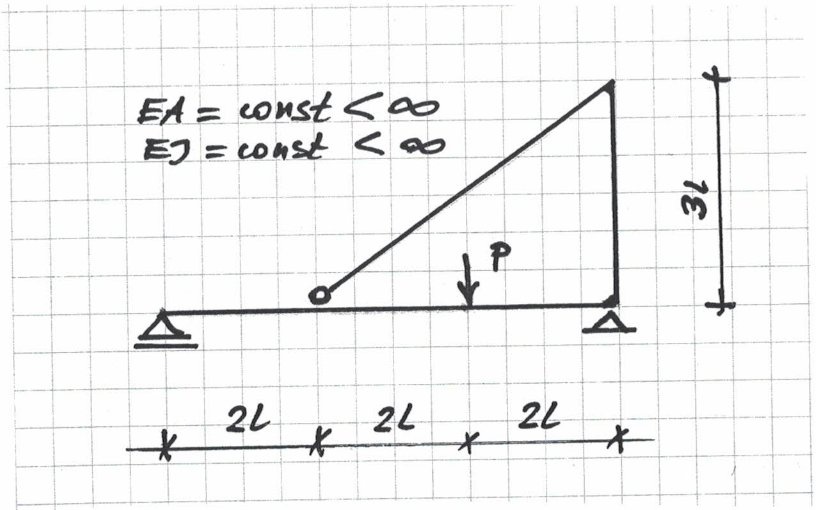
Egzamin pisemny z Mechaniki Konstrukcji I, 1 II 2023 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1

Dana jest rama płaska obciążona jak na rysunku; zapisać układ równań macierzowej metody przemieszczeń.

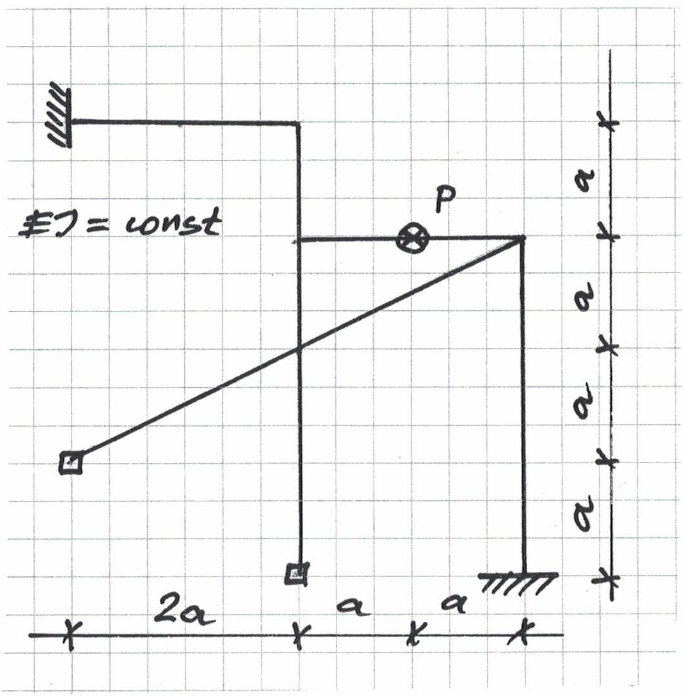
(For the given frame write down the equations of the displacement method in its matrix version)



Zadanie 2

Dany jest ruszt przegubowy, obciążony jak na rysunku. Sporządzić wykres momentów zginających.

