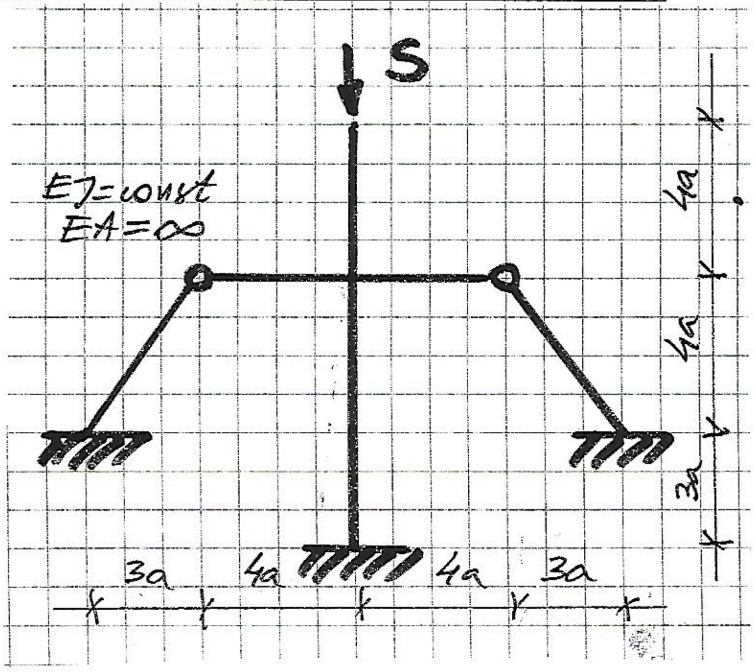
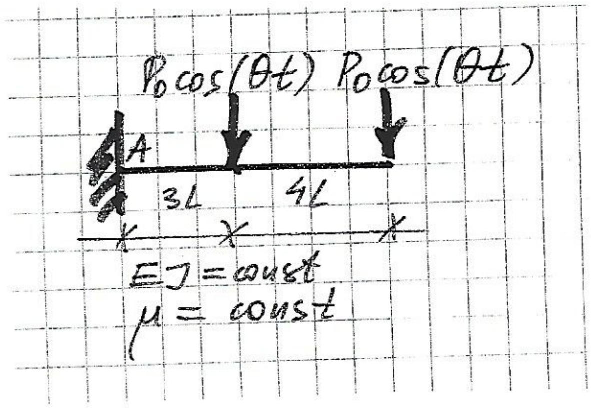
Egzamin pisemny z Mechaniki Konstrukcji II, 7 II 2017 roku.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

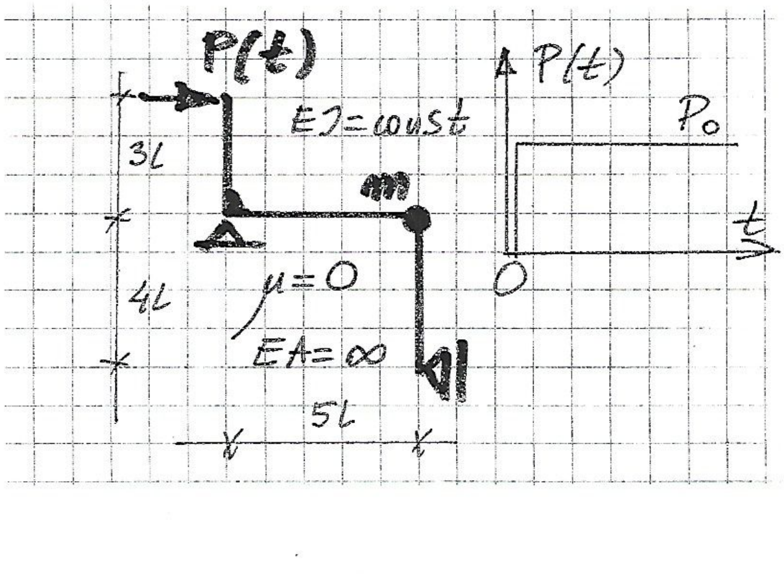
Zadanie 1
Dana jest rama płaska obciążona dużą siłą osiową. Ułożyć układ równań określający siłę krytyczną.
(*The given frame is subject to a big axial force. Write down the equations which determine the value of the critical force*).



