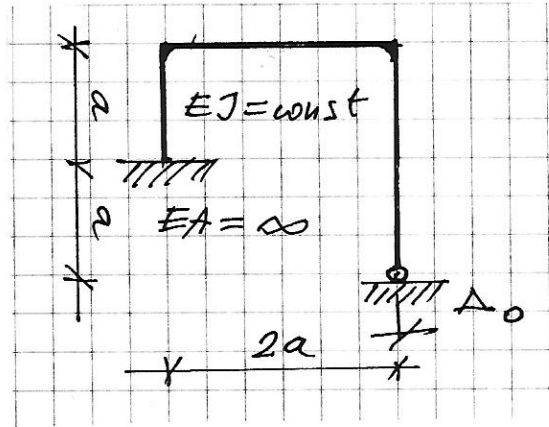


Egzamin pisemny z Mechaniki Konstrukcji I, 3 IX 2018 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna
				Data

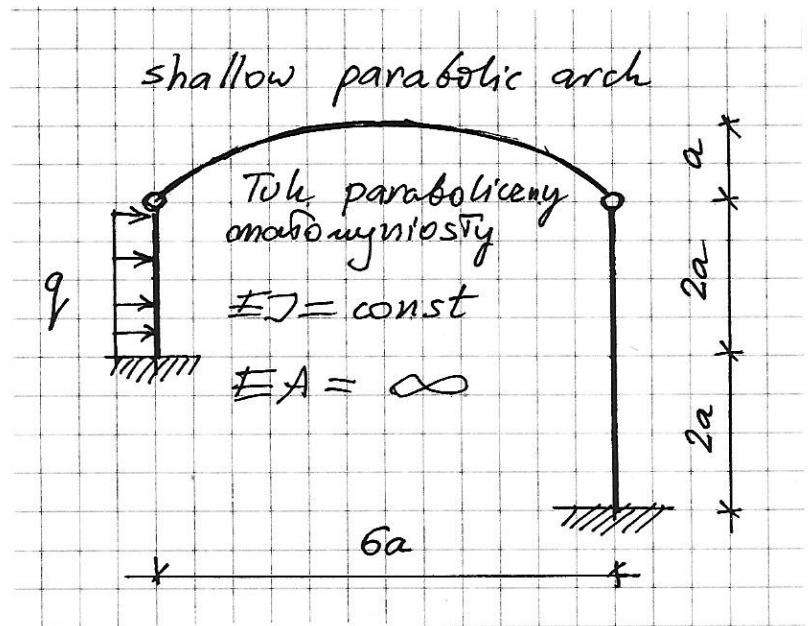
Zadanie 1

Dana jest rama płaska obciążona jak na rysunku; sporządzić wykres momentów zginających metodą przemieszczeń.
 (For the given frame construct the diagram of the bending moments by the displacement method)



Zadanie 2

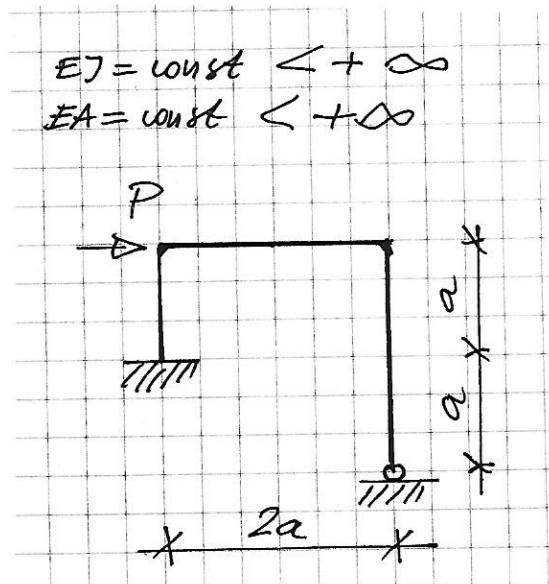
Dany jest ramołuk obciążony jak na rysunku. Sporządzić wykres momentów zginających
 (Consider the given frame with an arch loaded as shown in the figure. Find the diagram of the bending moments.)