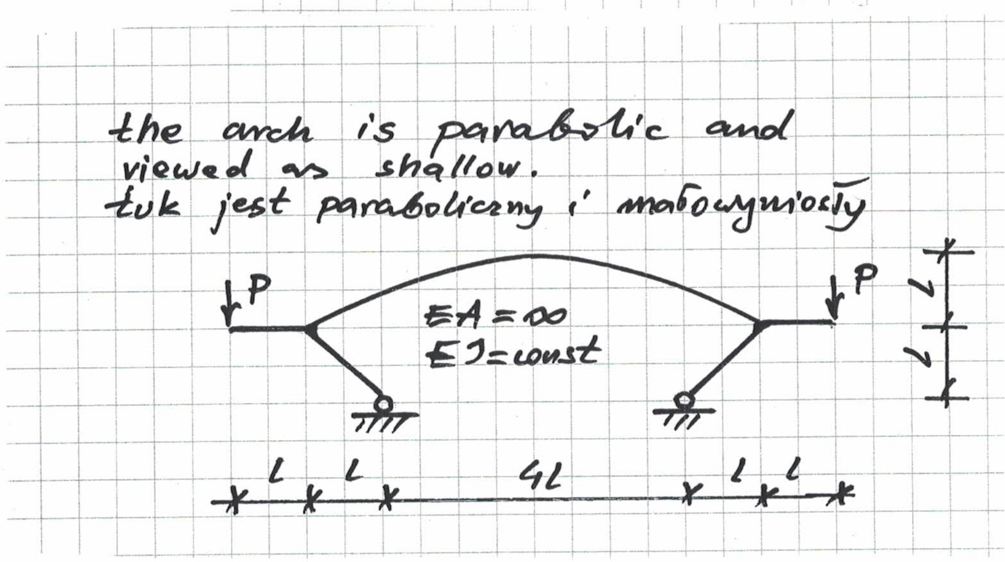
(There is given a system of beams, loaded as shown in the figure. Find the diagram of the bending moments).



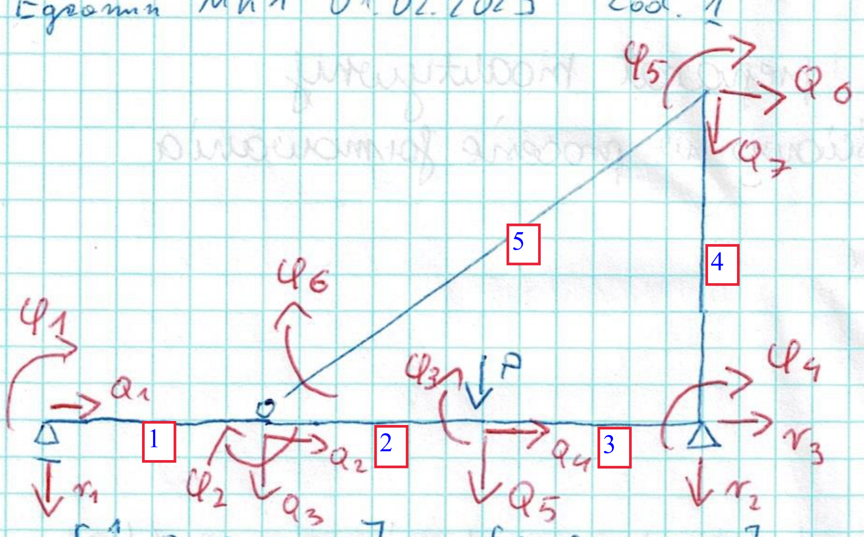
Zadanie 3.

Znaleźć rozkład momentów zginających w danym ramołuku

(Find the diagram of the bending moments in a given archframe).



Egzamin MK1 01.02.2023 Zad. 1



$$\mathbf{q} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} \end{bmatrix} \frac{EA}{L} \qquad \mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{5} \end{bmatrix} \frac{EJ}{L}$$

$$\Psi_1 = \frac{Q_3 - r_1}{2L} \qquad \Psi_2 = \frac{Q_5 - Q_3}{2L}$$

$$\Psi_3 = \frac{r_2 - Q_5}{2L} \qquad \Psi_4 = \frac{Q_6 - r_3}{3L}$$

$$\Psi_5 = \frac{\frac{4}{5} Q_7 + \frac{3}{5} Q_6 - \frac{4}{5} Q_3 - \frac{3}{5} Q_2}{5L}$$

