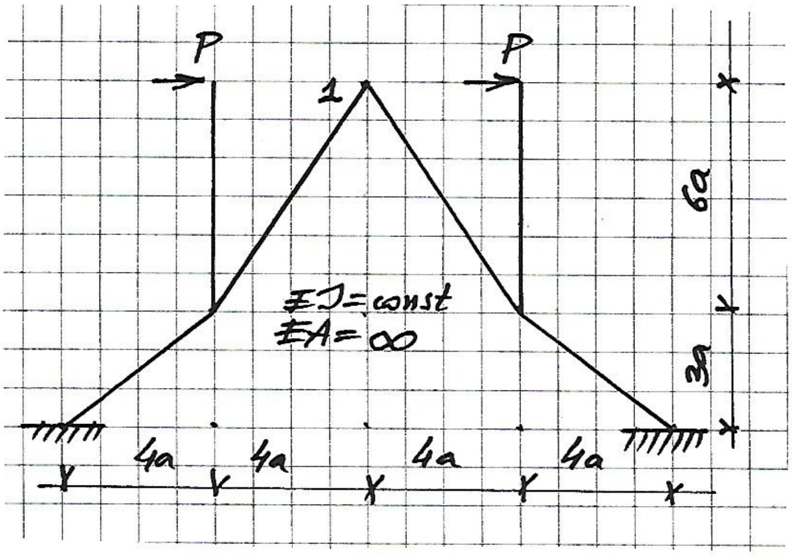
Egzamin pisemny z Mechaniki Konstrukcji I, 30 stycznia 2019 r.

NAZWISKO i Imię:				
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egz. pis.	Ocena Ostateczna
				Ocena łączna Data

Zadanie 1

Dana jest rama z prętów nieściśliwych; EJ=const. Dane jest obciążenie statyczne, por. Rys. Znaleźć przemieszczenie poziome u_1 węzła 1 metodą przemieszczeń.

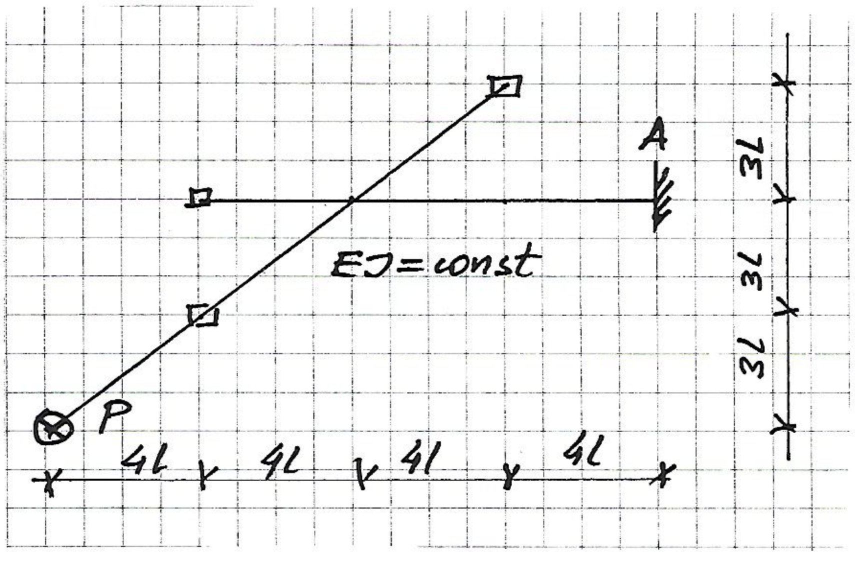
The given frame of inextensible bars is subjected to the static load, cf. Figure. Find the horizontal displacement u_1 *of the node 1 by the displacement method..*



Zadanie 2

Dany jest ruszt przegubowy. Znajdź wartość M_A

Consider the given system of beams. Find the value M_A



Zadanie 3

Wyprowadzić ogólne związki „transformacyjne" prostego pręta pryzmatycznego (schemat dwu utwierdzeń).

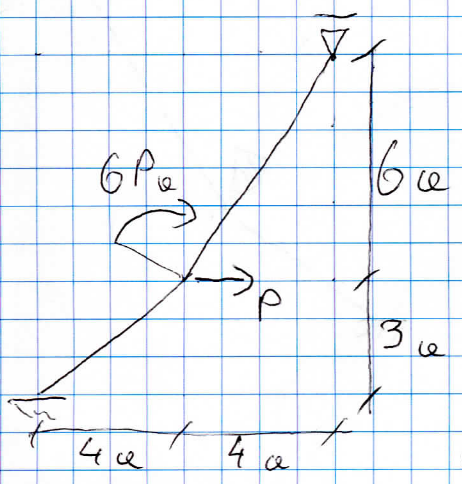
Derive the general slope –deflection equations for a prismatic and straight bar (the scheme of two clamped ends).

