Egzamin pisemny z Mechaniki Konstrukcji II, 8 IX 2016 r.

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Data</td></tr>
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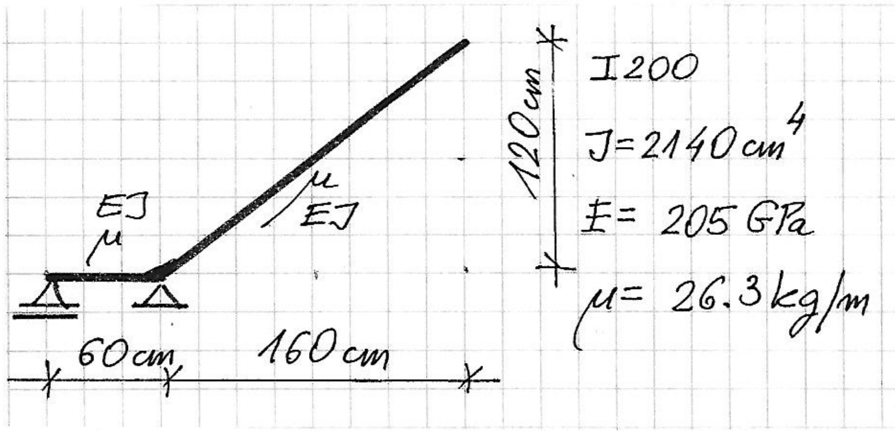
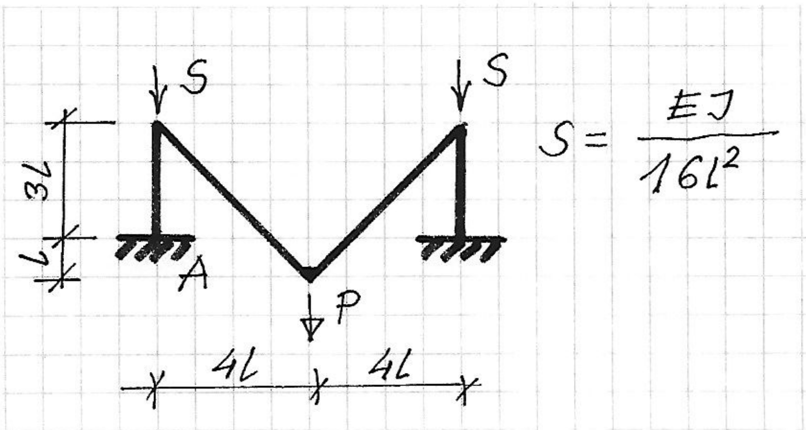
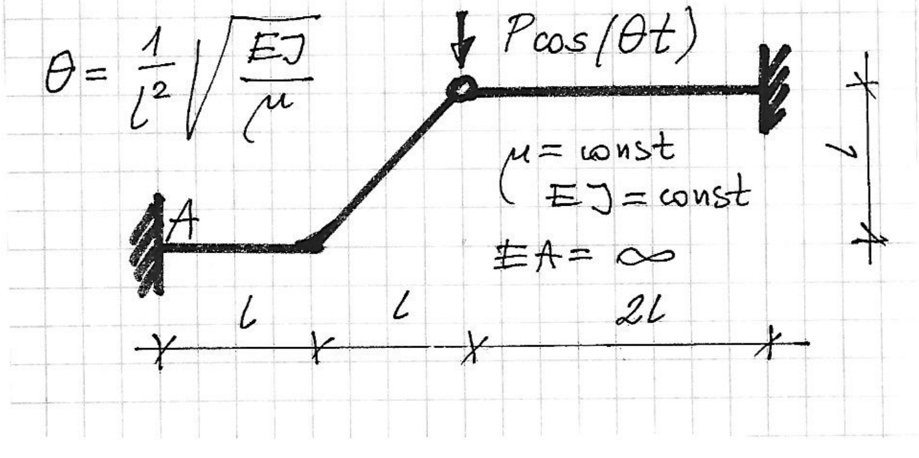
Zadanie 1
Znaleźć
pierwszą częstość
drgań własnych danej ramy.
*(Compute
the first eigenfrequency
of the given frame)*



