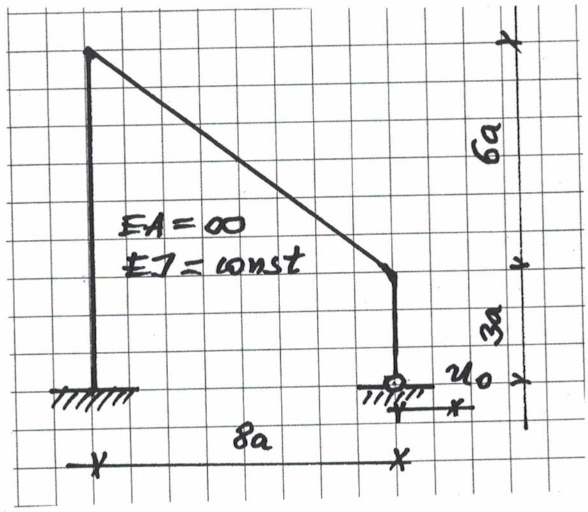
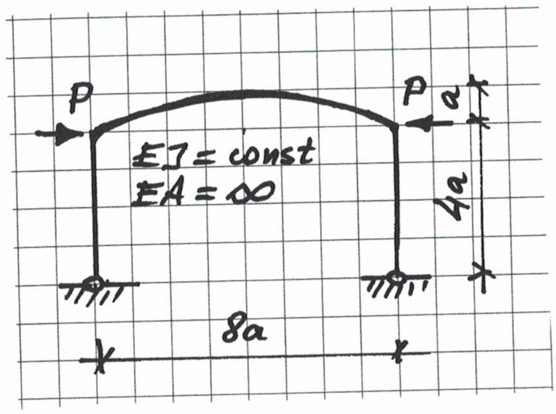
Egzamin pisemny z Mechaniki Konstrukcji I, 5 II 2020 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1
Dana jest rama płaska obciążona jak na rysunku. Sporządzić wykres momentów zginających metodą przemieszczeń.
(For the given frame construct the diagram of the bending moments by the displacement method)

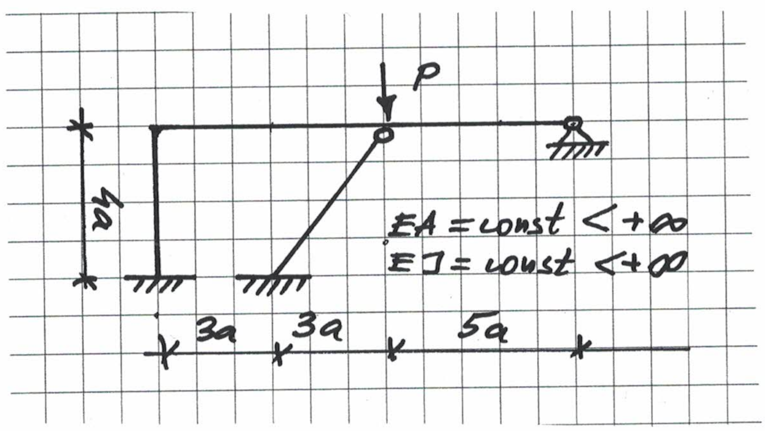


Zadanie 2
Dany jest ramołuk (z łukiem parabolicznym małowyniosłym) obciążony jak na rysunku. Sporządzić wykres momentów zginających metodą sił.
(Consider the given frame with an arch (being shallow and parabolic) loaded as shown in the figure. Find the diagram of bending moments by the force method).

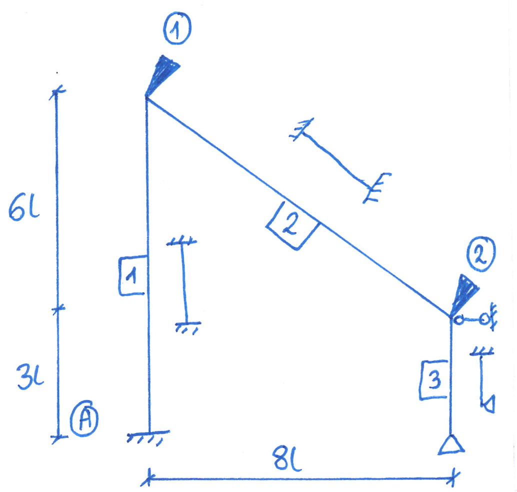


Zadanie 3
Rozważmy ramę obciążoną jak na rysunku. Zapisać równania macierzowej metody przemieszczeń.