Egzamin z MK 1 30 I 2019 r.

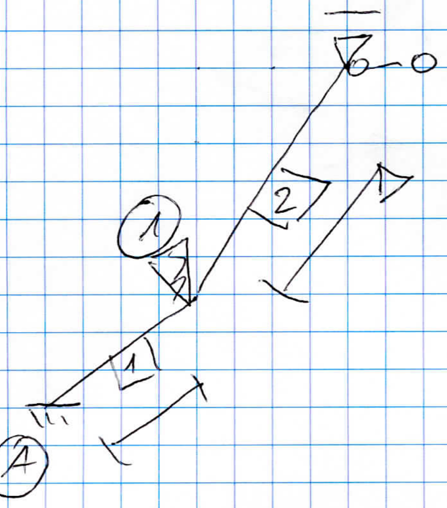
Zad 1

Zredukowany

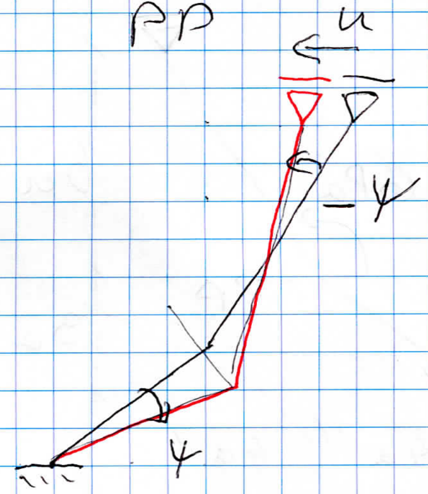
Schemat połówkowy



UGW



PP



$$u = 3a\psi + 6a(-\psi) = -3a\psi$$

RR:

$$\begin{cases} -\bar{\Phi}_1^1 - \bar{\Phi}_1^2 + 6Pa = 0 \\ \left(\bar{\Phi}_A^1 + \bar{\Phi}_1^1\right)\bar{\psi} + \bar{\Phi}_1^2(-\bar{\psi}) + P \cdot 3a\bar{\psi} = 0 \end{cases}$$

$$\bar{\Phi}_A^1 = \frac{EJ}{a}\left(\frac{2}{5}\varphi_1 - \frac{6}{5}\psi\right) ; \quad \bar{\Phi}_1^1 = \frac{EJ}{a}\left(\frac{4}{5}\varphi_1 - \frac{6}{5}\psi\right)$$

$$\bar{\Phi}_1^2 = \frac{EJ}{a}\left(\frac{3\sqrt{13}}{26}\varphi_1 + \frac{3\sqrt{13}}{26}\psi\right)$$

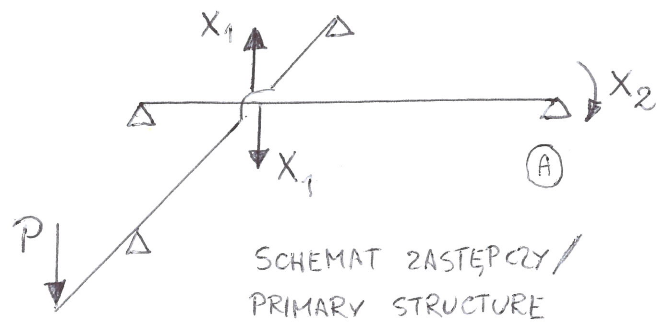
$$\frac{EJ}{a}\begin{bmatrix} -1{,}216 & 0{,}784 \\ 0{,}784 & -2{,}816 \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \end{bmatrix} Pa$$