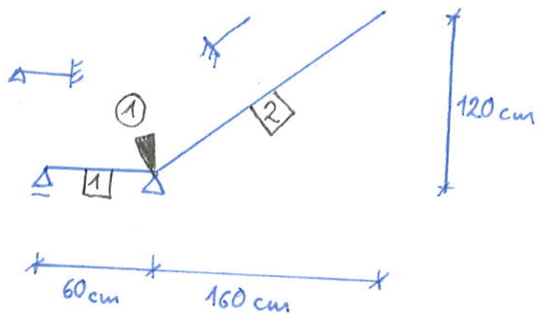
Zadanie 2
Rozważamy ramę
poddaną dużym siłom osiowym S
i zginaną siłą P.
Znaleźć moment zginający
w utwierdzeniu A.
*(Consider the frame
subject to a big axial force S
and bent by the force P.
Find
the bending moment
at the clamped edge A).*



Zadanie 3
Dana jest rama
obciążona
siłą harmonicznie zmienną w czasie,
por. rys.
Znaleźć amplitudę M_A.
*(The given frame
is subject
to a harmonic load,
see fig.
Compute the amplitude of M_A.)*



Zadanie 1. Problem 1



$$J = 2140\ cm^4 = 2{,}14 \cdot 10^{-5}\ m^4$$

$$E = 205\ GPa = 205 \cdot 10^6\ \frac{kN}{m^2}$$

$$\mu = 26{,}3\ \frac{kg}{m} \approx 26{,}3\ \frac{N}{m} \cdot \frac{s^2}{m} = 0{,}0263\ \frac{kN\,s^2}{m^2}$$

$$\left[N = kg \cdot \frac{m}{s^2}\right]$$

Przyjmijmy l=1m. Let l=1m

$$\lambda = l\sqrt[4]{\frac{\mu\omega^2}{EJ}}$$

$$l_1 = 0{,}6l \qquad \lambda_1 = 0{,}6l\sqrt[4]{\frac{\mu\omega^2}{EJ}} = 0{,}6\lambda$$

$$l_2 = 2l \qquad \lambda_2 = 2l\sqrt[4]{\frac{\mu\omega^2}{EJ}} = 2\lambda$$

$$q = [\varphi_1]$$

$$\Phi_1^1 + \Phi_1^2 = 0$$

$$\Phi_1^1 = \frac{EJ}{l_1}\left[\alpha'(\lambda_1)\varphi_1\right] = \frac{EJ}{0{,}6l}\alpha'(0{,}6\lambda)\varphi_1 = \frac{5}{3}\frac{EJ}{l}\alpha'(0{,}6\lambda)\varphi_1$$

$$\Phi_1^2 = \frac{EJ}{l_2}\left[\alpha''(\lambda_2)\varphi_1\right] = \frac{EJ}{2l}\alpha''(2\lambda)\varphi_1$$

$$\frac{EJ}{l}\left[\frac{5}{3}\alpha'(0{,}6\lambda) + \frac{1}{2}\alpha''(2\lambda)\right]\varphi_1 = 0$$

Szukamy rozwiązania nietrywialnego. We look for the non-trivial solution $\varphi_1 \neq 0$.

$$\frac{5}{3}\alpha'(0{,}6\lambda) + \frac{1}{2}\alpha''(2\lambda) = 0$$

$$\alpha'(0{,}6\lambda) + \frac{3}{10}\alpha''(2\lambda) = 0 \Rightarrow \alpha'(0{,}6\lambda) = -\frac{3}{10}\alpha''(2\lambda)$$

Korzystamy z tablic. We use the tables.