Zadanie 2
Dana belka ($EJ = const, \mu = const$) jest poddana obciążeniu harmonicznemu jak na rysunku; $\theta = \frac{1}{100l^2}\sqrt{\frac{EJ}{\mu}}$.
Zapisz równania określające amplitudę momentu M_A
(Given beam ($EJ = const, \mu = const$) *is subject to a harmonic load as shown in the figure.*
$\theta = \frac{1}{100l^2}\sqrt{\frac{EJ}{\mu}}$.
Write down the equations which determine the amplitude M_A)



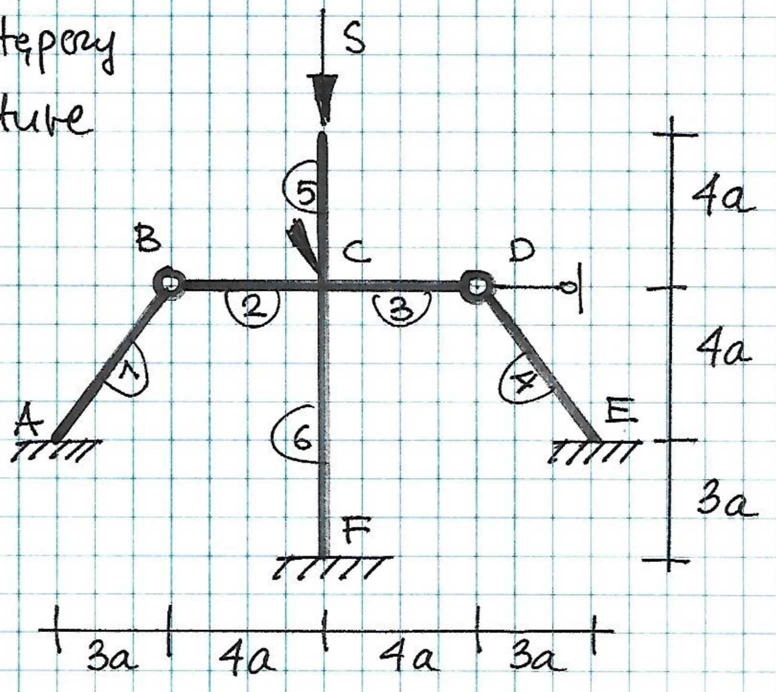
Zadanie 3
Dana rama jest obciążona nagle przyłożonym obciążeniem o wartości P_0. Przyjąć jednorodne warunki początkowe. Znaleźć przemieszczenie masy w dowolnej chwili czasu.
(The force P_0 is abruptly applied to the given frame. Find the displacement of the mass at arbitrary time instant. The initial conditions are homogeneous)