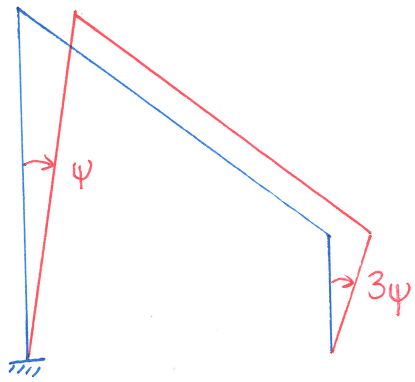
(Consider the frame loaded as shown in the figure. Write down the equations of the matrix version of the displacement method).



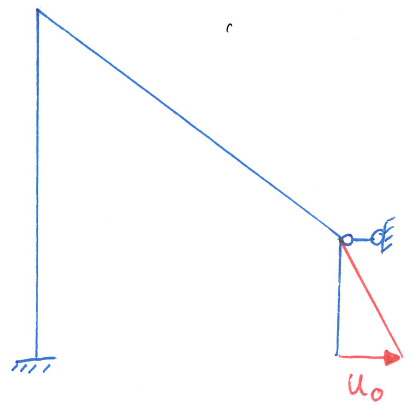
ZADANIE 1



UGW / PRIMARY STRUCTURE



PLAN PRZEMIESZCZEŃ / TRANSLATION PLAN



„WYJŚCIOWY" PLAN PRZEM. / „INITIAL" TRANSLATION PLAN

1) $\Phi_1^1 + \Phi_1^2 = 0$

2) $\Phi_2^2 + \Phi_2^3 = 0$

3) $(\Phi_A^1 + \Phi_1^1)\bar{\psi} + \Phi_2^3 \cdot 3\bar{\psi} = 0$

$$q = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{bmatrix}$$

$$\Phi_2^{03} = \frac{3EJ}{3l}\left(-\left(-\frac{u_0}{3l}\right)\right) = \frac{EJ}{l}\frac{u_0}{3l}$$

$$\Phi_A^1 = \frac{2EJ}{9l}(\varphi_1 - 3\psi)$$

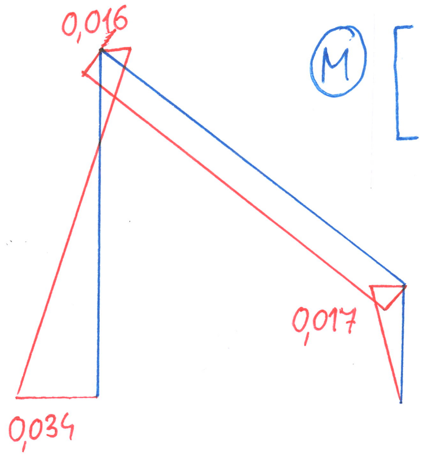
$$\Phi_1^1 = \frac{2EJ}{9l}(2\varphi_1 - 3\psi)$$

$$\Phi_1^2 = \frac{2EJ}{10l}(2\varphi_1 + \varphi_2)$$

$$\Phi_2^2 = \frac{2EJ}{10l}(\varphi_1 + 2\varphi_2)$$

$$\Phi_2^3 = \frac{3EJ}{3l}(\varphi_2 - 3\psi) + \Phi_2^{03}$$

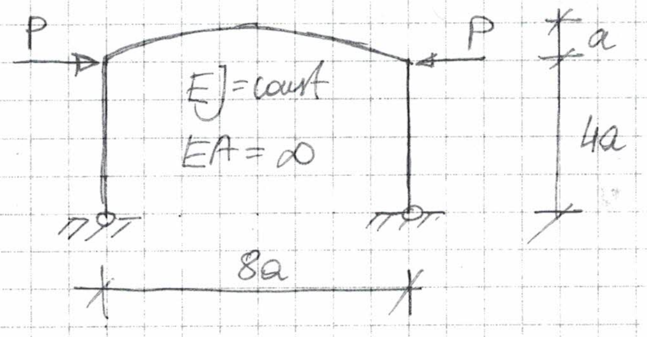
$$\mathbb{K} = \frac{EJ}{l}\begin{bmatrix} \frac{38}{45} & \frac{1}{5} & -\frac{2}{3} \\ \frac{1}{5} & \frac{7}{5} & -3 \\ -\frac{2}{3} & -3 & \frac{31}{3} \end{bmatrix}$$