$\lambda = 0{,}8$:

$$\alpha'(0{,}6 \cdot 0{,}8) \approx \alpha'(0{,}5) = 2{,}999$$

$$-\frac{3}{10}\alpha''(2 \cdot 0{,}8) = -\frac{3}{10}\alpha''(1{,}6) = 1{,}373$$

$\lambda = 0{,}9$:

$$\alpha'(0{,}6 \cdot 0{,}9) \approx \alpha'(0{,}7) = 2{,}995$$

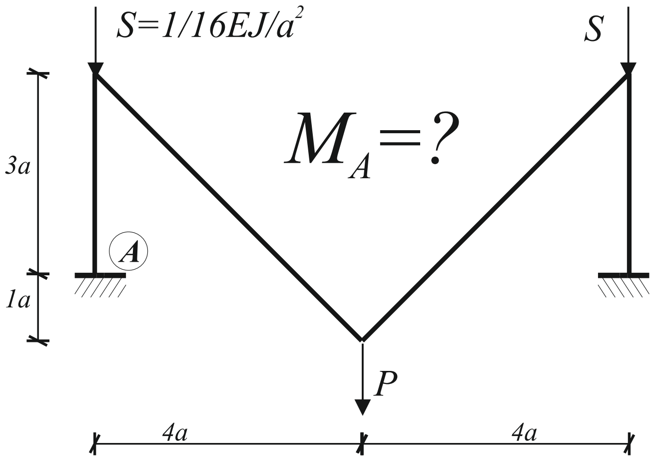
$$-\frac{3}{10}\alpha''(2 \cdot 0{,}9) = -\frac{3}{10}\alpha''(1{,}8) = 6{,}787$$

Rozwiązanie. Solution. $\lambda \in (0{,}8;\ 0{,}9)$. Przyjmijmy Let $\lambda = 0{,}85$

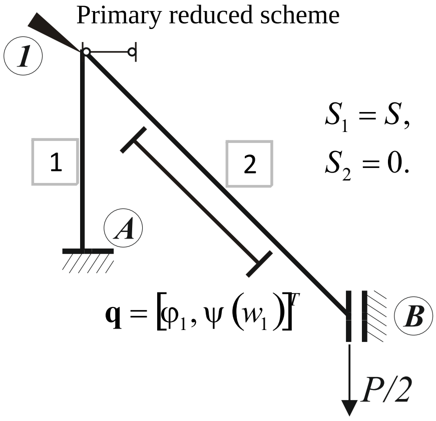
$$\lambda = l\sqrt[4]{\frac{\mu\omega^2}{EJ}} \Rightarrow \omega = \frac{\lambda^2}{l^2}\sqrt{\frac{EJ}{\mu}} = \left(\frac{0{,}85}{1{,}0m}\right)^2\sqrt{\frac{205 \cdot 10^6\ \frac{kN}{m^2} \cdot 2{,}14 \cdot 10^{-5}\ m^4}{0{,}0263\ \frac{kN\,s^2}{m^2}}}$$

$$\boxed{\omega \approx 295{,}1\ [Hz]}$$

Zadanie 2 Problem 2



Zastępczy schemat zredukowany
Primary reduced scheme



$S_1 = S,$

$S_2 = 0.$

$\mathbf{q} = [\varphi_1, \psi\,(w_1)]^T$

Plan przesunięć
Translation plan

$\psi_1 = \psi\,,$

$\psi_2 = \frac{3}{4}\psi\,,$

$\overline{L}_z = \frac{P}{2}4a\left(\frac{3}{4}\overline{\psi}\right) = \frac{3}{2}Pa\overline{\psi}\,,$

$\overline{L}_S = S3a\psi\overline{\psi} = \frac{3}{16}\frac{EJ}{a}\psi\overline{\psi}\,.$

Równania równowagi. Equilibrium equations

$$1.\ \Phi_1^1 + \Phi_1^2 = 0$$

$$2.\{\overline{\psi} = -1\}\ \left[\Phi_A^1 + \Phi_1^1\right](-1) + \left[\Phi_1^2 + \Phi_B^2\right]\left(-\frac{3}{4}\right) - \frac{3}{16}\frac{EJ}{a}\psi - \frac{3}{2}Pa = 0$$

Wzory transformacyjne.
Slope-deflection equations.

$\sigma_1 = 3a\sqrt{\frac{S}{EJ}} == 0.75,$

$\sigma_2 = 0.$

	$\frac{EJ}{a}\varphi_1$	$\frac{EJ}{a}\psi$				
F11	1.30815	-1.9812	+		=	-0.2348
FA1	0.67303	-1.9812	+		=	-1.3838
F12	0.70711	-0.7955	+		=	0.2348
FB2	0.35355	-0.7955	+		=	-0.4049

Pa

Układ równań i rozwiązanie.
System of equations and its solution.