$$\mathbf{B} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4/5 & \frac{3}{5} & 0 & 0 & \frac{4}{5} & -\frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$^{*}\mathbf{B} = \begin{bmatrix} 0 & 0 & -\frac{1}{2L} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2L} & 0 & -\frac{1}{2L} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2L} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{3L} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{25L} & 4/25L & 0 & 0 & -\frac{3}{25L} & -\frac{4}{25L} & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^{*}\mathbf{S} = \mathbf{S}^{*} = \begin{bmatrix} \frac{1}{2L} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{1}{2L} & 0 \\ 0 & 0 & \frac{1}{3L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}^{*} = \left[\begin{array}{c|cccccc} & 0 & 1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 1 & 0 & 0 & 0 \\ -//- & 0 & 0 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \qquad \begin{aligned} \mathbf{K}\mathbf{q} &= \mathbf{Q} \\ \mathbf{q} &= \mathbf{K}^{-1}\mathbf{Q} \end{aligned}$$

$$\mathbf{K} = \mathbf{B}^T \mathbf{E} \mathbf{B} + 2\, {}^{*}\mathbf{B}^T \mathbf{D}\, {}^{*}\mathbf{B} + 2\, \mathbf{B}^{*T} \mathbf{D} \mathbf{B}^{*} + {}^{*}\mathbf{B}^T \mathbf{D} \mathbf{B}^{*} + \mathbf{B}^{*T} \mathbf{D}\, {}^{*}\mathbf{B}$$

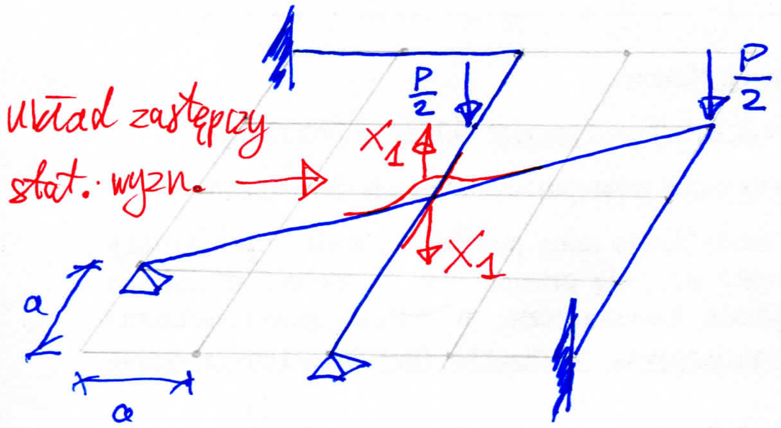
$$\mathbf{N} = \mathbf{E} \mathbf{B} \mathbf{q}$$

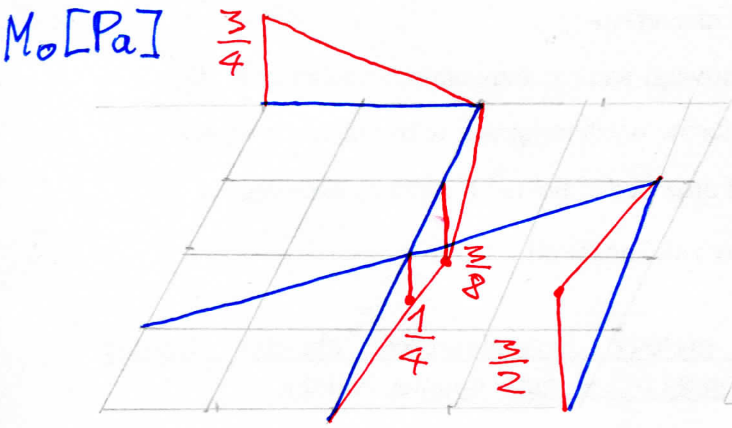
$$^{*}\mathbf{\Phi} = \mathbf{D}\left(2\, {}^{*}\mathbf{B} \mathbf{q} + \mathbf{B}^{*} \mathbf{q}\right) \qquad \mathbf{\Phi}^{*} = \mathbf{D}\left({}^{*}\mathbf{B} \mathbf{q} + 2 \mathbf{B}^{*} \mathbf{q}\right)$$

