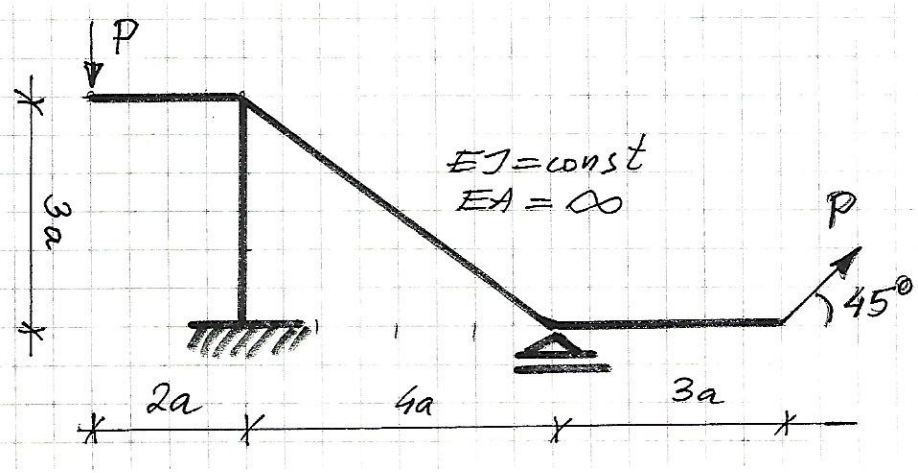
Egzamin pisemny z Mechaniki Konstrukcji I, 21 III 2018 r.

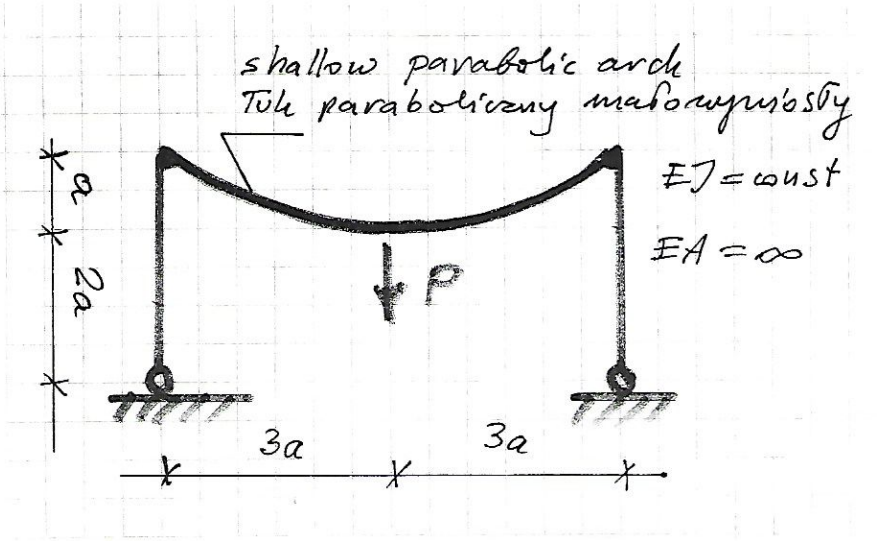
NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1
Dana jest rama płaska obciążona jak na rysunku. Sporządzić wykres momentów zginających metodą przemieszczeń.

(For the given frame construct the diagram of the bending moments by the displacement method)

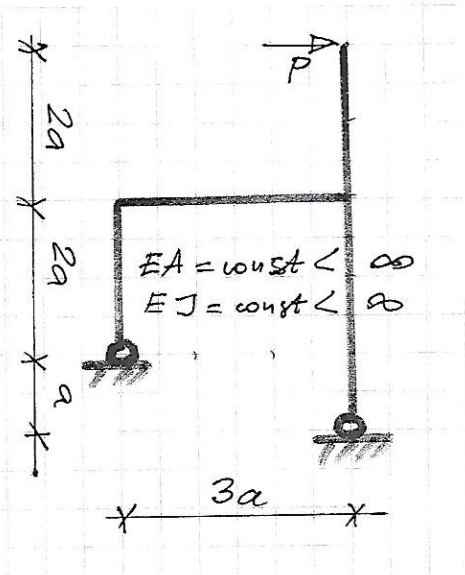


Zadanie 2
Dany jest ramołuk obciążony jak na rysunku. Sporządzić wykres momentów zginających metodą sił.
(Consider the given frame with an arch loaded as shown in the figure. Find the diagram of bending moments by the force method).

