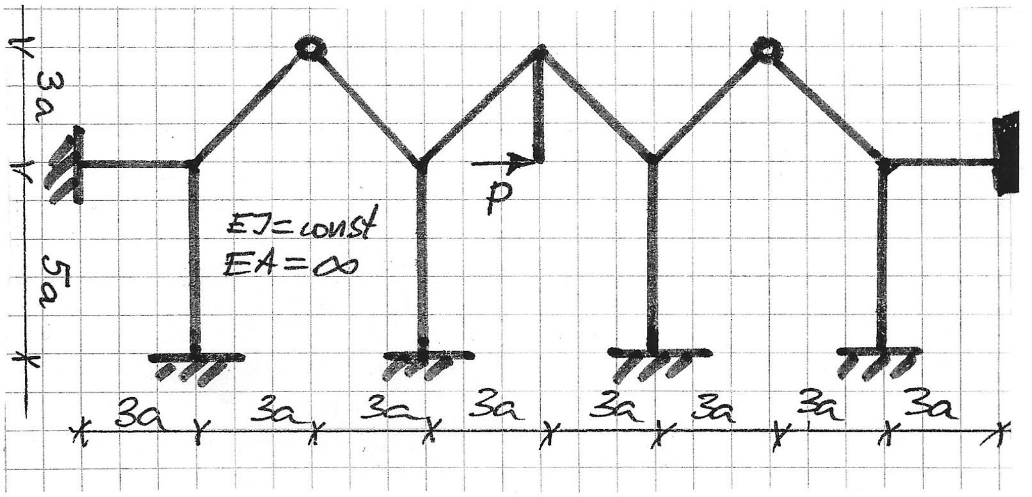
Egzamin pisemny z Mechaniki Konstrukcji I, 7 II 2018 r.

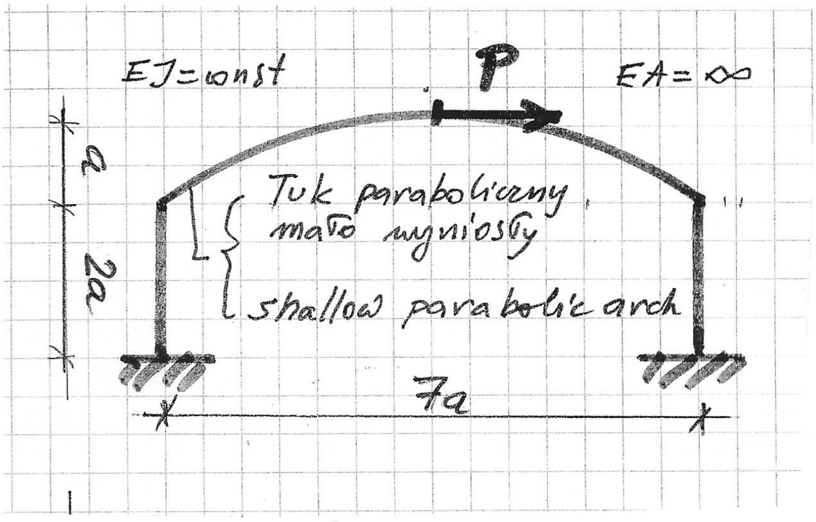
NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1
Dana jest rama płaska obciążona jak na rysunku. Sporządzić wykres momentów zginających metodą przemieszczeń.

(For the given frame construct the diagram of the bending moments by the displacement method)