$\frac{EJ}{a}$

2.0153E+00	-2.7767E+00	**q**	=		Pa
-2.7767E+00	**4.9681E+00**		=	1 1/2	

Rozwiązanie. Solution.

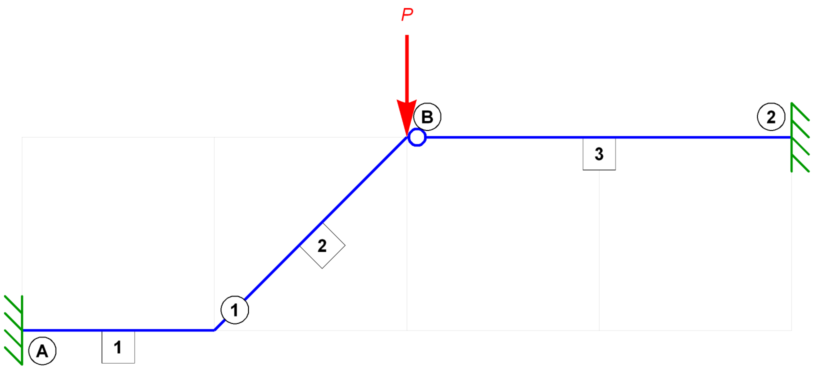
q =

1.809E+00
1.313E+00

$\frac{Pa^2}{EJ}$

Oprac. K. Hetmański

Zadanie 3. Problem 3.



$\lambda_1 = 1$

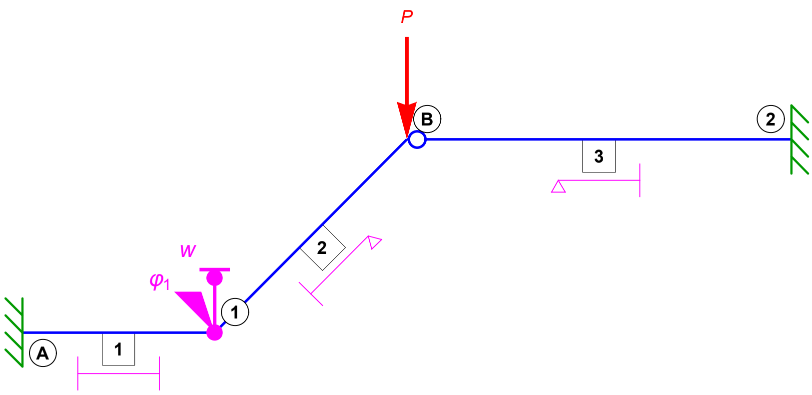
$\lambda_2 = \sqrt{2}$

$\lambda_3 = 2$

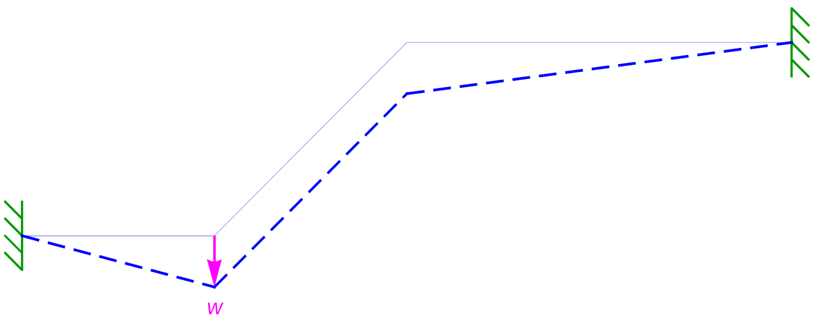
Wektor niewiadomych.
Vector of unknowns.

$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \frac{w}{1} \end{pmatrix}$$

Układ zastępczy.
The primary structure.