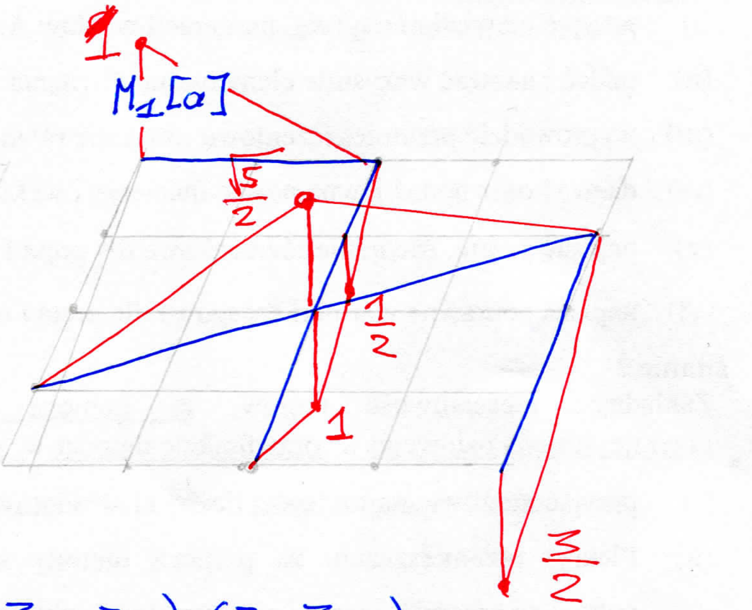
$$\mathbf{R} = \mathbf{S}^T \mathbf{N} + {}^{*}\mathbf{S}^T\, {}^{*}\mathbf{\Phi} + S^{*T} \mathbf{\Phi}^{*}$$

Egzamin z Mechaniki Konstrukcji 1 1.02.2023r Zadanie 2

Wygodnie jest zastąpić belkę stat. wyznaczalną obciążeniem stat. równoważnym:







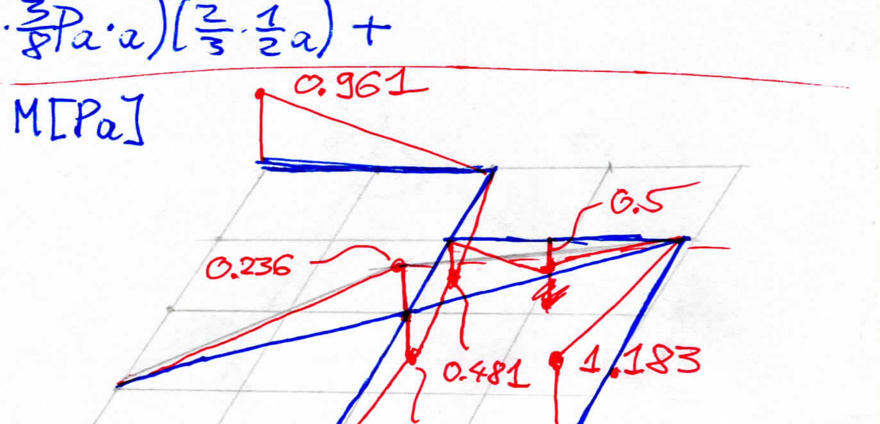
$$\delta_{11}=\frac{1}{EJ}\Big[\left(\tfrac{1}{2}a\cdot 2a\right)\cdot\left(\tfrac{2}{3}a\right)+\left(\tfrac{1}{2}\cdot\tfrac{3}{2}a\cdot 3a\right)\cdot\left(\tfrac{2}{3}\cdot\tfrac{3}{2}a\right)+2\cdot\left(\tfrac{1}{2}\cdot a\cdot 2a\right)\cdot\left(\tfrac{2}{3}a\right)+2\cdot\left(\tfrac{1}{2}\cdot\tfrac{\sqrt{5}}{2}a\cdot\sqrt{5}a\right)\cdot\left(\tfrac{2}{3}\cdot\tfrac{\sqrt{5}}{2}a\right)\Big]$$