Zadanie 1 / Problem #1

Schemat zastępczy

Primary structure



$$q = \begin{bmatrix} \varphi_C \\ \psi \end{bmatrix}$$

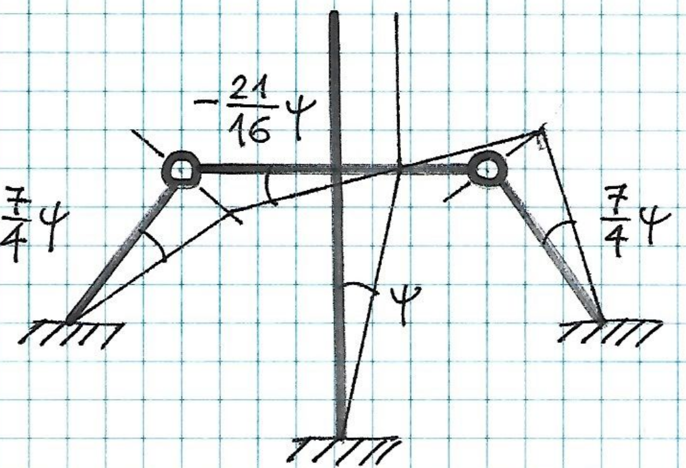
$$S^{(5)} = S^{(6)} = S$$

$$\sigma^{(5)} = 4\sigma$$

$$\sigma^{(6)} = 7\sigma$$

$$\sigma = a\sqrt{\frac{S}{EJ}}$$

Plan przesunięć / Translation plan



Równania równowagi / Equilibrium equations

$$\begin{cases} \Phi_C^{(2)} + \Phi_C^{(3)} + \Phi_C^{(5)} + \Phi_C^{(6)} = 0 \\ \Phi_A^{(1)} \cdot \frac{7}{4}\overline{\psi} + \Phi_C^{(2)} \cdot \left(-\frac{21}{16}\overline{\psi}\right) + \Phi_C^{(3)} \cdot \left(-\frac{21}{16}\overline{\psi}\right) + \Phi_E^{(4)} \cdot \frac{7}{4}\overline{\psi} \\ \quad + \left(\Phi_C^{(6)} + \Phi_F^{(6)}\right) \cdot \overline{\psi} + S \cdot 7a \cdot \psi \cdot \overline{\psi} = 0 \end{cases}$$

Ze względu na symetrię / By symmetry

$$\Phi_A^{(1)} = \Phi_E^{(4)}$$

$$\Phi_C^{(2)} = \Phi_C^{(3)}$$

$$\longrightarrow \begin{cases} \Phi_C^{(2)} + \frac{1}{2}\left(\Phi_C^{(5)} + \Phi_C^{(6)}\right) = 0 \\ -\Phi_A^{(1)} \cdot \frac{7}{4} + \Phi_C^{(2)} \cdot \frac{21}{16} - \frac{1}{2}\left(\Phi_C^{(6)} + \Phi_F^{(6)}\right) - \frac{7}{2} S a \psi = 0 \end{cases}$$

$$\Phi_A^{(1)} = \frac{3EJ}{5a}\left[-\frac{7}{4}\psi\right]$$

$$\Phi_C^{(2)} = \frac{3EJ}{4a}\left[\varphi_C + \frac{21}{16}\psi\right]$$

$$\Phi_C^{(5)} = \frac{EJ}{4a}\left[\alpha'''(4\sigma)\varphi_C\right]$$

$$\Phi_C^{(6)} = \frac{EJ}{7a}\left[\alpha(7\sigma)\varphi_C - \vartheta(7\sigma)\psi\right]$$

$$\Phi_F^{(6)} = \frac{EJ}{7a}\left[\beta(7\sigma)\varphi_C - \vartheta(7\sigma)\psi\right]$$

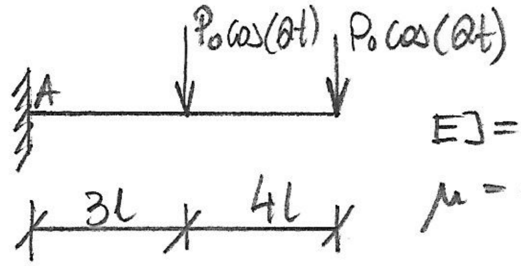
Równania równowagi w postaci macierzowej

Matrix form of the equilibrium equations

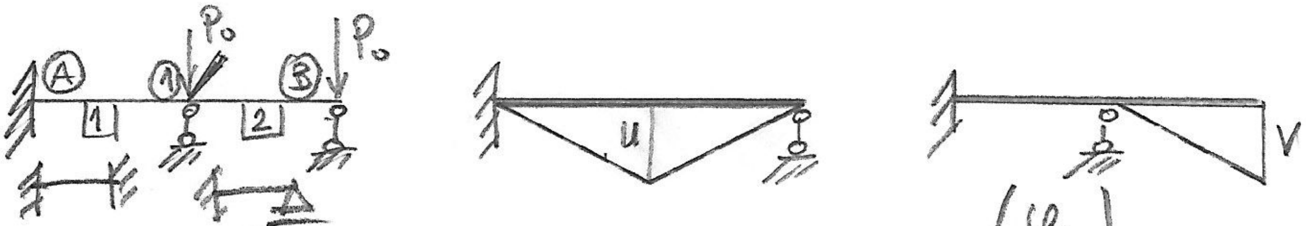
$$\mathbb{K}(\sigma)\,\mathbf{q} = \mathbf{0}$$

$$\mathbb{K}(\sigma) = \frac{EJ}{a}\left[\begin{array}{c|c} \frac{3}{4} + \frac{1}{8}\alpha'''(4\sigma) + \frac{1}{14}\alpha(7\sigma) & \frac{63}{64} - \frac{1}{14}\vartheta(7\sigma) \\ \hline \frac{63}{64} - \frac{1}{14}\vartheta(7\sigma) & \frac{16023}{5120} + \frac{1}{7}\vartheta(7\sigma) - \frac{7}{2}\sigma^2 \end{array}\right]$$

