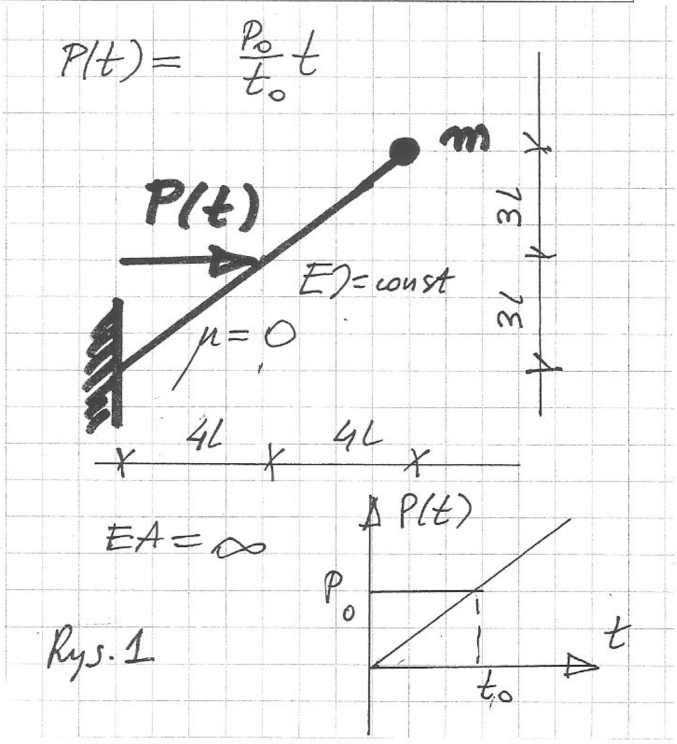
Egzamin pisemny z Mechaniki Konstrukcji II, 29 VI 2016 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1

Dany jest wspornik nieważki z masą skupioną na końcu, obciążony jak na Rys.1. Znaleźć przemieszczenie masy w chwili *t*. Przyjąć jednorodne warunki początkowe.

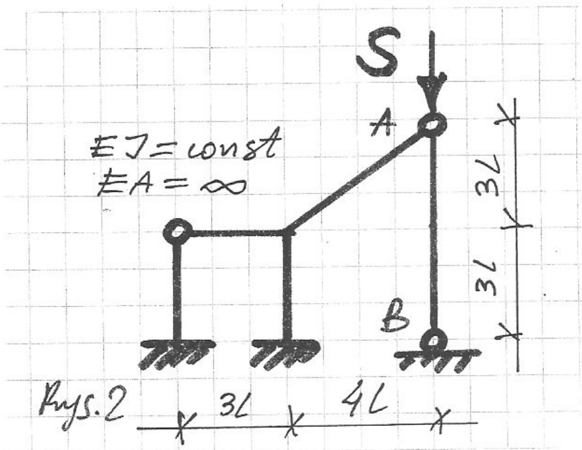
(The given weightless cantilever with a concentrated mass at its end is subject to the load described in Fig.1. Find the displacement of the mass at time instant t. Assume the homogeneous initial conditions.)



Zadanie 2

Znaleźć siłę krytyczną i sprawdzić wyboczenie lokalne pręta AB.

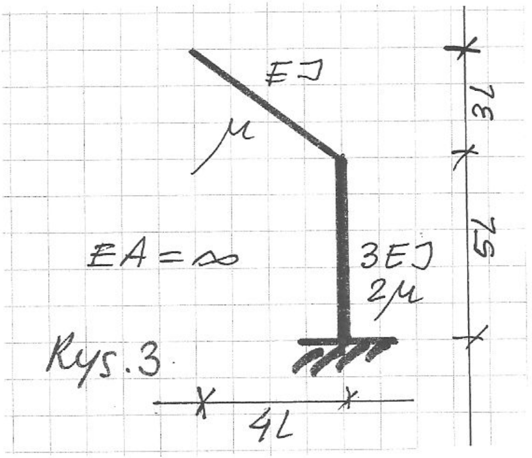
(Compute the critical force and check the local stability of the bar AB)



Zadanie 3

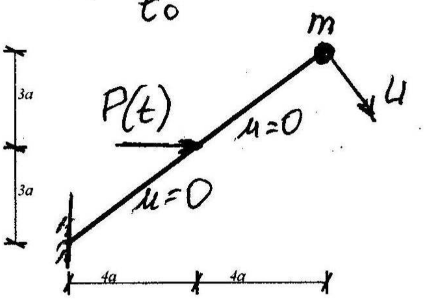
Zapisać równania określające częstości drgań własnych .

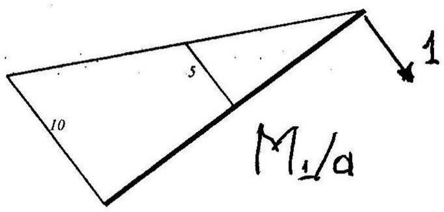
(Write down the equations which determine the eigenfrequencies)



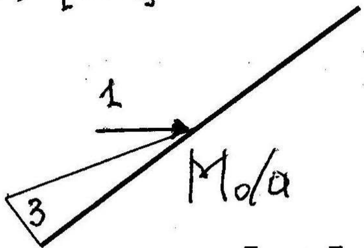
$P(t) = \frac{P_0}{t_0} t$

zad. 1/egzamin MK2/29.06.2016.