$$=\left(\frac{17}{4}+\frac{5\sqrt{5}}{6}\right)\frac{a^3}{EJ}$$

$$\delta_{10}=\frac{1}{EJ}\Big[\left(\tfrac{1}{2}\cdot\tfrac{3}{4}Pa\cdot 2a\right)\left(\tfrac{2}{3}a\right)+\left(\tfrac{1}{2}\cdot\tfrac{3}{2}Pa\cdot 3a\right)\left(-\tfrac{2}{3}\cdot\tfrac{3}{2}a\right)$$
$$+\left(\tfrac{1}{2}\cdot\tfrac{1}{4}Pa\cdot 2a\right)\left(\tfrac{2}{3}a\right)+\left(\tfrac{1}{2}\cdot\tfrac{3}{8}Pa\cdot a\right)\left(\tfrac{2}{3}\cdot\tfrac{1}{2}a\right)+$$
$$+\left(\tfrac{1}{2}\cdot\tfrac{1}{4}Pa\cdot a\right)\cdot\left(\tfrac{2}{3}\cdot a+\tfrac{1}{3}\cdot\tfrac{a}{2}\right)$$
$$+\left(\tfrac{1}{2}\cdot\tfrac{3}{8}Pa\cdot a\right)\left(\tfrac{1}{3}a+\tfrac{2}{3}\cdot\tfrac{a}{2}\right)\Big]$$
$$=-\frac{31}{24}\frac{Pa^3}{EJ}$$

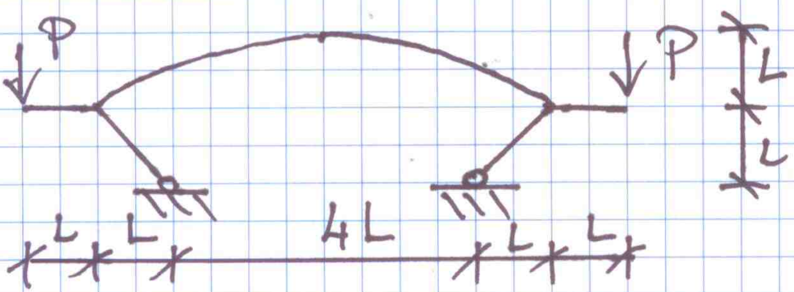
$$\delta_{11}X_1+\delta_{10}=0$$

$$\Rightarrow X_1=0.21128P$$



Rozwiązanie przygotował Karol Bołbotowski

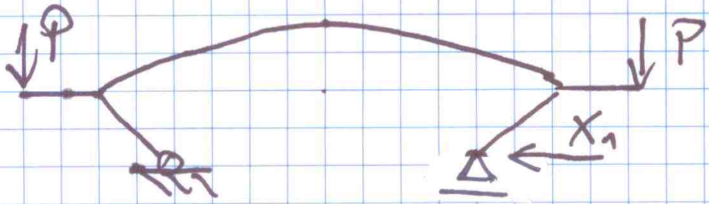
$EA \to \infty \quad EJ = \mathrm{const}$



$$y = \frac{4f}{(L_c)^2}\, x\,(L_c - x)^2 =$$

$$= \frac{x}{9L}\,(6L - x)$$

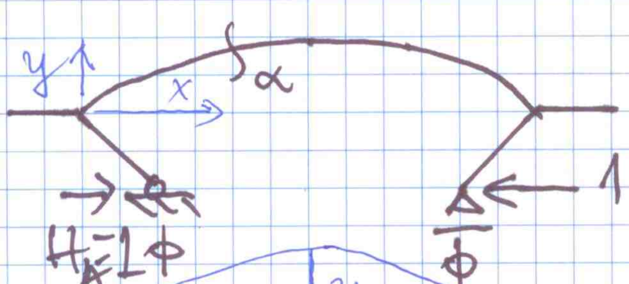
$ds \cong dx$ (Łuk nałow.)