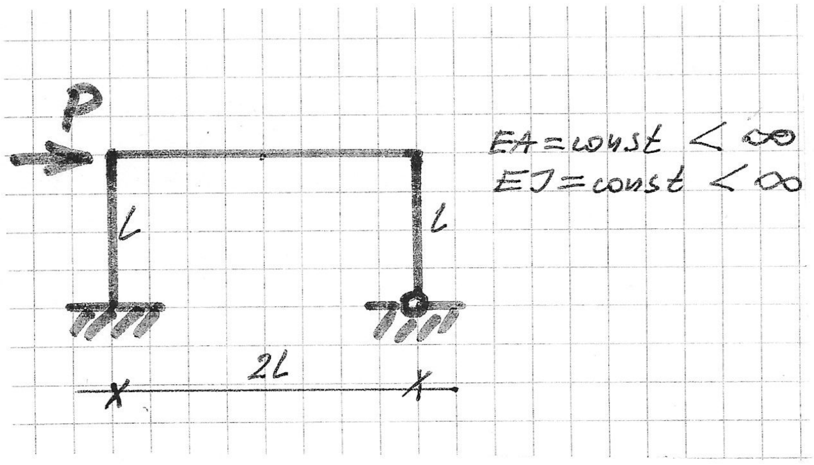


Zadanie 2
Dany jest ramołuk obciążony jak na rysunku. Sporządzić wykres momentów zginających metodą sił.
(Consider the given frame with an arch loaded as shown in the figure. Find the diagram of bending moments by the force method).



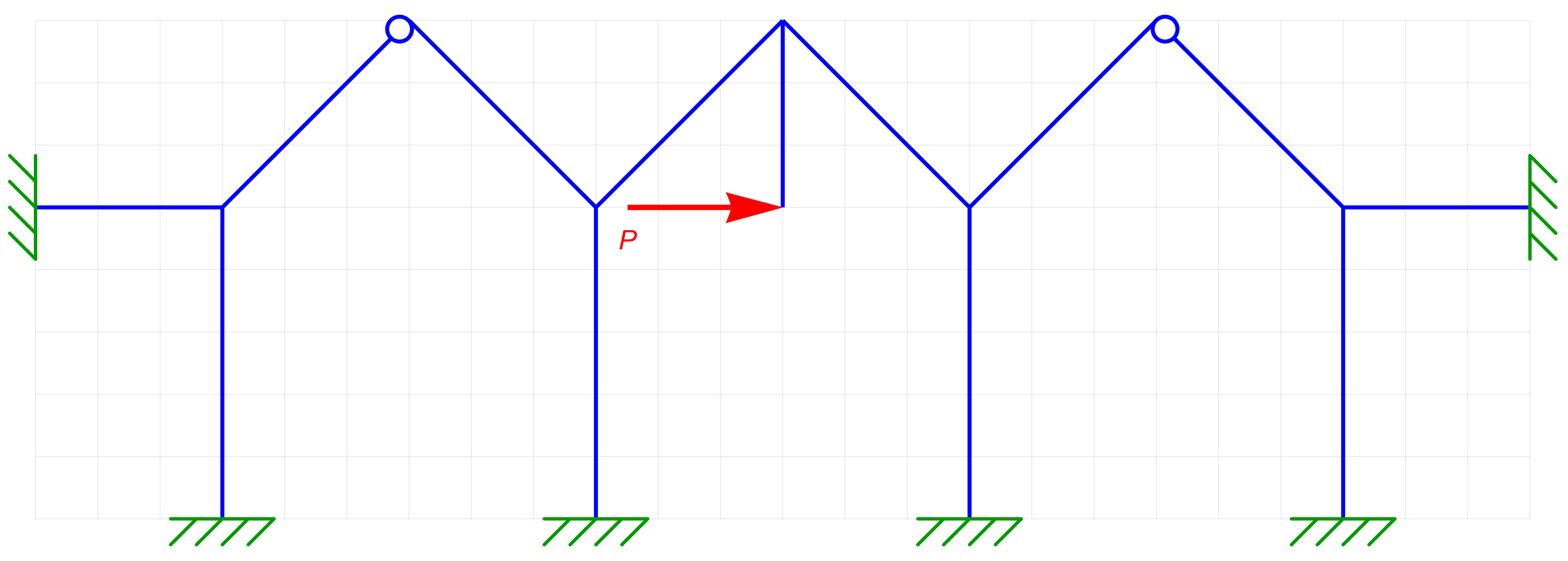
Zadanie 3
Rozważamy ramę obciążoną jak na rysunku. Zapisać równania macierzowej metody przemieszczeń.