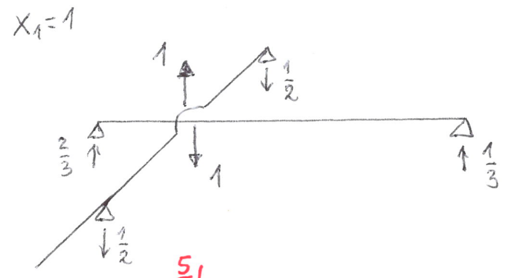
$$\varphi_1 = 6{,}85\frac{Pa^2}{EJ}$$

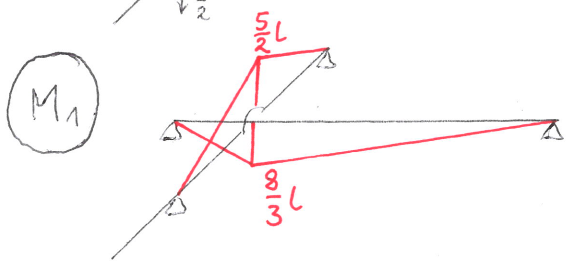
$$\psi = 2{,}97\frac{Pa^2}{EJ}$$

$$u = -3a\psi = -8{,}91\frac{Pa^3}{EJ}$$

ZADANIE 2



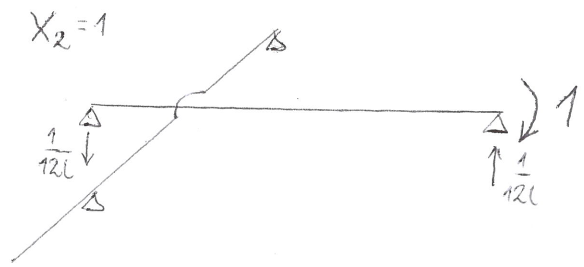




$$\begin{cases} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{10} = 0 \\ \delta_{12} X_1 + \delta_{22} X_2 + \delta_{20} = 0 \end{cases}$$

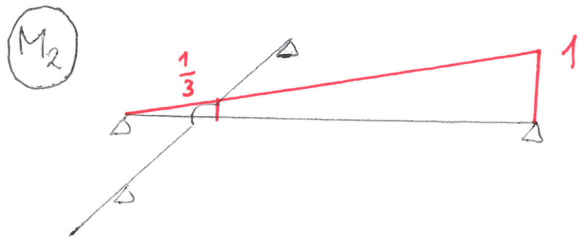
$$\delta_{11} = \frac{1}{EJ}\left[2 \cdot \frac{1}{2} \cdot 5l \cdot \frac{5}{2}l \cdot \frac{2}{3} \cdot \frac{5}{2}l + \frac{1}{2} \cdot 4l \cdot \frac{8}{3}l \cdot \frac{2}{3} \cdot \frac{8}{3}l + \right.$$

$$\left. + \frac{1}{2} \cdot 8l \cdot \frac{8}{3}l \cdot \frac{2}{3} \cdot \frac{8}{3}l \right] = 49{,}28 \frac{l^3}{EJ}$$



$$\delta_{12} = \frac{1}{EJ}\left[\frac{1}{2} \cdot 8l \cdot \frac{8}{3}l \cdot \left(-\frac{2}{3} \cdot \frac{1}{3} - \frac{1}{3} \cdot 1\right) + \right.$$

$$\left. + \frac{1}{2} \cdot 4l \cdot \frac{8}{3}l \cdot \left(-\frac{2}{3} \cdot \frac{1}{3}\right)\right] = -7{,}11 \frac{l^2}{EJ}$$

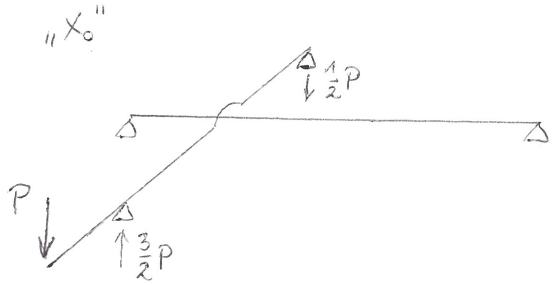


$$\delta_{22} = \frac{1}{EJ}\left[\frac{1}{2} \cdot 12l \cdot 1 \cdot \frac{2}{3} \cdot 1\right] = 4 \frac{l}{EJ}$$

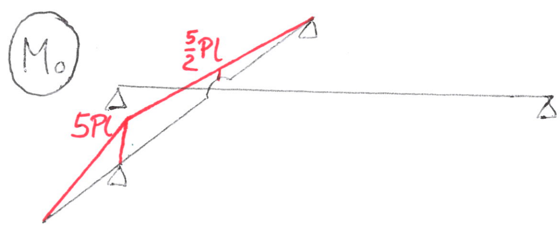
$$\delta_{10} = \frac{1}{EJ}\left[\frac{1}{2} \cdot 5l \cdot \frac{5}{2}l \cdot \frac{2}{3} \cdot \frac{5}{2}Pl + \frac{1}{2} \cdot 5l \cdot \frac{5}{2}l \cdot \left(\frac{2}{3} \cdot \frac{5}{2}Pl + \right.\right.$$

$$\left.\left. + \frac{1}{3} \cdot 5Pl\right)\right] = 31{,}25 \frac{Pl^3}{EJ}$$

$$\delta_{20} = 0$$



$$\begin{bmatrix} 49{,}28 \frac{l^3}{EJ} & -7{,}11 \frac{l^2}{EJ} \\ -7{,}11 \frac{l^2}{EJ} & 4 \frac{l}{EJ} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 31{,}25 \frac{Pl^3}{EJ} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