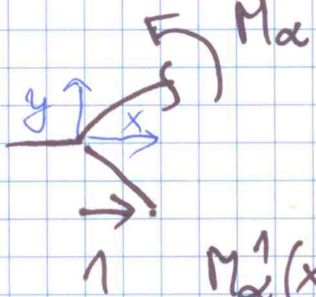


$$\delta_{11} X_1 + \delta_{10} = 0$$

s.s.W

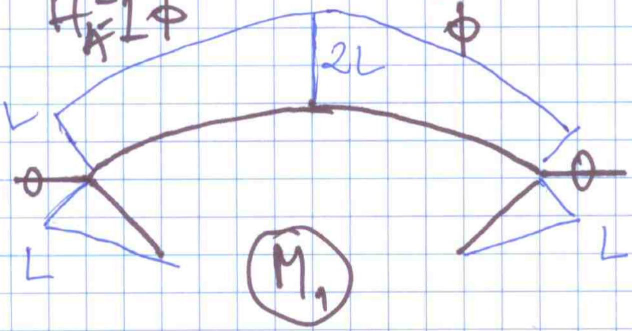
Stan $X_1 = 1$





$$M_\alpha^1(x) = -1\cdot(y + L) =$$

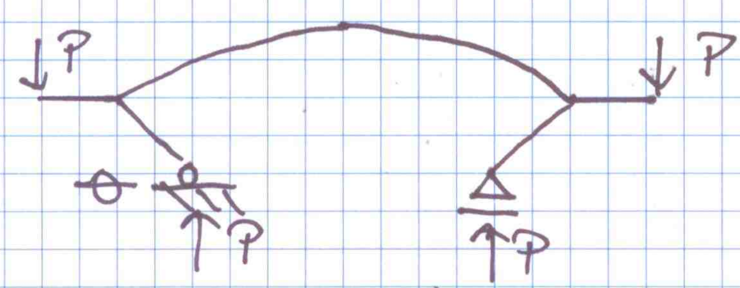
$$= -\left[\frac{x}{9L}(6L - x) + L\right]$$

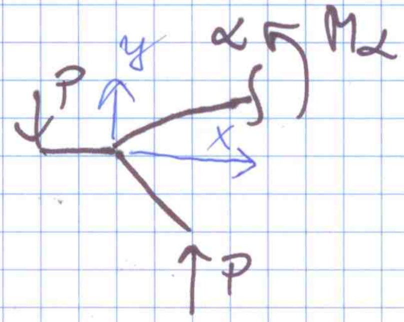


$$\delta_{11} = \int \frac{(M_1)^2}{EJ}\, ds = \frac{1}{EJ}\left[2\cdot\frac{1}{2}L\cdot\sqrt{2}L\cdot\frac{2}{3}L\right] +$$

$$+ \int_0^{6L} \frac{1}{EJ}\left[\frac{x}{9L}(6L - x) + L\right]^2 dx = 18{,}143\,\frac{L^3}{EJ}$$

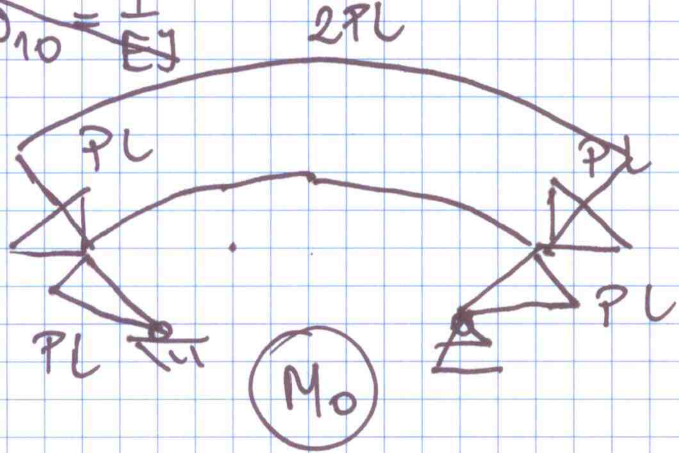
stan „0"





$$M_0(x) = -P(x+L) + P(x-L) = $$
$$= -2PL$$

$\cancel{\delta_{10} = \frac{1}{EJ}}$



$$\delta_{10} = \frac{1}{EJ}\left[2\cdot\frac{1}{2}PL\cdot L\cdot 0 + 2\cdot\frac{1}{2}\cdot PL\cdot\sqrt{2}L\cdot\frac{2}{3}L + \int_0^{6L}(-2PL)\left[-\frac{x}{9L}(6L-x)\,l\right]dx\right] = 20{,}943\,\frac{PL^3}{EJ}$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -1{,}154\,P$$

$$M(x) = M_0(x) + X_1\cdot M_1(x)$$