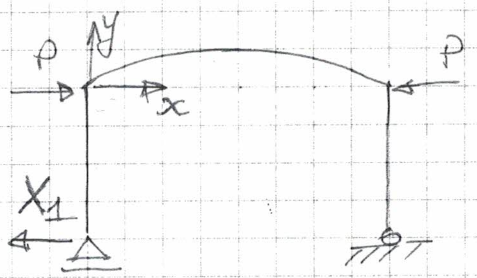
M $\left[\frac{EJu_0}{l^2}\right]$

$$Q = \begin{bmatrix} 0 \\ \frac{1}{3} \\ -1 \end{bmatrix}\frac{EJ}{l}\frac{u_0}{l} \qquad \mathbb{K}q + Q = 0 \Rightarrow q = \begin{bmatrix} 0{,}081 \\ -0{,}082 \\ 0{,}078 \end{bmatrix}\frac{u_0}{l}$$

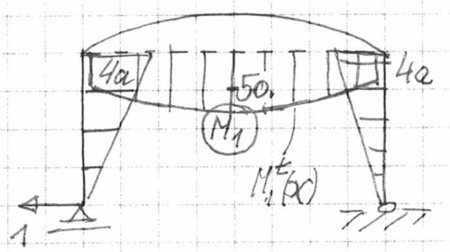
Egzamin MK1 5 II 2020 Zadanie 2



Ramołuk jest jednokrotnie statycznie niewyznaczalny

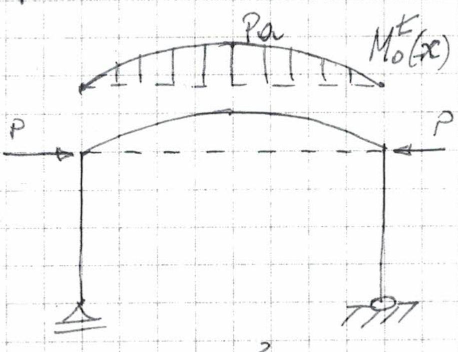


$$y(x) = \frac{4a}{(8a)^2}\, x(8a - x) = \frac{1}{16a}\, x(8a - x) =$$
$$= \frac{1}{2}x - \frac{1}{16}\frac{x^2}{a}$$



$$M_1^t(x) = 4a + 1 \cdot y(x)$$

$$M_0^t(x) = -P \cdot y(x)$$



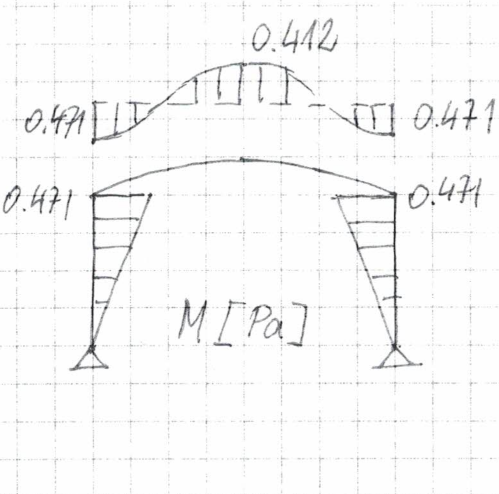
$$\delta_{11} = 2 \cdot \frac{1}{EJ} \cdot \frac{1}{2} \cdot 4a \cdot 4a \cdot \frac{2}{3} \cdot 4a + 2\int_0^{4a} \frac{M_1^t(x)\, M_1^t(x)}{EJ} dx$$

