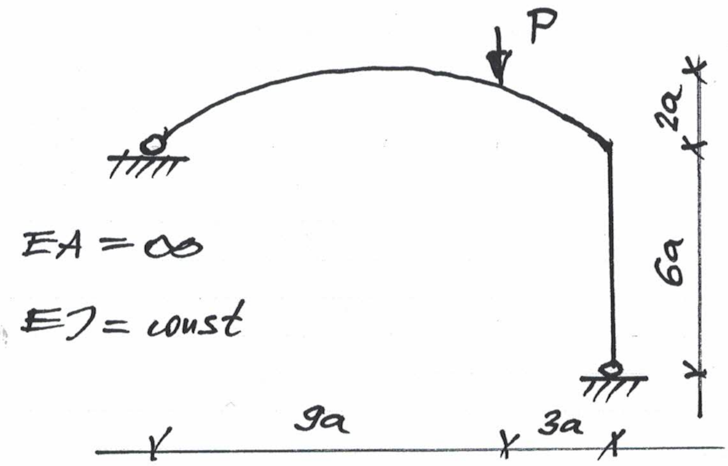
Egzamin pisemny z Mechaniki Konstrukcji I, 3 lutego 2025 r.

Imię i NAZWISKO				
Nazwisko prowadzącego ćwiczenia				Nr albumu
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egz. pis.	Ocena Ostateczna Data

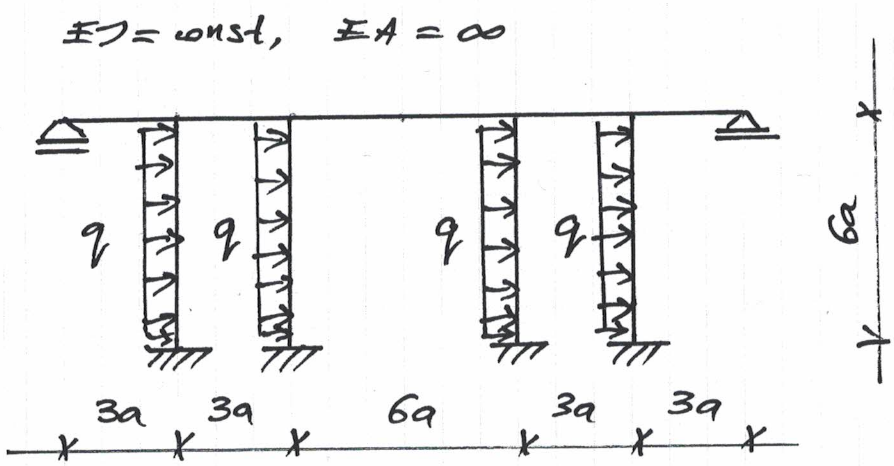
Zadanie 1

Dany jest ramołuk z łukiem parabolicznym, małowyniosłym obciążony jak na rysunku. Znaleźć moment zginający w kluczu łuku. (*Given is a frame with a shallow parabolic arch loaded as shown in the figure; compute the bending moment in the middle of the arch*)



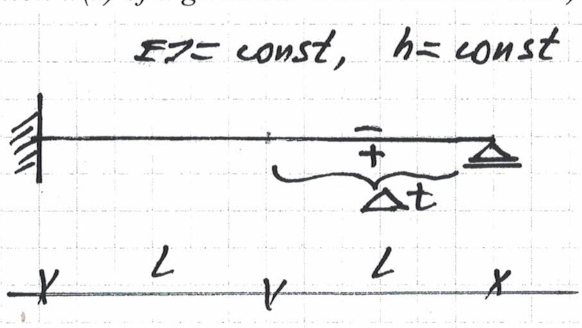
Zadanie 2

Dana jest rama płaska. Znaleźć wykres momentów zginających ręczną metodą przemieszczeń (*Given is a planar frame. Construct the diagram of bending moments by the displacement method*)



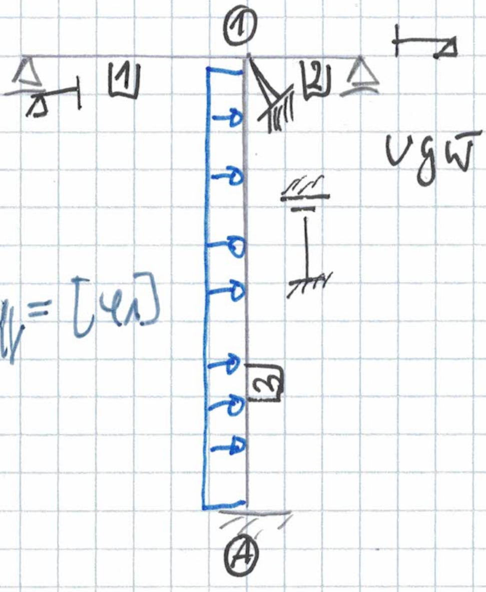
Zadanie 3

Skonstruować linię ugięcia danego pręta obciążonego termicznie.
(*Construct the deflection function w(x) of a given bar with a thermal load*)