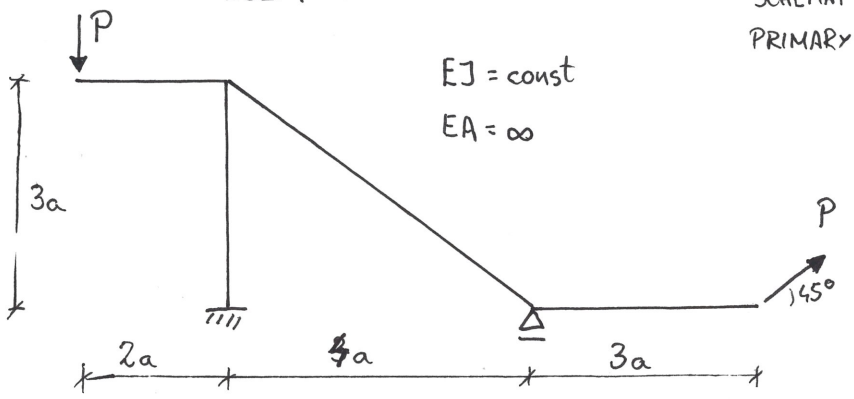


Zadanie 3
Rozważamy ramę obciążoną jak na rysunku. Zapisać równania macierzowej metody przemieszczeń.

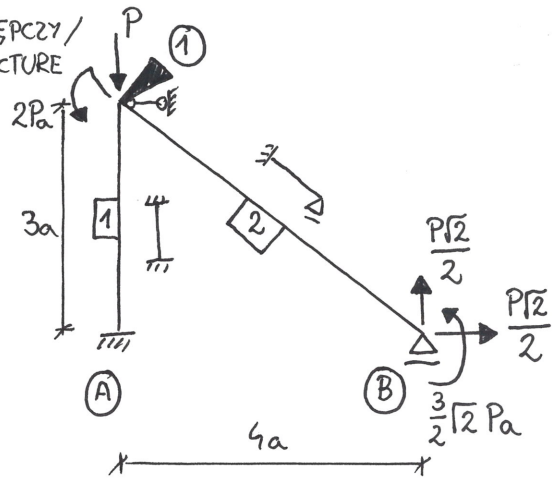
(Consider the frame loaded as shown in the figure. Write down the equations of the matrix version of the displacement method).



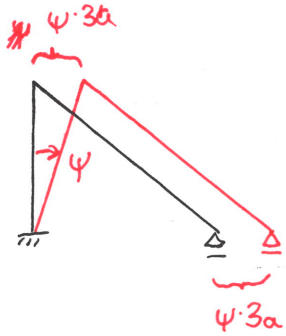
ZADANIE 1 / PROBLEM 1



SCHEMAT ZASTĘPCZY / PRIMARY STRUCTURE



PLAN PRZEMIESZCZEŃ / DISPLACEMENT PLAN



RÓWNANIA RÓWNOWAGI / EQUILIBRIUM EQUATIONS

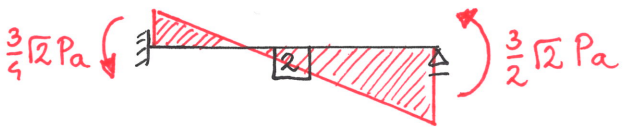
$$q = \begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix}$$