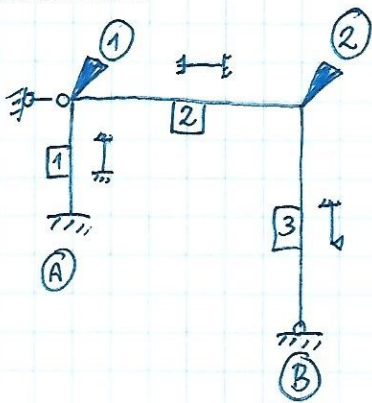
Zadanie 3

Zapisać równania macierzowej metody przemieszczeń.

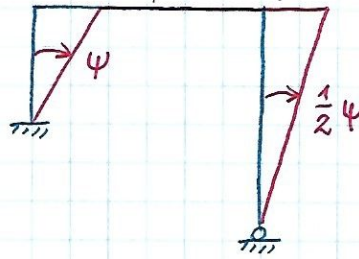
(Write down the equations of the displacement method in the matrix version).



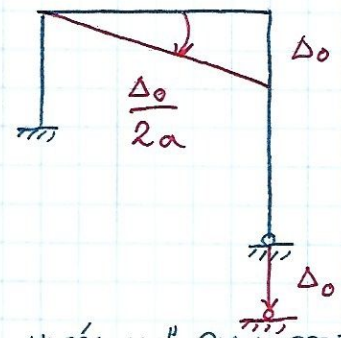
ZADANIE 1



SCHEMAT ZASTĘPCZY
PRIMARY STRUCTURE



PLAN PRZEMIESZCZEŃ
TRANSLATION PLAN



„WYJŚCIOWY” PLAN PRZEMIESZCZEŃ
„INITIAL” TRANSLATION PLAN

$$q = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{bmatrix}$$

$$1) \bar{\Phi}_1^1 + \bar{\Phi}_1^2 = 0$$

$$2) \bar{\Phi}_2^2 + \bar{\Phi}_2^3 = 0$$

$$3) (\bar{\Phi}_A^1 + \bar{\Phi}_1^1) \cdot \bar{\psi} + \bar{\Phi}_2^3 \cdot \frac{1}{2} \bar{\psi} = 0$$

$$\bar{\Phi}_A^1 = \frac{2EJ}{a} (\varphi_1 - 3\psi)$$

$$\bar{\Phi}_1^1 = \frac{2EJ}{a} (2\varphi_1 - 3\psi)$$

$$\bar{\Phi}_1^2 = \frac{2EJ}{2a} (2\varphi_1 + \varphi_2) + \frac{2EJ}{2a} \left(-3 \cdot \frac{\Delta_0}{2a}\right)$$

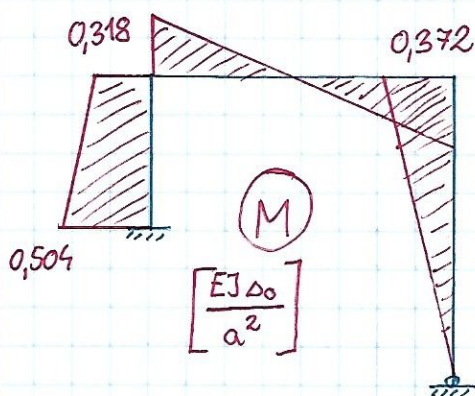
$$\bar{\Phi}_2^2 = \frac{2EJ}{2a} (\varphi_1 + 2\varphi_2) + \frac{2EJ}{2a} \left(-3 \cdot \frac{\Delta_0}{2a}\right)$$

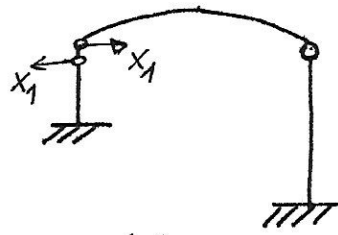
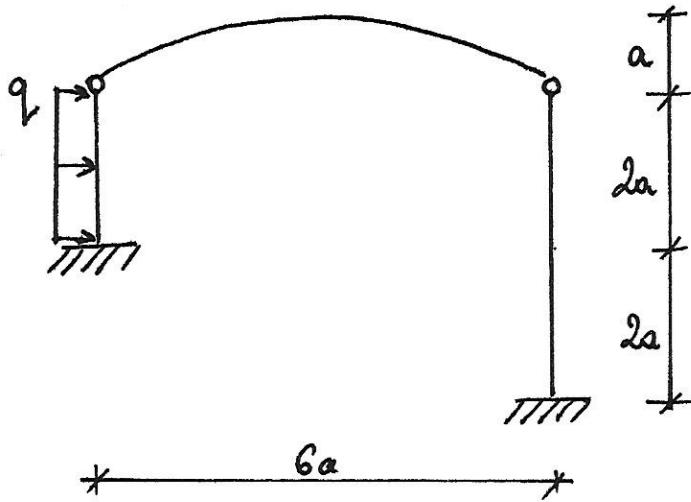
$$\bar{\Phi}_2^3 = \frac{3EJ}{2a} \left(\varphi_2 - \frac{1}{2}\psi\right)$$