ZADANIE 2

2x redukcja + kondensacja statyczna



$q = [\varphi_1]$

Momenty wyjściowe

[3]

$$\bar{\Phi}_A^{(3)} = -\frac{q(6l)^2}{3} = -12\,ql^2$$

$$\bar{\Phi}_1^{(3)} = -\frac{q(6l)^2}{6} = -6\,ql^2$$

WT

$$\Phi_1^{(1)} = \frac{EJ}{l}\varphi_1$$

$$\Phi_1^{(2)} = \frac{EJ}{l}[2\varphi_1]$$

$$\Phi_1^{(3)} = \frac{EJ}{l}\left[\frac{1}{6}\varphi_1\right] - 6ql^2$$

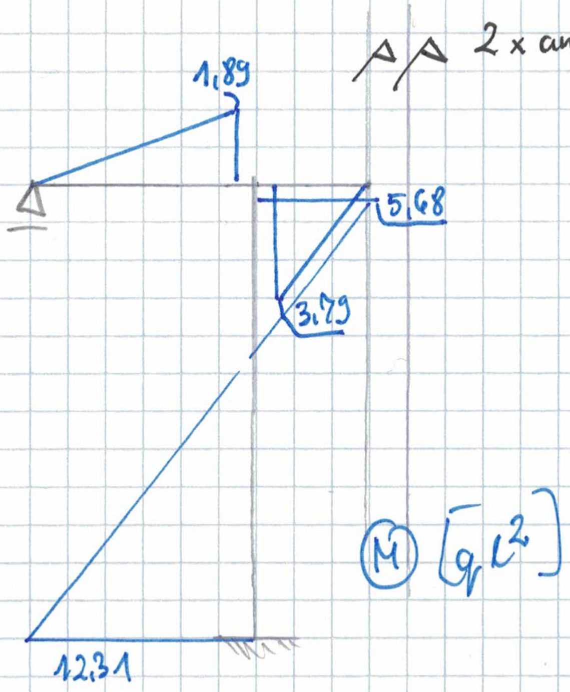
$$\Phi_A^{(3)} = \frac{EJ}{l}\left[-\frac{1}{6}\varphi_1\right] - 12ql^2$$

r. r. MP $\quad \Phi_1^{(1)} + \Phi_1^{(2)} + \Phi_1^{(3)} = 0$

$$\frac{EJ}{l}\left[1 + 2 + \frac{1}{6}\right]\varphi_1 = 6ql^2$$

$$\varphi_1 = \frac{36}{19}\frac{ql^3}{EJ}$$

2 x antysymetria



ZAD. 3.   EGZ. MK1 ST   03.02.2025

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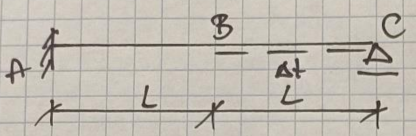
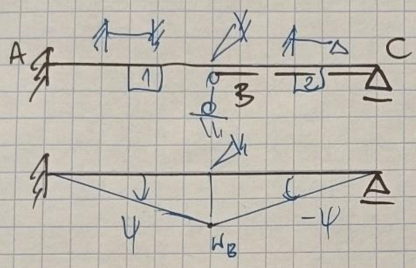


$w(x) = ?$

$$w(x) = \begin{cases} w_1(x) & AB \\ w_2(x) & BC \end{cases}$$

I wariant



$$q = \begin{bmatrix} \varphi_B \\ \psi \end{bmatrix}$$

$$w_B = \psi \cdot L$$

równanie M.?

1) $\Sigma M_B = 0 \Rightarrow \phi_B^{(1)} + \phi_B^{(2)} = 0$

2) t.k $\Rightarrow (\phi_A^{(1)} + \phi_B^{(1)})\,\psi + \phi_B^{(2)} \cdot (-\psi) = 0 \Rightarrow -\phi_A^{1} - \phi_B^{1} + \phi_B^{2} = 0$

wzory transf.