(Consider the frame loaded as shown in the figure. Write down the equations of the matrix version of the displacement method).



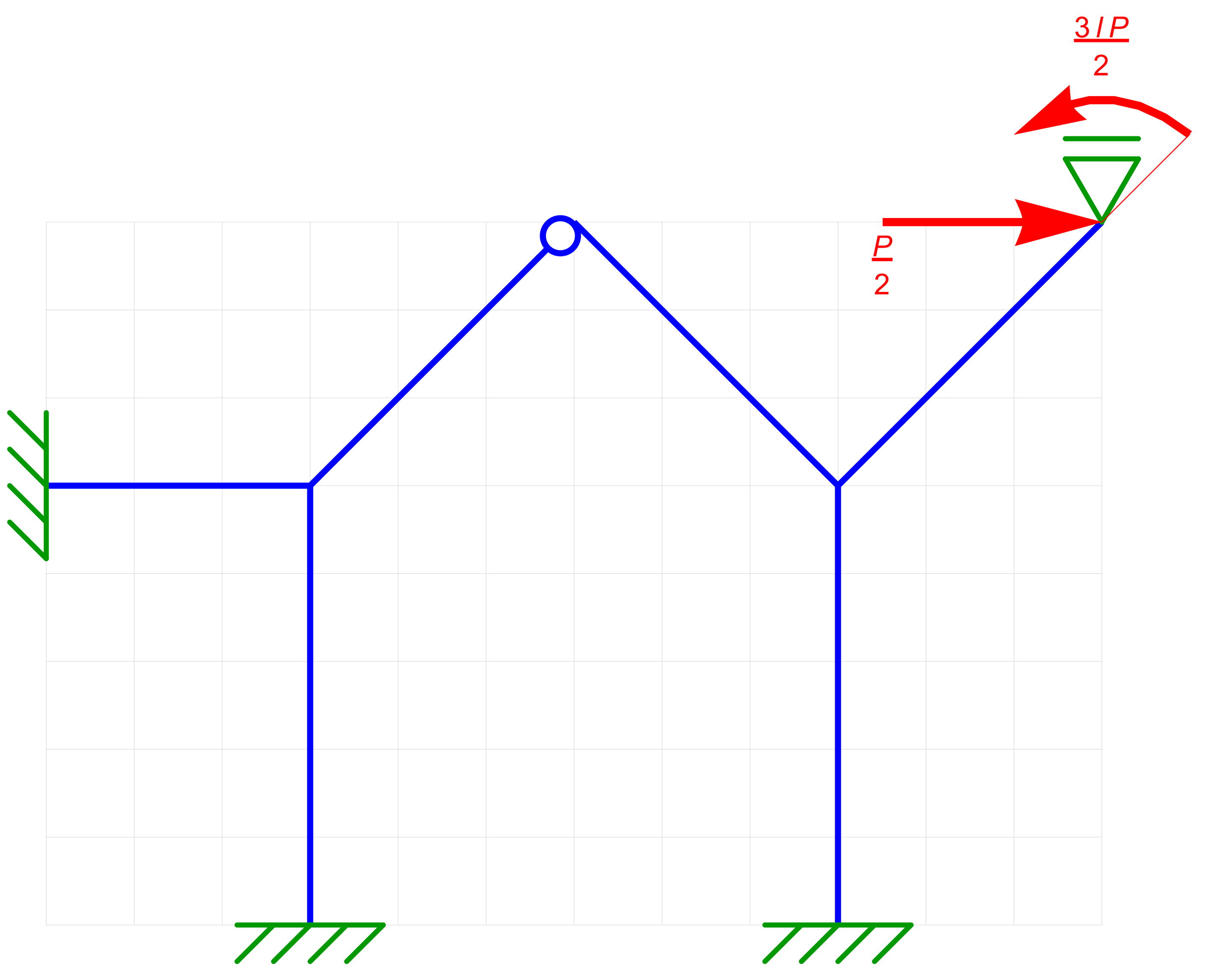
Egzamin MKI 07.02.18 Zadanie 1.