1) $\Phi_1^1 + \bar{\Phi}_1^2 + 2Pa = 0$

2) $(\Phi_A^1 + \Phi_1^1)\bar{\psi} + \bar{L}_2 = 0$

$\bar{L}_2 = P\frac{\sqrt{2}}{2}\cdot\bar{\psi}\cdot 3a = \frac{3}{2}\sqrt{2}\,Pa\,\bar{\psi}$

MOMENT WYJŚCIOWY NA PRĘCIE 2 / INITIAL MOMENT ON BAR 2:



$$\Phi_A^1 = \frac{2EJ}{3a}(\varphi_1 - 3\psi)$$

$$\Phi_1^1 = \frac{2EJ}{3a}(2\varphi_1 - 3\psi)$$

$$\bar{\Phi}_1^2 = \frac{3EJ}{5a}\varphi_1 + \Phi_1^{02} = \frac{3EJ}{5a}\varphi_1 - \frac{3}{4}\sqrt{2}\,Pa$$

1) $\frac{2EJ}{3a}(2\varphi_1 - 3\psi) + \frac{3EJ}{5a}\varphi_1 - \frac{3}{4}\sqrt{2}\,Pa + 2Pa = 0$

2) $\frac{2EJ}{3a}(\varphi_1 - 3\psi) + \frac{2EJ}{3a}(2\varphi_1 - 3\psi) + \frac{3}{2}\sqrt{2}\,Pa = 0 \quad \| \cdot(-1)$

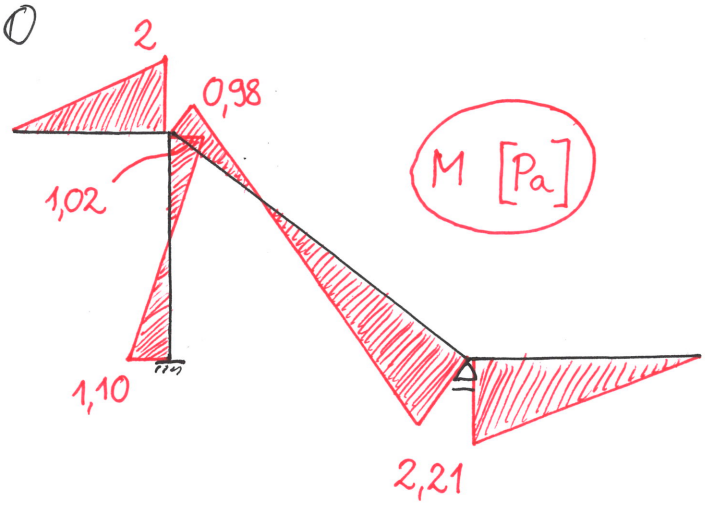
$$\mathbb{K} = \frac{EJ}{a}\begin{bmatrix} 1{,}933 & -2 \\ -2 & 4 \end{bmatrix}$$

$$\mathbb{Q} = \begin{bmatrix} 0{,}939 \\ -2{,}121 \end{bmatrix} Pa$$

$$\frac{EJ}{a}\begin{bmatrix} \frac{29}{15} & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix} + \begin{bmatrix} 2 - \frac{3}{4}\sqrt{2} \\ -\frac{3}{2}\sqrt{2} \end{bmatrix} Pa = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbb{K}q + \mathbb{Q} = 0$$

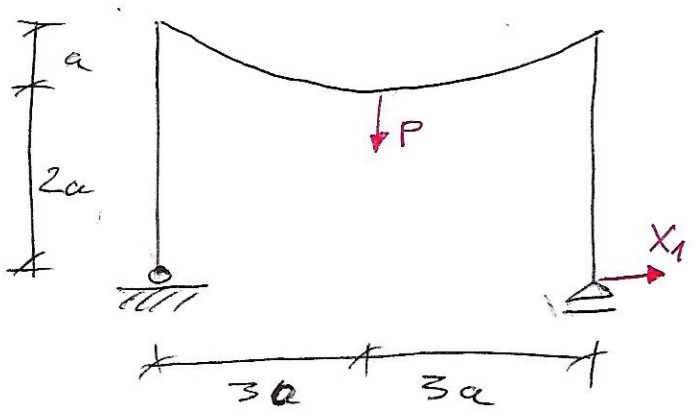
$$\boxed{\begin{array}{l} \varphi_1 = 0{,}130 \\ \psi = 0{,}595 \end{array}} \left[\frac{Pa^2}{EJ}\right]$$