$q_V(t) = [u(t)]$



Macierz mas:

$$\underbrace{m \; [\,1\,]}_{m_{11}}$$

Macierz podatności:

$$\underbrace{\left[\frac{a^3}{EJ}\right] \cdot [\,3.3333E+02\,]}_{d_{11}}$$

$$d_{10} = [\,6.250E+01\,] \left[\frac{a^3}{EJ}\right]$$

Różniczkowe równanie rozwiązujące:

$u(t) = -d_{11} m_{11} \ddot{u}(t) + P(t) d_{10} \; /: m_{11} d_{11}$

$\ddot{u}(t) + \underbrace{\frac{1}{m_{11} d_{11}}}_{= \omega^2} u(t) = \underbrace{\frac{P_0 d_{10}}{m_{11} d_{11} t_0}}_{= b} t$, gdzie: $\begin{cases} \omega^2 = 3 \cdot 10^{-3} \frac{EJ}{ma^3} \\ b = 0.1875 \frac{P_0}{m t_0} \end{cases}$.

$\ddot{u}(t) + \omega^2 u(t) = b \cdot t$

CORJ: $u_{og}(t) = A \sin\omega t + B \cos\omega t$

CSRN postaci $u_s(t) = A_s + B_s t$ → podstawienie do równania różniczkowego $\omega^2 A_s + \omega^2 B_s t = bt \Rightarrow \begin{cases} A_s = 0 \\ B_s = \frac{b}{\omega^2} = 62.5 \frac{P_0 a^3}{t_0 EJ} \end{cases}$

Zatem: rozwiązanie jest postaci:

$u(t) = u_{og}(t) + u_s(t) = A \sin\omega t + B \cos\omega t + B_s t$

Warunki początkowe:

$u(0) = 0 \Rightarrow B = 0$.

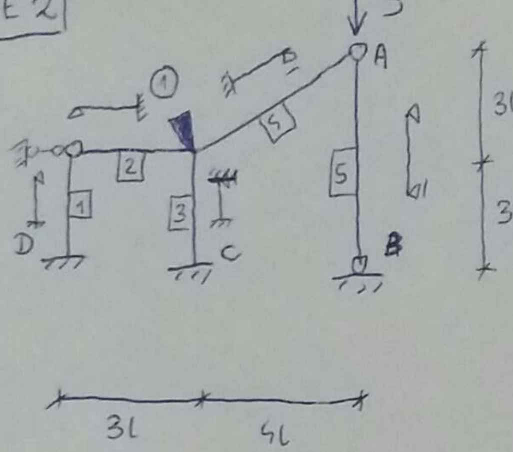
$\dot{u}(t) = \omega A \cos\omega t + B_s$; $\dot{u}(0) = 0 \Rightarrow \omega A + B_s = 0 \Rightarrow$ $A = -\frac{B_s}{\omega}$

Odpowiedź:

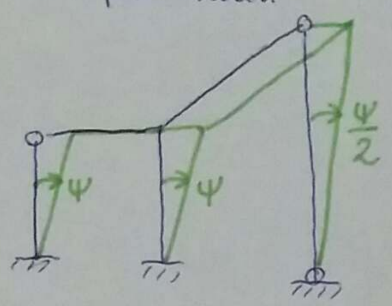
$$u(t) = B_s \left(t - \frac{\sin\omega t}{\omega} \right).$$

oprac.: K. Hetmański

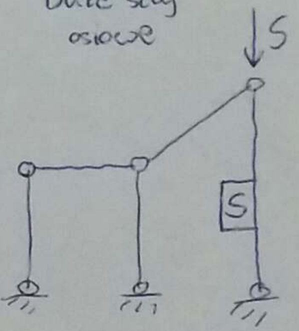
ZADANIE 2



Plan przemieszczeń



Duże siły osiowe



$$q = \begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix}$$

$$\sigma = l\sqrt{\frac{S}{EJ}} \Rightarrow S = \frac{\sigma^2}{l^2} EJ$$

	l	ψ	S	σ
1.	$3l$	ψ	0	0
2.	$3l$	0	0	0
3.	$3l$	ψ	0	0
4.	$5l$	0	0	0
5.	$6l$	$\frac{\psi}{2}$	S	6σ

Równania równowagi:

1) $\Phi_1^2 + \Phi_1^3 + \Phi_1^4 = 0$

2) $\Phi_D^1 \cdot \bar{\psi} + (\Phi_C^3 + \Phi_1^3) \cdot \bar{\psi} + \bar{L}_S = 0 \quad \| \cdot (-1)$

$$\bar{L}_S = 6l \cdot S \cdot \frac{\psi}{2} \cdot \frac{\bar{\psi}}{2} = \frac{3}{2} S l \psi \bar{\psi} = \frac{3}{2}\sigma^2 \frac{EJ}{l} \psi \cdot \bar{\psi}$$