$$\delta_{10} = 2 \cdot \int_0^{4a} \frac{M_1^t(x)\, M_0^t(x)}{EJ} dx$$

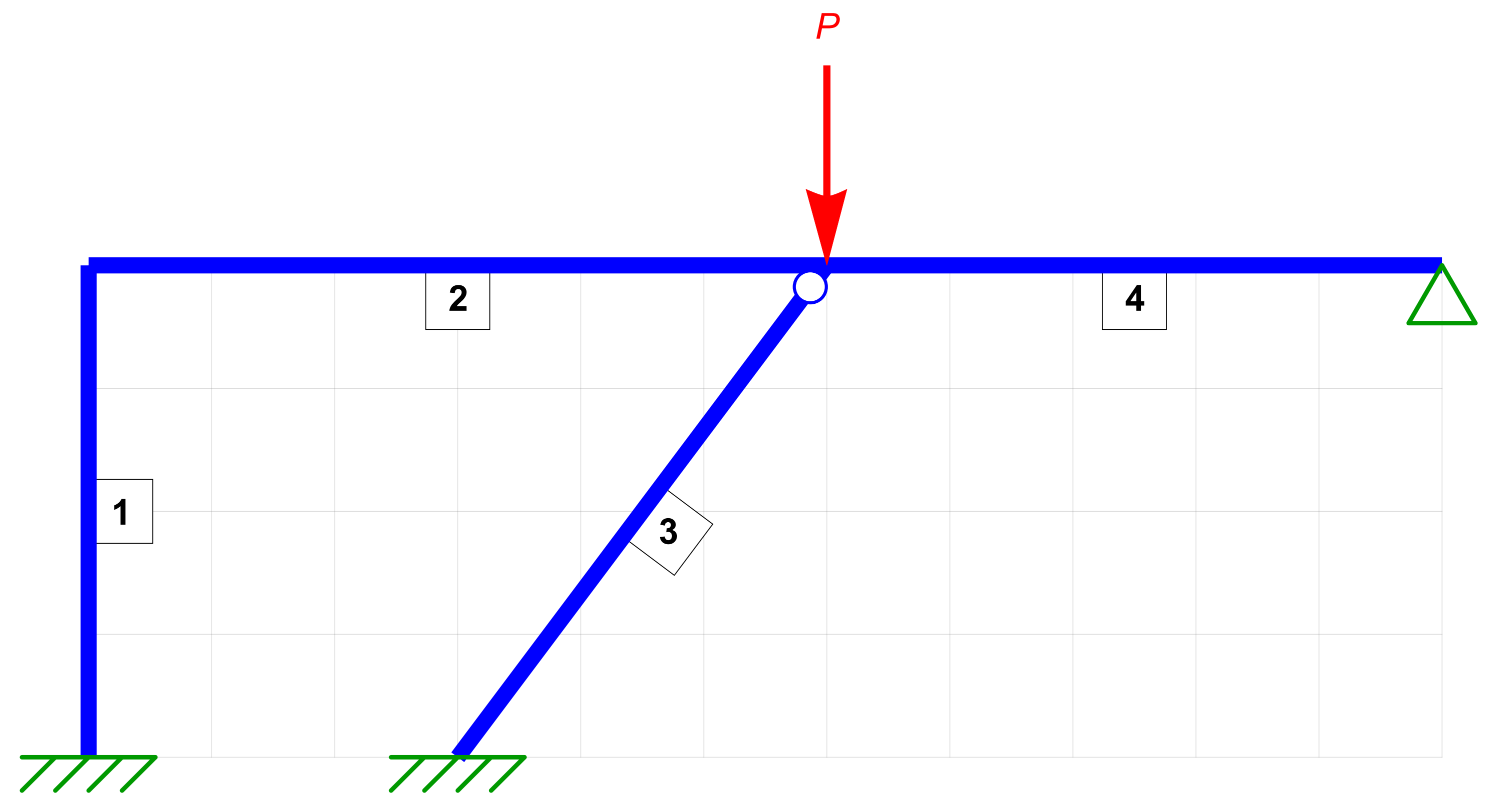
$$\delta_{11} = \frac{1088a^3}{5EJ}$$

$$\delta_{10} = -\frac{128Pa^3}{5EJ}$$

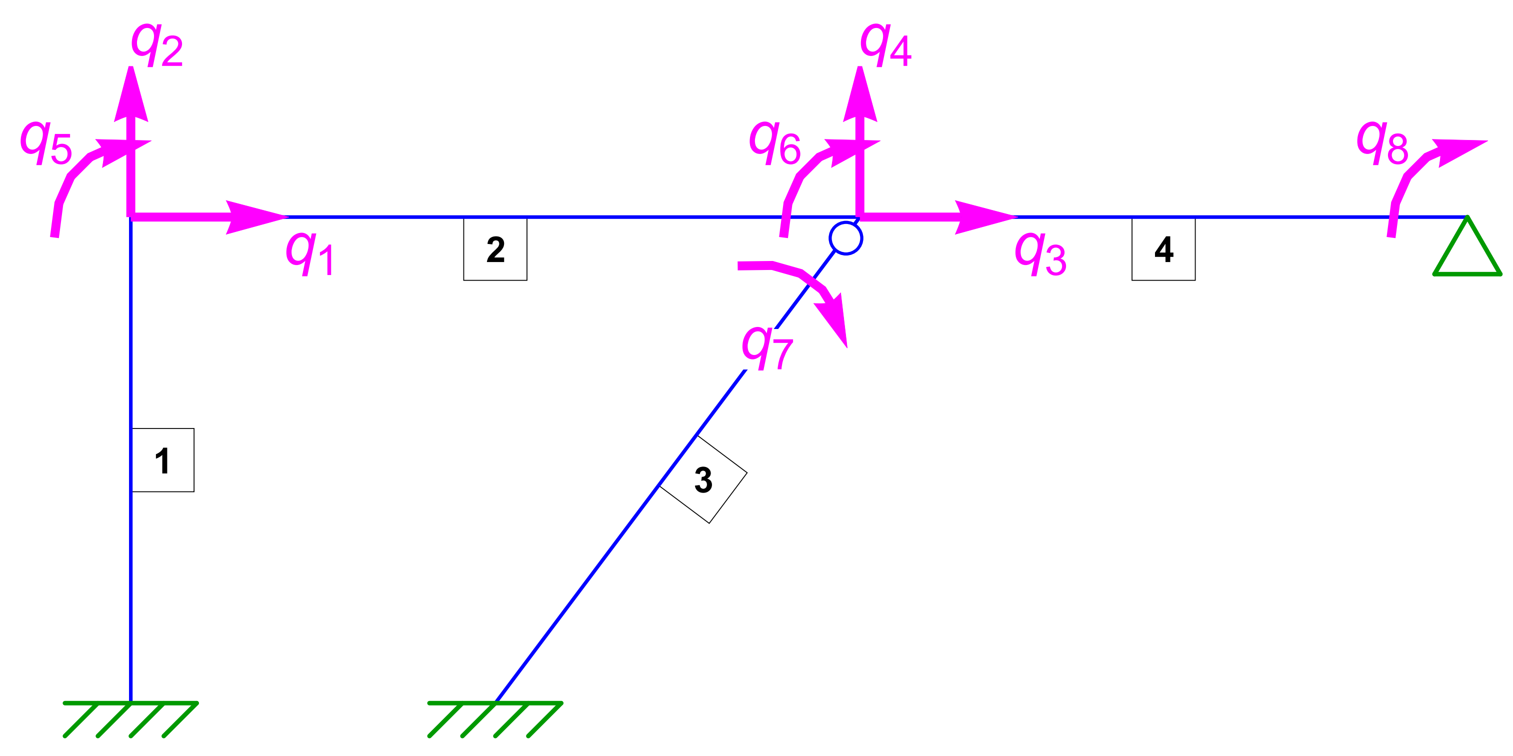
$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{2}{17}P$$



Geometria oraz obciazenia konstrukcji:



Niewiadome kinematyczne:



Ustalono 8 niewiadomych kinematycznych.

Równania geometryczne kratownicy ($\triangle, {}^*\chi$, χ^* – wektory odkształceń prętów; B, ${}^*\beta$, β^* – macierze geometryczne):

$$\mathbf{\Delta} = \mathbf{B}\,\mathbf{q}$$

$${}^*\boldsymbol{\chi} = {}^*\boldsymbol{\beta}\,\mathbf{q}$$

$$\boldsymbol{\chi}^* = \boldsymbol{\beta}^*\,\mathbf{q}$$

czyli:

$$\begin{pmatrix} \triangle_1 \\ \triangle_2 \\ \triangle_3 \\ \triangle_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{5} & \frac{4}{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{pmatrix}$$

$$\begin{pmatrix} {}^*\chi_1 \\ {}^*\chi_2 \\ {}^*\chi_3 \\ {}^*\chi_4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4\,a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{6\,a} & 0 & \frac{1}{6\,a} & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{4}{25\,a} & \frac{3}{25\,a} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{5\,a} & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{pmatrix}$$

$$\begin{pmatrix} \chi_1^* \\ \chi_2^* \\ \chi_3^* \\ \chi_4^* \end{pmatrix} = \begin{pmatrix} -\frac{1}{4\,a} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{6\,a} & 0 & \frac{1}{6\,a} & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{4}{25\,a} & \frac{3}{25\,a} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{5\,a} & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{pmatrix}$$