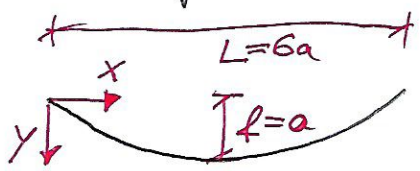
EGZAMIN MKI 21.03.18 Zadanie 2:

Wyznaczyć wykres M.

Układ zastępczy:

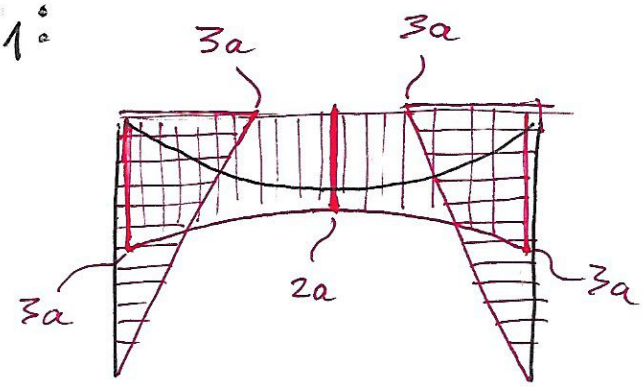


Równanie paraboli

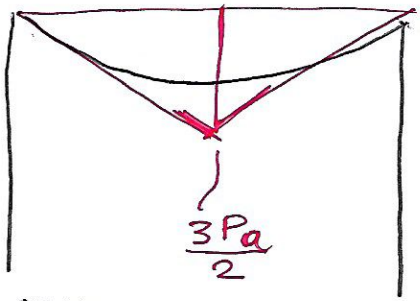


$$y(x) = \frac{4f}{L^2} x(L-x) = \frac{1}{9a} x(6a-x)$$

M_1:



M_0:



Funkcja momentu na łuku (wartości dodatnia oznacza rozciąganie włókien dolnych):

$$M_1^{\text{łuk}}(x) = 3a - 1 \cdot y(x) = 3a - \frac{1}{9a} x(6a-x) \quad \text{dla } x \in [0, 6a]$$

$$M_0^{\text{łuk}}(x) = \frac{P}{2} \cdot x \quad \text{dla } x \in [0, 3a]$$

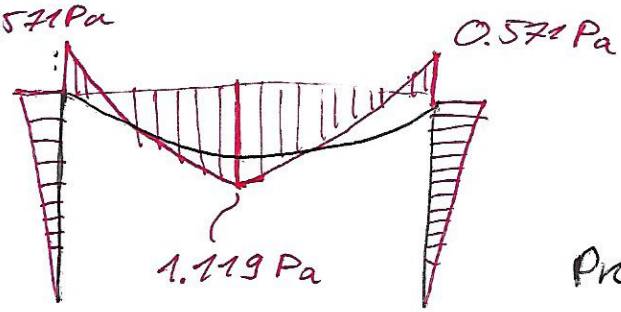
$$\delta_{11} = 2 \cdot \frac{1}{EJ}\left(\frac{1}{2} \cdot 3a \cdot 3a\right)\left(\frac{2}{3} \cdot 3a\right) + \frac{1}{EJ} \int_0^{6a} \left[3a - \frac{1}{9a} x(6a-x)\right]^2 \cdot \boxed{1} \cdot dx = \frac{256}{5} \frac{L^3}{EJ}$$

$$\delta_{10} = 2 \cdot \frac{1}{EJ} \int_0^{3a} \left[\frac{P}{2} x\right]\left[3a - \frac{1}{9a} x(6a-x)\right] \cdot \boxed{1} \cdot dx = \frac{39}{4} \frac{PL^3}{EJ}$$