Zadanie 2 / Problem #2



$EJ = const$ $\quad \mu = const$ $\quad \theta = \frac{1}{100 l^2}\sqrt{\frac{EJ}{\mu}}$ $\quad \lambda_1 = 0{,}3 \quad \lambda_2 = 0{,}4$



$$q = \begin{pmatrix} \varphi_1 \\ u/l \\ V/l \end{pmatrix}$$

1) $\phi_1^1 + \phi_1^2 = 0$

2) $-W_1^1 \bar{u} - W_1^2 \bar{u} + P_0 \bar{u} = 0$

3) $-W_B^2 \bar{V} + P_0 \bar{V} = 0$

$$\phi_A^1 = \frac{EJ}{3l}\left(\beta(0{,}3)\varphi_1 - \delta(0{,}3)\frac{u}{3l}\right)$$

$$\phi_1^1 = \frac{EI}{3l}\left(\alpha(0{,}3)\varphi_1 - \theta(0{,}3)\frac{u}{3l}\right)$$

$$\phi_1^2 = \frac{EI}{4l}\left(\alpha'(0{,}4)\varphi_1 + \theta'(0{,}4)\frac{u}{4l} - \delta'(0{,}4)\frac{V}{4l}\right)$$

$$W_A^1 = \frac{EI}{9l^2}\left(\delta(0{,}3)\varphi_1 - \varepsilon(0{,}3)\frac{u}{3l}\right)$$

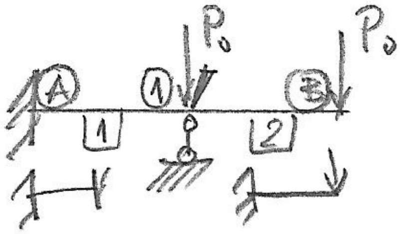
$$W_1^1 = -\frac{EI}{9l^2}\left(\theta(0{,}3)\varphi_1 - \gamma(0{,}3)\frac{u}{3l}\right)$$

$$W_1^2 = \frac{EI}{16l^2}\left(\theta'(0{,}4)\varphi_1 + \gamma'(0{,}4)\frac{u}{4l} - \varepsilon'(0{,}4)\frac{V}{4l}\right)$$

Z rozwiązania układu równań wynika

$\varphi_1 = 21{,}42 \frac{Pl^2}{EI}$ $\quad \frac{u}{l} = 36{,}72 \frac{Pl^2}{EI}$ $\quad V = 144{,}10 \frac{Pl^2}{EI}$

$$M_A = \phi_A^1$$



1) $\phi_1^1 + \phi_1^2 = 0$

2) $-\overline{W}_1^1 \bar{u} - \overline{W}_1^2 \bar{u} + P_0 \bar{u} = 0$

$$\phi_A^1 = \frac{EI}{3l}\left(\beta(0{,}3)\varphi_1 - \delta(0{,}3)\frac{u}{3l}\right)$$

$$\phi_1^1 = \frac{EI}{3l}\left(\alpha(0{,}3)\varphi_1 - \sigma(0{,}3)\frac{u}{3l}\right)$$

$$\phi_1^2 = \frac{EI}{4l}\left(\alpha''(0{,}4)\varphi_1 + \sigma(0{,}4)\frac{u}{4l}\right) + \phi_1^{02}$$

$$\overline{W}_A^1 = \frac{EI}{9l^2}\left(\delta(0{,}3)\varphi_1 - \varepsilon(0{,}3)\frac{u}{3l}\right)$$

$$\overline{W}_1^1 = -\frac{EI}{9l^2}\left(\sigma(0{,}3) - \gamma(0{,}3)\frac{u}{3l}\right)$$

$$\overline{W}_1^2 = \frac{EI}{16l^2}\left(\sigma''(0{,}4)\varphi_1 + \gamma''(0{,}4)\frac{u}{4l}\right) + \overline{W}_1^{02}$$