$$\phi_A^{(1)} = \frac{2EJ}{L}\left[\varphi_B - 3\psi\right] = \frac{EJ}{L}\left[2\varphi_B - 6\psi\right]$$

$$\phi_B^{(1)} = \frac{2EJ}{L}\left[2\varphi_B - 3\psi\right] = \frac{EJ}{L}\left[4\varphi_B - 6\psi\right]$$

$$\phi_B^{(2)} = \frac{3EJ}{L}\left[\varphi_B - (-\psi)\right] + \phi_B^{\alpha(2)} = \frac{EJ}{L}\left[3\varphi_B + 3\psi\right] - \frac{3}{2}\frac{EJ\alpha_t\Delta t}{h}$$

$$\frac{EJ}{L}\begin{bmatrix} 4+3 & -6+3 \\ -2-4+3 & 6+6+3 \end{bmatrix}\begin{bmatrix} \varphi_B \\ \psi \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}\frac{EJ\alpha_t\Delta t}{h}$$

$$\frac{EJ}{L}\begin{bmatrix} 7 & -3 \\ -3 & 15 \end{bmatrix}\begin{bmatrix} \varphi_B \\ \psi \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}\frac{EJ\alpha_t\Delta t}{h}$$

ZAD.3. EGZ. MK1 ST 03.02.2025

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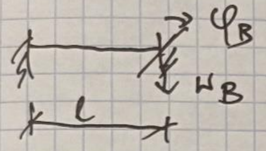
$$\begin{bmatrix}\varphi_B\\ \psi\end{bmatrix} = \begin{bmatrix}0{,}28125\\ 0{,}15625\end{bmatrix}\frac{\alpha_t \Delta t \cdot l}{h} \Rightarrow w_B = 0{,}15625\, l \,\frac{\alpha_t \Delta t}{h}\, l$$

ugięcie belki „1" (A – B)



$$w_1(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$$

$$\varphi_1(x) = C_1 + 2C_2 x + 3C_3 x^2$$

$$w_1(0) = 0 \Rightarrow C_0 = 0$$

$$\varphi_1(0) = 0 \Rightarrow C_1 = 0$$

$$w(l) = w_B = 0{,}15625 \frac{\alpha_t \Delta t\, l^2}{h} \Rightarrow C_2 \cdot l^2 + C_3 \cdot l^3 = 0{,}15625 \frac{\alpha_t \Delta t\, l^2}{h}$$

$$\varphi(l) = 0{,}28125 \frac{\alpha_t \Delta t\, l}{h} \Rightarrow 2C_2 l + 3C_3 l^2 = 0{,}28125 \frac{\alpha_t \Delta t\, l}{h}$$

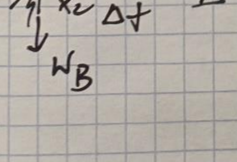
$$\begin{cases} -2C_2 - 2C_3 l = -0{,}3125 \dfrac{\alpha_t \Delta t}{h} \\ 2C_2 + 3C_3 l = 0{,}28125 \dfrac{\alpha_t \Delta t}{h} \end{cases}$$

$$C_3 = -0{,}03125 \frac{\alpha_t \Delta t}{h \cdot l}$$

$$C_2 = -C_3 l + 0{,}15625 \frac{\alpha_t \Delta t}{h} = 0{,}125 \frac{\alpha_t \Delta t}{h}$$

$$w_1(x) = \frac{\alpha_t \Delta t}{h}\left[0{,}125 x^2 - 0{,}03125 \frac{1}{l} x^3\right]$$

ugięcie belki „2" (B – C)



$$w_2(x_2) = D_0 + D_1 x_2 + D_2 x_2^2 + D_3 x_2^3$$

$$\varphi_2(x_2) = D_1 + 2D_2 x_2 + 3D_3 x_2^2$$

$$M_2(x_2) = EJ\left(\varkappa_2 - \frac{\alpha_t \Delta t}{h}\right) =$$

$$= EJ\left[-\frac{d^2 w_2}{dx_2^2} - \frac{\alpha_t \Delta t}{h}\right] =$$

2.3

$$M_2(x_2) = EJ\left[-2D_2 - 6D_3 x_2 - \frac{\alpha_t \Delta t}{h}\right]$$

war. brzegowe

$$w_2(0) = w_B = 0{,}15625 \frac{\alpha_t \Delta t\, l^2}{h} \Rightarrow D_0 = 0{,}15625 \frac{\alpha_t \Delta t\, l^2}{h}$$

$$\varphi_2(0) = \varphi_B = 0{,}28125 \frac{\alpha_t \Delta t\, l}{h} \Rightarrow D_1 = 0{,}28125 \frac{\alpha_t \Delta t\, l}{h}$$

$$(*)\quad w_2(l) = 0 \Rightarrow 0{,}15625 \frac{\alpha_t \Delta t\, l^2}{h} + 0{,}28125 \frac{\alpha_t \Delta t\, l^2}{h} + D_2 l^2 + D_3 l^3 = 0$$

$$(**)\quad M_2(l) = 0 \Rightarrow EJ\left[-2D_2 - 6D_3 l - \frac{\alpha_t \Delta t}{h}\right] = 0$$

z (*)

$$D_2 + D_3 \cdot l = -0{,}4375 \frac{\alpha_t \Delta t}{h}$$

z (**)

$$-D_2 - 3D_3 l = \frac{\alpha_t \Delta t}{2h}$$

$$-2D_3 l = 0{,}0625 \frac{\alpha_t \Delta t}{h} \Rightarrow D_3 = -0{,}03125 \frac{\alpha_t \Delta t}{h \cdot l}$$