2) $-\Phi_D^1 - \Phi_C^3 - \Phi_1^3 - \frac{3}{2}\sigma^2 \frac{EJ}{l}\psi = 0$

Wzory transformacyjne:

$$\Phi_1^2 = \frac{3EJ}{3l}\varphi_1 \qquad \Phi_1^4 = \frac{3EJ}{5l}\varphi_1$$

$$\Phi_1^3 = \frac{2EJ}{3l}(2\varphi_1 - 3\psi) \qquad \Phi_D^1 = \frac{3EJ}{3l}(-\psi)$$

$$\Phi_C^3 = \frac{2EJ}{3l}(\varphi_1 - 3\psi)$$

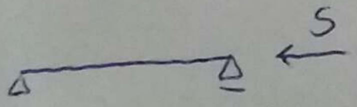
$$K = \frac{EJ}{l}\begin{bmatrix} \frac{44}{15} & -2 \\ -2 & 5 - \frac{3}{2}\sigma^2 \end{bmatrix}$$

$$\det K = \frac{44}{15}\left(5 - \frac{3}{2}\sigma^2\right) - 2 \cdot 2 = 0$$

$$\sigma^2 = \frac{80}{33} \approx 2{,}42$$

$$S_{kr} = \sigma^2 \frac{EJ}{l^2} = 2{,}42\frac{EJ}{l^2}$$

Wyboczenie lokalne pręta AB:

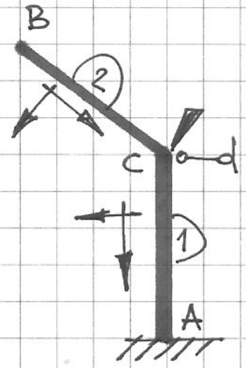


$$S_{kr}^{lok} = \frac{\pi^2 EJ}{(\mu L)^2} = \frac{3{,}14^2 \; EJ}{(1{,}0 \cdot 6l)^2} \approx 0{,}27\frac{EJ}{l^2}$$

$$\boxed{S_{kr}^{lok} = 0{,}27\frac{EJ}{l^2} < 2{,}42\frac{EJ}{l^2}}$$

Zadanie 3 / Problem #3

Primary structure / Schemat zastępczy



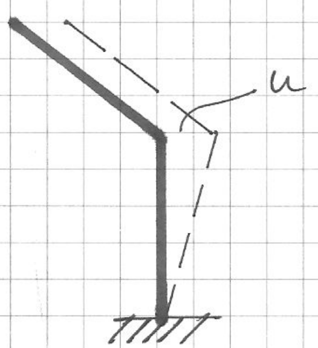
$$\lambda = L\sqrt[4]{\frac{\mu\omega^2}{EJ}}$$

$$\lambda^{(1)} = 5\sqrt[4]{2}\,\lambda = 4,5\,\lambda$$

$$\lambda^{(2)} = 5\lambda$$

$$q = \begin{bmatrix} \varphi_C \\ \frac{u}{L} \end{bmatrix}$$

Plan przesunięć



$$w_C^{(1)} = -u \qquad w_C^{(2)} = -\frac{3}{5}u$$

$$u^{(2)} = \frac{4}{5}u$$

Równania równowagi:

$$\Phi_C^{(1)} + \Phi_C^{(2)} = 0$$

$$W_C^{(1)}\cdot \bar{w}_C^{(1)} + W_C^{(2)}\cdot \bar{w}_C^{(2)} - B_{\parallel}^{(2)}\bar{u}^{(2)} = 0$$

Wzory transformacyjne:

$$\Phi_C^{(1)} = \frac{3EJ}{5L}\left[\alpha(4,5\lambda)\varphi_C - \vartheta(4,5\lambda)\cdot\frac{-u}{5L}\right]$$

$$\Phi_C^{(2)} = \frac{EJ}{5L}\left[\alpha''(5\lambda)\varphi_C - \vartheta''(5\lambda)\left(-\frac{3}{5}u\right)\cdot\frac{1}{5L}\right]$$

$$W_C^{(1)} = \frac{3EJ}{(5L)^2}\left[\vartheta(4,5\lambda)\varphi_C + \gamma(4,5\lambda)\cdot\frac{-u}{5L}\right]$$

$$W_C^{(2)} = -\frac{EJ}{(5L)^2}\left[\vartheta''(5\lambda)\varphi_C - \gamma''(5\lambda)\cdot\left(-\frac{3}{5}u\right)\cdot\frac{1}{5L}\right]$$

$$B_{\parallel}^{(2)} = \mu\cdot 5L\cdot\omega^2\cdot\frac{4}{5}u = \frac{EJ}{L^2}\left[4\lambda^4\frac{u}{L}\right]$$