Związki konstytutywne ramy (N, ${}^*\Phi$, Φ^* – wektory sił w prętach; E, D – macierze konstytutywne):

$$\mathbf{N} = \mathbf{E}\,\mathbf{\Delta}$$

$${}^*\mathbf{\Phi} = \mathbf{D}\,(\,2\,{}^*\boldsymbol{\chi} + \boldsymbol{\chi}^*\,)$$

$$\mathbf{\Phi}^* = \mathbf{D}\,(\,{}^*\boldsymbol{\chi} + 2\,\boldsymbol{\chi}^*\,)$$

czyli:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix} = \begin{pmatrix} \frac{EA}{4\,a} & 0 & 0 & 0 \\ 0 & \frac{EA}{6\,a} & 0 & 0 \\ 0 & 0 & \frac{EA}{5\,a} & 0 \\ 0 & 0 & 0 & \frac{EA}{5\,a} \end{pmatrix} \begin{pmatrix} \triangle_1 \\ \triangle_2 \\ \triangle_3 \\ \triangle_4 \end{pmatrix}$$

$$\begin{pmatrix} {}^*\Phi_1 \\ {}^*\Phi_2 \\ {}^*\Phi_3 \\ {}^*\Phi_4 \end{pmatrix} = \begin{pmatrix} \frac{EJ}{2\,a} & 0 & 0 & 0 \\ 0 & \frac{EJ}{3\,a} & 0 & 0 \\ 0 & 0 & \frac{2\,EJ}{5\,a} & 0 \\ 0 & 0 & 0 & \frac{2\,EJ}{5\,a} \end{pmatrix} \begin{pmatrix} 2\,{}^*\chi_1 + \chi_1^* \\ 2\,{}^*\chi_2 + \chi_2^* \\ 2\,{}^*\chi_3 + \chi_3^* \\ 2\,{}^*\chi_4 + \chi_4^* \end{pmatrix}$$

$$\begin{pmatrix} \Phi_1^* \\ \Phi_2^* \\ \Phi_3^* \\ \Phi_4^* \end{pmatrix} = \begin{pmatrix} \frac{EJ}{2\,a} & 0 & 0 & 0 \\ 0 & \frac{EJ}{3\,a} & 0 & 0 \\ 0 & 0 & \frac{2\,EJ}{5\,a} & 0 \\ 0 & 0 & 0 & \frac{2\,EJ}{5\,a} \end{pmatrix} \begin{pmatrix} {}^*\chi_1 + 2\,\chi_1^* \\ {}^*\chi_2 + 2\,\chi_2^* \\ {}^*\chi_3 + 2\,\chi_3^* \\ {}^*\chi_4 + 2\,\chi_4^* \end{pmatrix}$$

Równania równowagi ramy (Q – wektor statycznych obciążeń zewnętrznych):

$$\mathbf{B}^{\mathsf{T}}\,\mathbf{N} + ({}^*\boldsymbol{\beta})^{\mathsf{T}}\,({}^*\mathbf{\Phi}) + (\boldsymbol{\beta}^*)^{\mathsf{T}}\,(\mathbf{\Phi}^*) = \mathbf{Q}$$

czyli:

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{3}{5} & -1 \\ 0 & 0 & \frac{4}{5} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix} + \begin{pmatrix} -\frac{1}{4\,a} & 0 & 0 & 0 \\ 0 & -\frac{1}{6\,a} & 0 & 0 \\ 0 & 0 & -\frac{4}{25\,a} & 0 \\ 0 & \frac{1}{6\,a} & \frac{3}{25\,a} & -\frac{1}{5\,a} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} {}^*\Phi_1 \\ {}^*\Phi_2 \\ {}^*\Phi_3 \\ {}^*\Phi_4 \end{pmatrix} + \begin{pmatrix} -\frac{1}{4\,a} & 0 & 0 & 0 \\ 0 & -\frac{1}{6\,a} & 0 & 0 \\ 0 & 0 & -\frac{4}{25\,a} & 0 \\ 0 & \frac{1}{6\,a} & \frac{3}{25\,a} & -\frac{1}{5\,a} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Phi_1^* \\ \Phi_2^* \\ \Phi_3^* \\ \Phi_4^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Zadanie przygotował Karol Bołbotowski.