Plan przesunięć.
Translation plan.



	w_i^K	w_k^K	u^K
Pręt 1 :	$w_A^1 = 0$	$w_1^1 = w$	$u^1 = 0$
Pręt 2 :	$w_1^2 = \frac{1}{\sqrt{2}} w$	$w_B^2 = \frac{1}{\sqrt{2}} w$	$u^2 = -\frac{1}{\sqrt{2}} w$
Pręt 3 :	$w_B^3 = w$	$w_2^3 = 0$	$u^3 = 0$

Wyjściowe siły brzegowe.
Initial end forces.

$$W_B^{03} = - P$$

Wzory transformacyjne.
Slope-deflection equations.

$$\Phi_1^1 = \frac{EJ}{l} [\alpha(1) \varphi_1 - \vartheta(1) \frac{w}{l}] = \frac{EJ}{l} [3.990 \varphi_1 - 5.948 \frac{w}{l}]$$

$$\Phi_1^2 = \frac{EJ}{l} [\frac{1}{\sqrt{2}} \alpha'(\sqrt{2}) \varphi_1 + \{ \frac{1}{2\sqrt{2}} \vartheta'(\sqrt{2}) - \frac{1}{2\sqrt{2}} \delta'(\sqrt{2}) \} \frac{w}{l}] = \frac{EJ}{l} [2.067 \varphi_1 - 0.179 \frac{w}{l}]$$

$$W_1^1 = \frac{EJ}{l^2} [- \vartheta(1) \varphi_1 + \gamma(1) \frac{w}{l}] = \frac{EJ}{l^2} [- 5.948 \varphi_1 + 11.628 \frac{w}{l}]$$

$$W_1^2 = \frac{EJ}{l^2} [\frac{1}{2} \vartheta'(\sqrt{2}) \varphi_1 + \{ \frac{1}{4} \gamma'(\sqrt{2}) - \frac{1}{4} \varepsilon'(\sqrt{2}) \} \frac{w}{l}] = \frac{EJ}{l^2} [1.326 \varphi_1 - 0.633 \frac{w}{l}]$$

$$W_B^2 = \frac{EJ}{l^2} [- \frac{1}{2} \delta'(\sqrt{2}) \varphi_1 + \{ - \frac{1}{4} \varepsilon'(\sqrt{2}) + \frac{1}{4} \chi'(\sqrt{2}) \} \frac{w}{l}] = \frac{EJ}{l^2} [- 1.580 \varphi_1 - 0.380 \frac{w}{l}]$$

$$W_B^3 = \frac{EJ}{l^2} [\frac{1}{8} \chi'(2) \frac{w}{l}] - P = \frac{EJ}{l^2} [- 0.115 \frac{w}{l}] - P$$

Osiowe siły bezwładności.
Axial inertia forces.

$$B_{||}^2 = \Theta^2 \cdot \mu \cdot \sqrt{2}\, l \cdot (- \frac{1}{\sqrt{2}} w) = \frac{EJ}{l^2} [- \frac{w}{l}]$$

Równania równowagi.
Equations of equilibrium.

$$\Phi_1^1 + \Phi_1^2 = 0$$

$$- W_1^1 \cdot \overline{w} - W_1^2 \cdot \frac{1}{\sqrt{2}} \overline{w} - W_B^2 \cdot \frac{1}{\sqrt{2}} \overline{w} - W_B^3 \cdot \overline{w} + B_{||}^2 \cdot (- (1 / (\sqrt{2})) \overline{w}) = \overline{0}$$

$$\frac{EJ}{l} \begin{pmatrix} 6.057 & -6.127 \\ -6.127 & 10.090 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \frac{w}{l} \end{pmatrix} = l P \begin{pmatrix} 0 \\ 1.000 \end{pmatrix}$$

Rozwiązanie.
Solution.

$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \frac{w}{l} \end{pmatrix} = \frac{l^2 P}{EJ} \begin{pmatrix} 0.260 \\ 0.257 \end{pmatrix}$$

Siły i momenty brzegowe.
End forces and moments.

$$\Phi_A^1 = -1.028\, l\, P$$

$$\Phi_1^1 = -0.491\, l\, P$$

$$\Phi_1^2 = 0.491\, l\, P$$

$$\Phi_2^3 = 0.236\, l\, P$$

$$W_A^1 = -1.549\, P$$

$$W_1^1 = 1.442\, P$$

$$W_1^2 = 0.182\, P$$

$$W_B^2 = -0.508\, P$$

$$W_B^3 = -1.029\, P$$

$$W_2^3 = -0.174\, P$$