Narysować wykres momentów metodą przemieszczeń.



Redukcja części statycznie wyznaczalnej oraz schemat połówkowy:

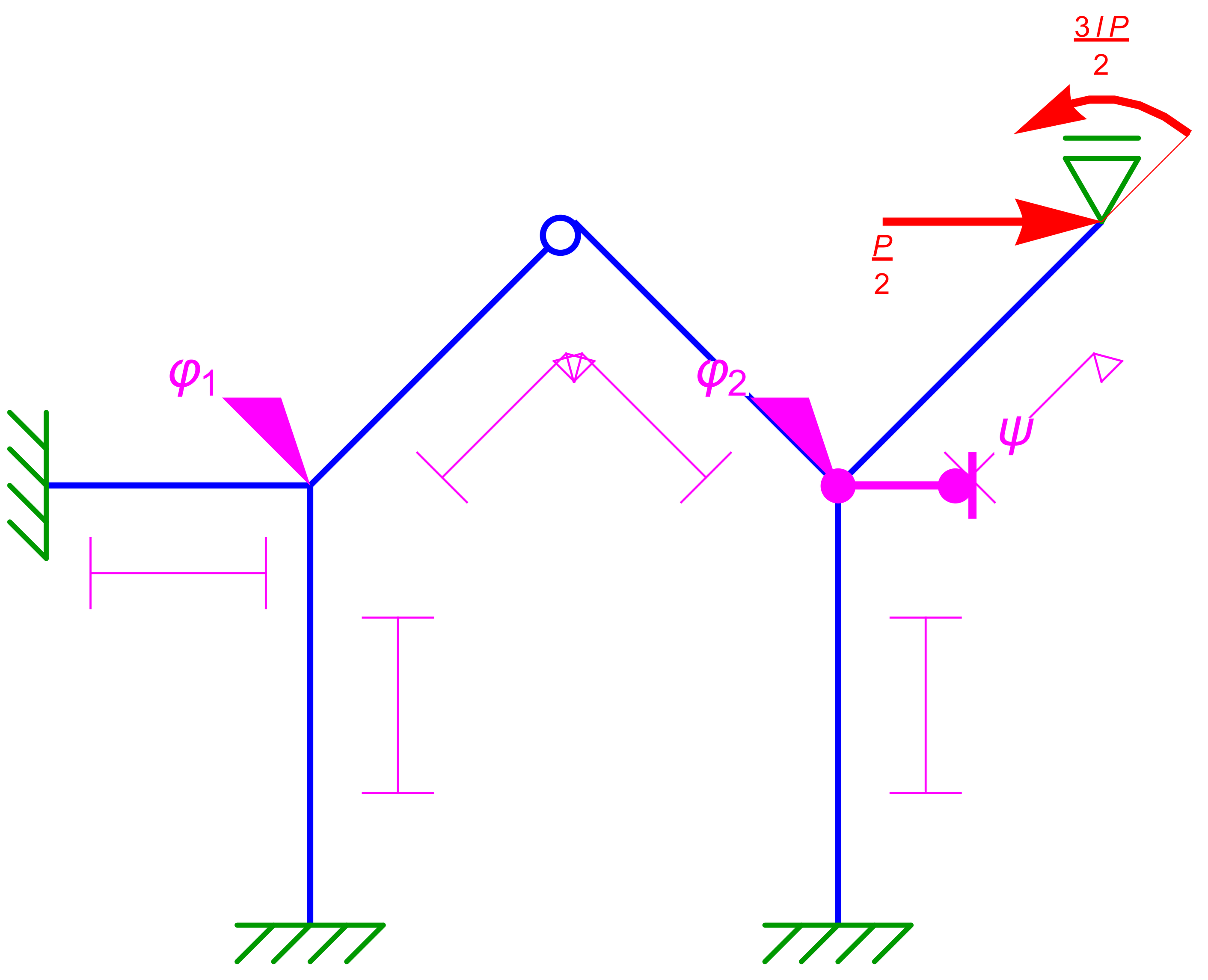
Geometria oraz obciążenia konstrukcji (wymiar oczka siatki – 1):