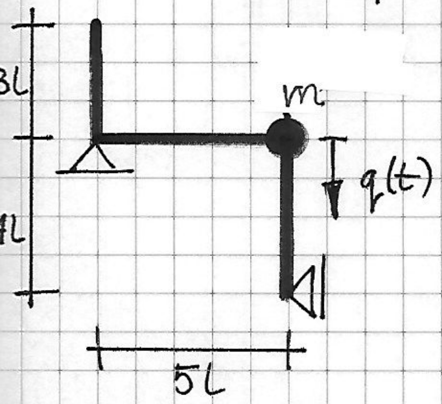
Wyjściowy moment ϕ_1^{02} oraz siłę $\overline{W}_1^{02}$ należy obliczyć stosując np. tw. Bettiego.

Zadanie 3 / Problem #3

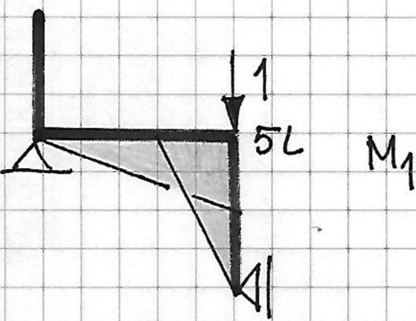
Równanie ruchu / Equation of motion

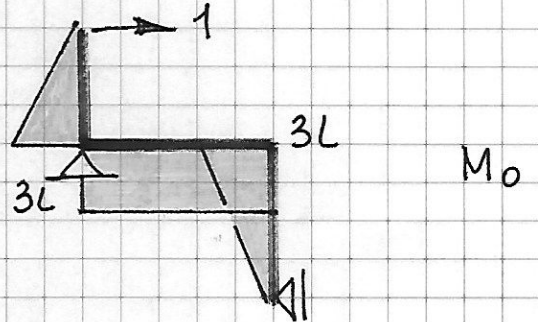
$$d m \ddot{q}(t) + q(t) = d_0 P(t)$$

$$\ddot{q}(t) + \frac{1}{dm} q(t) = \frac{d_0}{d} \cdot \frac{1}{m} P(t)$$



Wyznaczenie d, do / Determining d and do





$$d = \frac{1}{EJ}\left[\frac{1}{2}\cdot 5L \cdot 5L \cdot \frac{2}{3}\cdot 5L + \frac{1}{2}\cdot 4L \cdot 5L \cdot \frac{2}{3}\cdot 5L\right] = 75\,\frac{L^3}{EJ}$$

$$d_0 = \frac{1}{EJ}\left[\frac{1}{2}\cdot 5L \cdot 5L \cdot 3L + \frac{1}{2}\cdot 4L \cdot 5L \cdot \frac{2}{3}\cdot 3L\right] = \frac{115}{2}\,\frac{L^3}{EJ}$$

Stąd / From this

$$\ddot{q}(t) + \frac{EJ}{75mL^3} q(t) = \frac{23}{30m} P_0$$

CORJ / GSHE

$$q_0(t) = A_1 \sin(\omega t) + A_2 \cos(\omega t)\ , \quad \omega^2 = \frac{1}{dm}$$

CSRN / PSNHE

$$q_s(t) = B\ , \quad q(t) = q_0(t) + q_s(t)$$

Wyznaczamy A_1, A_2, B / We determine A_1, A_2, B

$$\frac{EJ}{75mL^3} q_s(t) = \frac{23}{30m} P_0 \longrightarrow B = \frac{115}{2}\,\frac{P_0 L^3}{EJ}$$

Jednorodne warunki początkowe / Homogeneous initial conditions

$$\left.\begin{aligned} q(0)&=0 \qquad & A_2 + B &= 0 \\ \dot{q}(0)&=0 \qquad & A_1\omega &= 0 \end{aligned}\right\} \qquad \begin{aligned} A_2 &= -\frac{115}{2}\frac{P_0 l^3}{EJ} \\ A_1 &= 0 \end{aligned}$$

Ostatecznie / Finally

$$q(t) = \frac{115}{2}\frac{P_0 l^3}{EJ}\left[1-\cos(\omega t)\right] \quad , \quad \omega = \sqrt{\frac{EJ}{75ml^3}}$$