$\boxed{1}$ – mała wyniosłość

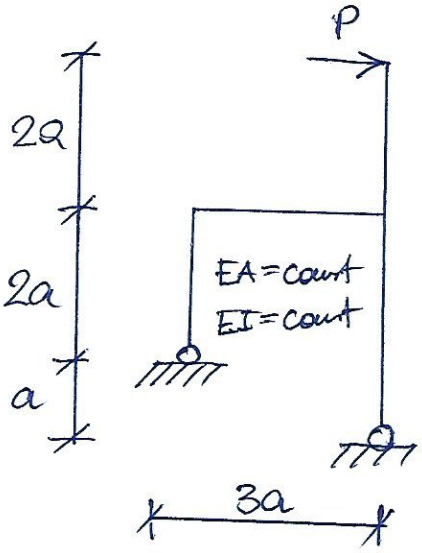
$$\delta_{11} X_1 + \delta_{10} = 0 \Rightarrow \boxed{X_1 \approx -0{,}190\,P}$$

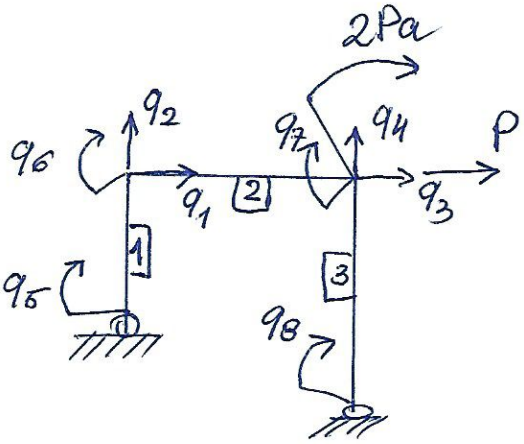
$M = M_1 X_1 + M_0$:



Przygotował:
Karol Bobowski

Egzamin z MK1, 21 III 2018, zadanie 3





$$q = \begin{bmatrix} q_1 \\ \vdots \\ q_8 \end{bmatrix}$$

$$\Delta = \mathbb{B} q \qquad \mathbb{N} = \mathbb{E}\mathbb{B} q$$

$$^*\mathbb{X} = {}^*\mathbb{B} q \qquad {}^*\Phi = \mathbb{D}(2\,{}^*\mathbb{B} q + \mathbb{B}^* q)$$

$$\mathbb{X}^* = \mathbb{B}^* q \qquad \Phi^* = \mathbb{D}({}^*\mathbb{B} q + 2\mathbb{B}^* q)$$

$$\mathbb{K} q = \mathbb{Q} \rightarrow q$$

$$\mathbb{K} = \mathbb{B}^T \mathbb{E} \mathbb{B} + 2({}^*\mathbb{B})^T \mathbb{D}\, {}^*\mathbb{B} + ({}^*\mathbb{B})^T \mathbb{D} \mathbb{B}^* + (\mathbb{B}^*)^T \mathbb{D}\, {}^*\mathbb{B} + 2(\mathbb{B}^*)^T \mathbb{D}\, \mathbb{B}^*$$

$$\mathbb{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbb{E} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \frac{EA}{l}$$

$${}^*\mathbb{B} = \begin{bmatrix} -\frac{1}{2a} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3a} & 0 & \frac{1}{3a} & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{3a} & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbb{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix} \frac{EI}{l}$$

$$\mathbb{B}^* = \begin{bmatrix} -\frac{1}{2a} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{3a} & 0 & \frac{1}{3a} & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{3a} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbb{Q} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 2a \\ 0 \end{bmatrix} P$$