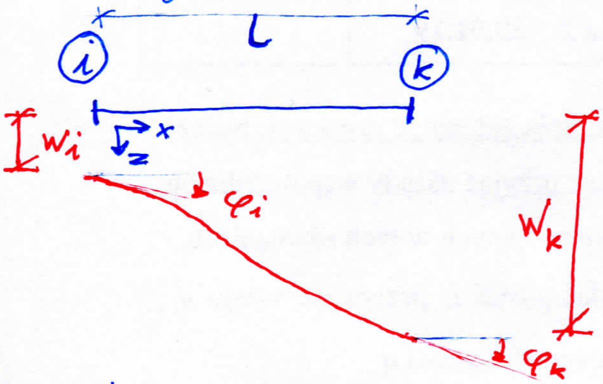
$$\mathbb{D}\mathbb{X} + \mathbb{D}_0 = \mathbb{O}$$

$$\mathbb{X} = \begin{bmatrix} -0{,}853 \; P \\ -1{,}516 \; Pl \end{bmatrix}$$

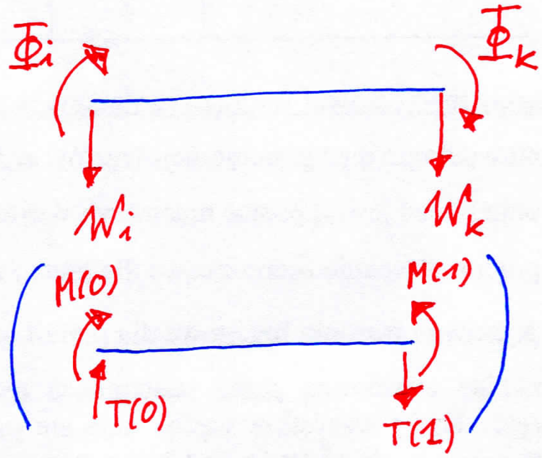
$$\boxed{M_A = X_2 = -1{,}516 \, Pl}$$

Egzamin MK I 30.I.2019 zadanie 3:

Kinematyka (dane):



Statyka (szukane):



Różniczkowe, przemieszczeniowe rów. równ.:
(zmienna $\xi = \frac{x}{L}$)

$$EJ \frac{1}{L^4} \frac{d^4 w}{d\xi^4}(\xi) = 0$$

$$\Downarrow$$

$$w(\xi) = C_0 + C_1 \xi + C_2 \xi^2 + C_3 \xi^3$$

$\oplus$ warunki brzegowe:

$w(0) = w_i$	$w(1) = w_k$
$\varphi(0) = \varphi_i$	$\varphi(1) = \varphi_k$

gdzie $\varphi(\xi) = \frac{1}{L} \frac{dw}{d\xi}(\xi)$

$$\Downarrow$$

$$w(\xi) = w_i + (L\varphi_i)\xi + \left(-2L\varphi_i - L\varphi_k + 3(w_k - w_i)\right)\xi^2 + \left(L\varphi_i + L\varphi_k - 2(w_k - w_i)\right)\xi^3 =$$

$$= w_i(1 - 3\xi^2 + 2\xi^3) + w_k(3\xi^2 - 2\xi^3) + L\varphi_i(\xi - 2\xi^2 + \xi^3) + L\varphi_k(-\xi^2 + \xi^3)$$

$$M(\xi) = EJ \cdot \varkappa(\xi) = EJ\left(-\frac{1}{L^2}\frac{d^2 w}{d\xi^2}(\xi)\right) = -\frac{EJ}{L^2}\left(w_i(-6+12\xi) + w_k(6-12\xi) + L\varphi_i(-4+6\xi) + L\varphi_k(-2+6\xi)\right)$$

$$= \left\{\psi := \frac{w_k - w_i}{L}\right\} = \frac{EJ}{L}\left((+4-6\xi)\varphi_i + (2-6\xi)\varphi_k + (-6+12\xi)\psi\right)$$

$$T(\xi) = \frac{1}{L}\frac{dM}{d\xi}(\xi) = \frac{EJ}{L^2}\left(-6\varphi_i - 6\varphi_k + 12\psi\right)$$

Ostatecznie:

$$\Phi_i = M(0) = \frac{EJ}{L}\left(4\varphi_i + 2\varphi_k - 6\psi\right)$$

$$\Phi_k = -M(1) = \frac{EJ}{L}\left(2\varphi_i + 4\varphi_k - 6\psi\right)$$

$$W_i = -T(0) = -\frac{EJ}{L^2}\left(-6\varphi_i - 6\varphi_k + 12\psi\right)$$

$$W_k = T(1) = \frac{EJ}{L^2}\left(-6\varphi_i - 6\varphi_k + 12\psi\right)$$

Rozwiązanie przygotował:
Karol Bołbotowski