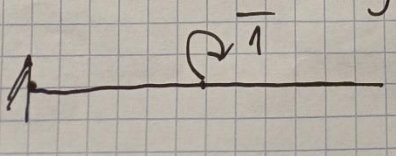
$$D_2 = -0{,}4375 \frac{\alpha_t \Delta t}{h} + 0{,}03125 \frac{\alpha_t \Delta t}{h} = -0{,}40625 \frac{\alpha_t \Delta t}{h}$$

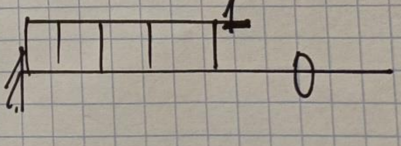
$$w_2(x_2) = \frac{\alpha_t \Delta t\, l^2}{h}\left[0{,}15625 + 0{,}28125 \frac{x_2}{l} - 0{,}40625 \frac{x_2^2}{l^2} - 0{,}03125 \frac{x_2^3}{l^3}\right.$$

II wariant rozwiązania

można φ_B i w_B ustalić ze wzorów MAXWELLA - MOHRA

stan wirtualny φ_B:

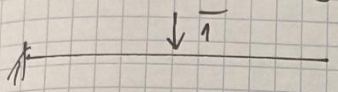


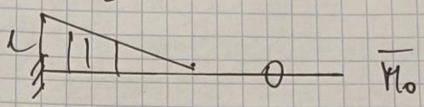


$\overline{M}_0$

$\bar{1}\,\varphi_B = \int \bar{M}_0 \cdot \frac{M}{EJ}\, ds$

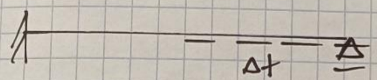
Stan wirtualny do w_B:



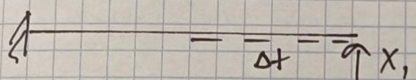


$$\bar{1}\cdot w_B = \int \bar{M}_0 \frac{M}{EJ}\, ds + \underbrace{\int \bar{M}_0 \frac{\alpha_t \Delta t}{h}\, ds}_{=0}$$ ponieważ $\bar{M}_0 = 0$

Stan rzeczywisty

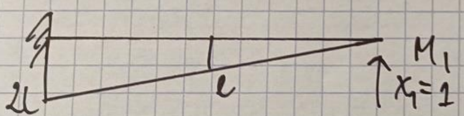


SSN



$$\delta_{11} X_1 + \delta_{10} = 0$$

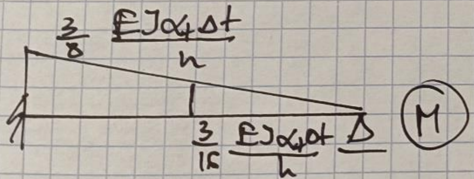
Stan $X_1 = 1$



$$\delta_{11} = \int \frac{M_1^2}{EJ}\, ds = \frac{1}{EJ}\left[\frac{1}{2} 2l \cdot 2l \cdot \frac{2}{3} 2l\right] = \frac{8}{3}\frac{l^3}{EJ}$$

$$\delta_{10} = \frac{\alpha_t \Delta t}{h}\cdot \frac{1}{2} l \cdot l = \frac{1}{2}\frac{\alpha_t \Delta t\, l^2}{h}$$

$$X_1 = -\frac{1}{2}\cdot\frac{3}{8}\frac{\alpha_t \Delta t\, l^2}{h}\cdot\frac{EJ}{l^3} = -\frac{3}{16}\frac{EJ\,\alpha_t \Delta t}{h\cdot l}$$



$$\varphi_B = \frac{1}{EJ}\left[1\cdot l\cdot\frac{1}{2}\left(\frac{3}{8}+\frac{3}{16}\right)\frac{EJ\alpha_t\Delta t}{h}\right] = \frac{9}{32}\frac{\alpha_t \Delta t\, l}{h} = 0{,}28125\frac{\alpha_t\Delta t\, l}{h}$$

$$w_B = \frac{1}{EJ}\left[\frac{1}{2} l\cdot l\cdot\left(\frac{2}{3}\frac{3}{8}+\frac{1}{3}\frac{3}{16}\right)\frac{EJ\alpha_t\Delta t}{h}\right] = \frac{5}{32}\frac{\alpha_t\Delta t\cdot l^2}{h} = 0{,}15625\frac{\alpha_t\Delta t\, l^2}{h}$$

dalej jak na str. „2" i „3"