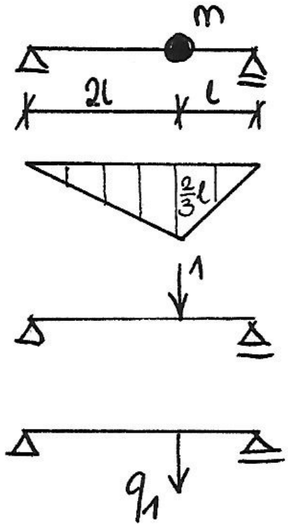
$$B^{(2)} = \mu\cdot 5L\cdot\omega^2\cdot\frac{3}{5}u = \frac{EJ}{L^2}\left[3\lambda^4\frac{u}{L}\right]$$

$$\mathbb{K} = \frac{EJ}{L}\begin{bmatrix} \frac{1}{16}\alpha(8\lambda) + \frac{1}{5}\alpha''(5\lambda) & \frac{1}{128}\vartheta(8\lambda) + \frac{4}{125}\vartheta''(5\lambda) \\ \frac{1}{128}\vartheta(8\lambda) + \frac{4}{125}\vartheta''(5\lambda) & \frac{1}{1024}\gamma(8\lambda) + \frac{16}{3125}\gamma''(5\lambda) - \frac{9}{5}\lambda^4 \end{bmatrix}$$

$\det(\mathbb{K}) = 0 \Rightarrow \lambda \Rightarrow \omega$

Zadanie 3

a)



$$\frac{h}{\omega} = \frac{1}{10} \qquad 2h = \frac{c}{m}$$

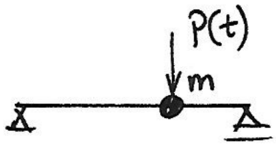
$$\delta_{11} = \frac{4}{9}\frac{l^3}{EI} \qquad m^* = m$$

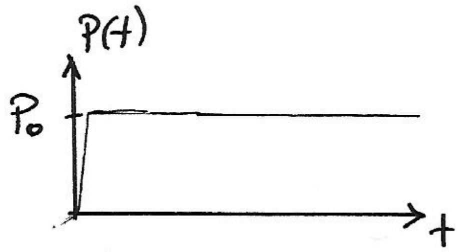
$$(1 - \omega^2 \delta_{11} m^*) q_1 = 0$$

$$\omega^2 = \frac{1}{\delta_{11} m^*} = \frac{9}{4}\frac{EI}{ml^3}$$

$$\omega = \frac{3}{2}\sqrt{\frac{EI}{ml^3}}$$

b)





$$m\ddot{q}(t) + c\dot{q}(t) + kq(t) = P(t) \quad / \cdot \frac{1}{m}$$

$$\ddot{q}(t) + 2h\dot{q}(t) + \omega^2 q(t) = \frac{1}{m} P(t) \qquad P(t) = P_0$$

$$q(t) = q_0(t) + q_s(t)$$

$$q_0(t) = e^{-ht}\left(C_1 \sin(\tilde{\omega} t) + C_2 \cos(\tilde{\omega} t)\right) \qquad \tilde{\omega} = \sqrt{\omega^2 - h^2}$$

$$q_s(t) = \frac{P_0}{m\omega^2}$$

$$q(0) = 0 \rightarrow \quad C_2 = -\frac{P_0}{m\omega^2}$$

$$\dot{q}(0) = 0 \rightarrow \quad C_1 = -\frac{P_0}{10 m\omega\tilde{\omega}}$$

$$q(t) = e^{-\frac{\omega t}{10}}\left(-\frac{1}{10\omega\tilde{\omega}} \sin(\tilde{\omega} t) - \frac{1}{\omega^2}\cos(\tilde{\omega} t)\right) \cdot \frac{P_0}{m}$$