$$K = \begin{bmatrix} 6 & 1 & -6 \\ 1 & \frac{7}{2} & -\frac{3}{4} \\ -6 & -\frac{3}{4} & \frac{99}{8} \end{bmatrix} \frac{EJ}{a^2}$$

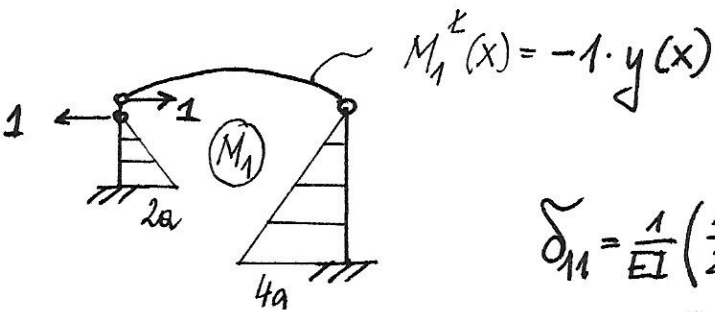
$$Q = \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \\ 0 \end{bmatrix} \frac{EJ \Delta_0}{a^2}$$

$$Kq + Q = 0 \Rightarrow q = \begin{bmatrix} 0,411 \\ 0,358 \\ 0,221 \end{bmatrix} \frac{\Delta_0}{a}$$

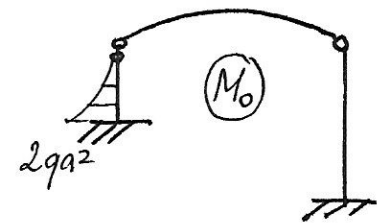




$$y(x) = \frac{4f}{L^2} x(L-x) = \frac{2}{3}x - \frac{1}{9a}x^2$$

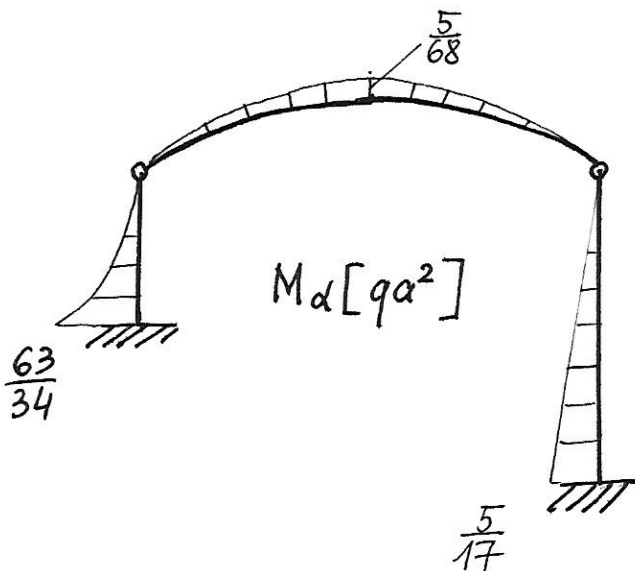


$$\begin{aligned} \delta_{11} &= \frac{1}{EI} \left(\frac{1}{2} \cdot 2a \cdot 2a \cdot \frac{2}{3} \cdot 2a + \frac{1}{2} \cdot 4a \cdot 4a \cdot \frac{2}{3} \cdot 4a + \int_0^{6a} [-y(x)]^2 dx \right) = \\ &= \frac{136}{5} \frac{qa^3}{EI} = 27,2 \frac{a^3}{EI} \end{aligned}$$

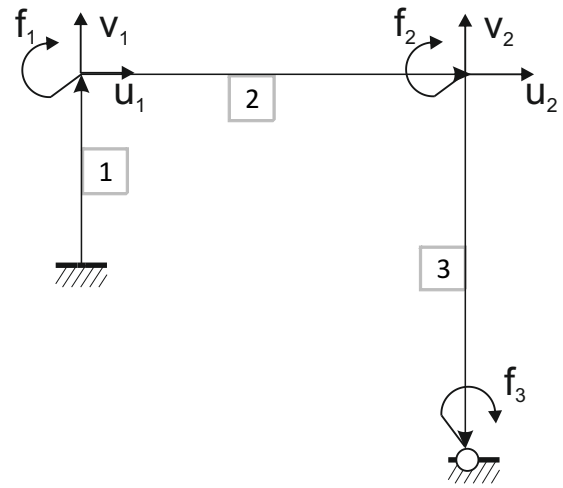
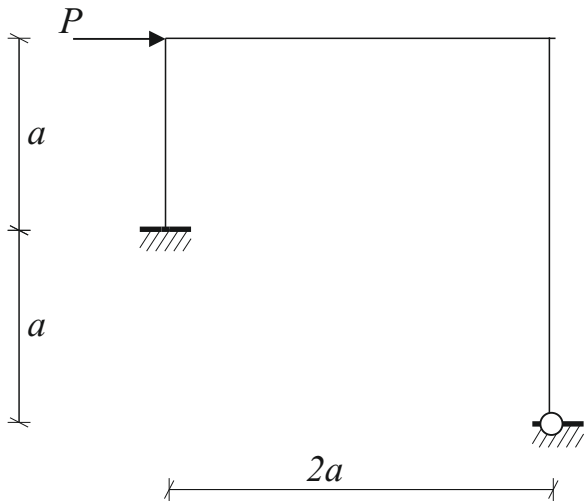


$$\begin{aligned} \delta_{10} &= \frac{1}{EI} \left[-\frac{1}{2} \cdot 2qa^2 \cdot 2a \cdot \frac{2}{3} \cdot 2a + \frac{2}{3} \cdot \frac{q(2a)^2}{8} \cdot 2a \cdot \frac{1}{2} \cdot 2a \right] = \\ &= -2 \frac{qa^4}{EI} \end{aligned}$$

$$x_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{5}{68} qa$$



Egz. MK_1/03.09.2018/zad_3:



Rys. Oznaczenia niewiadomych \mathbf{q} i orientacja prętów.

$$\mathbf{q}^T = \begin{bmatrix} f_1 & f_2 & f_3 & u_1/a & v_1/a & u_2/a & v_2/a \end{bmatrix}$$

$$\mathbf{B}[\mathbf{a}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$*\mathbf{B}[\mathbf{1}] = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & -0.5 & 0 & 0.5 \\ 3 & 0 & 0 & 0 & 0 & -0.5 & 0 \end{bmatrix}$$

$$\mathbf{B}^*[\mathbf{1}] = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & -0.5 & 0 & 0.5 \\ 3 & 0 & 0 & 1 & 0 & -0.5 & 0 \end{bmatrix}$$

$$\mathbf{E}[\mathbf{EA}/\mathbf{a}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$\mathbf{D}[\mathbf{EJ}/\mathbf{a}] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta = \mathbf{B} \mathbf{q}$$

$$*\chi = *\varphi - \psi \quad \chi^* = \varphi^* - \psi$$

$$*\chi = *\mathbf{B} \mathbf{q} \quad \chi^* = \mathbf{B}^* \mathbf{q}$$

$$\mathbf{K} = \mathbf{B}^T \mathbf{E} \mathbf{B} + 2 * \mathbf{B}^T \mathbf{D} * \mathbf{B} + 2 \mathbf{B}^* \mathbf{D} \mathbf{B}^* + \mathbf{B}^T \mathbf{D} \mathbf{B} + \mathbf{B}^* \mathbf{D} * \mathbf{B}$$

$$\mathbf{K} \mathbf{q} = \mathbf{Q}$$

$$\mathbf{Q}[\mathbf{Pa}] = \begin{bmatrix} f_1 & 0 \\ f_2 & 0 \\ f_3 & 0 \\ u_1/a & 1 \\ v_1/a & 0 \\ u_2/a & 0 \\ v_2/a & 0 \end{bmatrix}$$