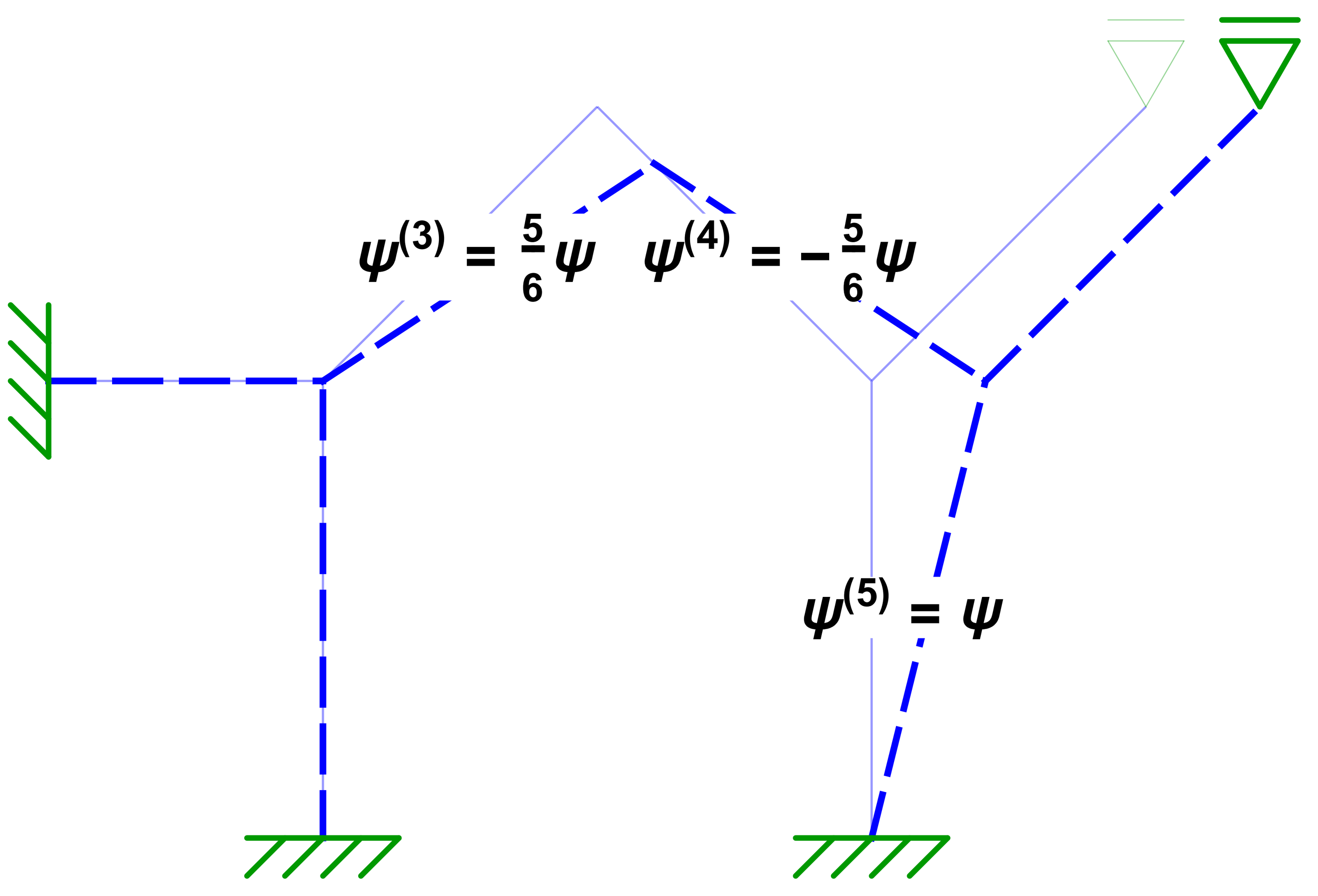
Wektor niewiadomych:

$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{pmatrix}$$

Układ geometrycznie wyznaczalny:



Plan przemieszczeń:



$\psi^{(1)} = 0$

$\psi^{(2)} = 0$

$\psi^{(3)} = \frac{5}{6}\,\psi$

$\psi^{(4)} = -\frac{5}{6}\,\psi$

$\psi^{(5)} = \psi$

$\psi^{(6)} = 0$

Momenty wyjściowe:

$\Phi_2^{06} = -\frac{3}{4}\,l\,P$

Wzory transformacyjne:

$\Phi_B^1 = \frac{EJ}{l}\left[\frac{2}{3}\varphi_1\right]$

$\Phi_1^1 = \frac{EJ}{l}\left[\frac{4}{3}\varphi_1\right]$

$\Phi_A^2 = \frac{EJ}{l}\left[\frac{2}{5}\varphi_1\right]$

$\Phi_1^2 = \frac{EJ}{l}\left[\frac{4}{5}\varphi_1\right]$

$\Phi_1^3 = \frac{EJ}{l}\left[\frac{1}{\sqrt{2}}\varphi_1 - \frac{5}{6\sqrt{2}}\psi\right]$

$\Phi_2^4 = \frac{EJ}{l}\left[\frac{1}{\sqrt{2}}\varphi_2 + \frac{5}{6\sqrt{2}}\psi\right]$

$\Phi_C^5 = \frac{EJ}{l}\left[\frac{2}{5}\varphi_2 - \frac{6}{5}\psi\right]$

$\Phi_2^5 = \frac{EJ}{l}\left[\frac{4}{5}\varphi_2 - \frac{6}{5}\psi\right]$

$\Phi_2^6 = \frac{EJ}{l}\left[\frac{1}{\sqrt{2}}\varphi_2\right] - \frac{3}{4}\,l\,P$

Równania równowagi:

$\Phi_1^1 + \Phi_1^2 + \Phi_1^3 = 0$

$\Phi_2^4 + \Phi_2^5 + \Phi_2^6 = 0$

$\Phi_1^3 \cdot \frac{5}{6}\overline{\psi} + \Phi_2^4 \cdot \left(-\frac{5}{6}\overline{\psi}\right) + \left(\Phi_C^5 + \Phi_2^5\right)\overline{\psi} + \frac{1}{2}P \cdot 5\,l\,\overline{\psi} = \overline{0}$

$$\frac{EJ}{l}\begin{pmatrix} \frac{32}{15} + \frac{1}{\sqrt{2}} & 0 & -\frac{5}{6\sqrt{2}} \\ 0 & \frac{4}{5} + \sqrt{2} & -\frac{6}{5} + \frac{5}{6\sqrt{2}} \\ -\frac{5}{6\sqrt{2}} & -\frac{6}{5} + \frac{5}{6\sqrt{2}} & \frac{12}{5} + \frac{25}{18\sqrt{2}} \end{pmatrix}\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{pmatrix} = l\,P\begin{pmatrix} 0 \\ \frac{3}{4} \\ \frac{5}{2} \end{pmatrix}$$

Rozwiązanie metody przemieszczeń:

$$\mathbf{q} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{pmatrix} = \frac{l^2 P}{EJ}\begin{pmatrix} 0.182 \\ 0.580 \\ 0.876 \end{pmatrix}$$

Momenty brzegowe:

$\Phi_B^1 = 0.121\,l\,P$

$\Phi_1^1 = 0.242\,l\,P$

$\Phi_A^2 = 0.073\,l\,P$

$\Phi_1^2 = 0.145\,l\,P$

$\Phi_1^3 = -0.388\,l\,P$

$\Phi_2^4 = 0.926\,l\,P$

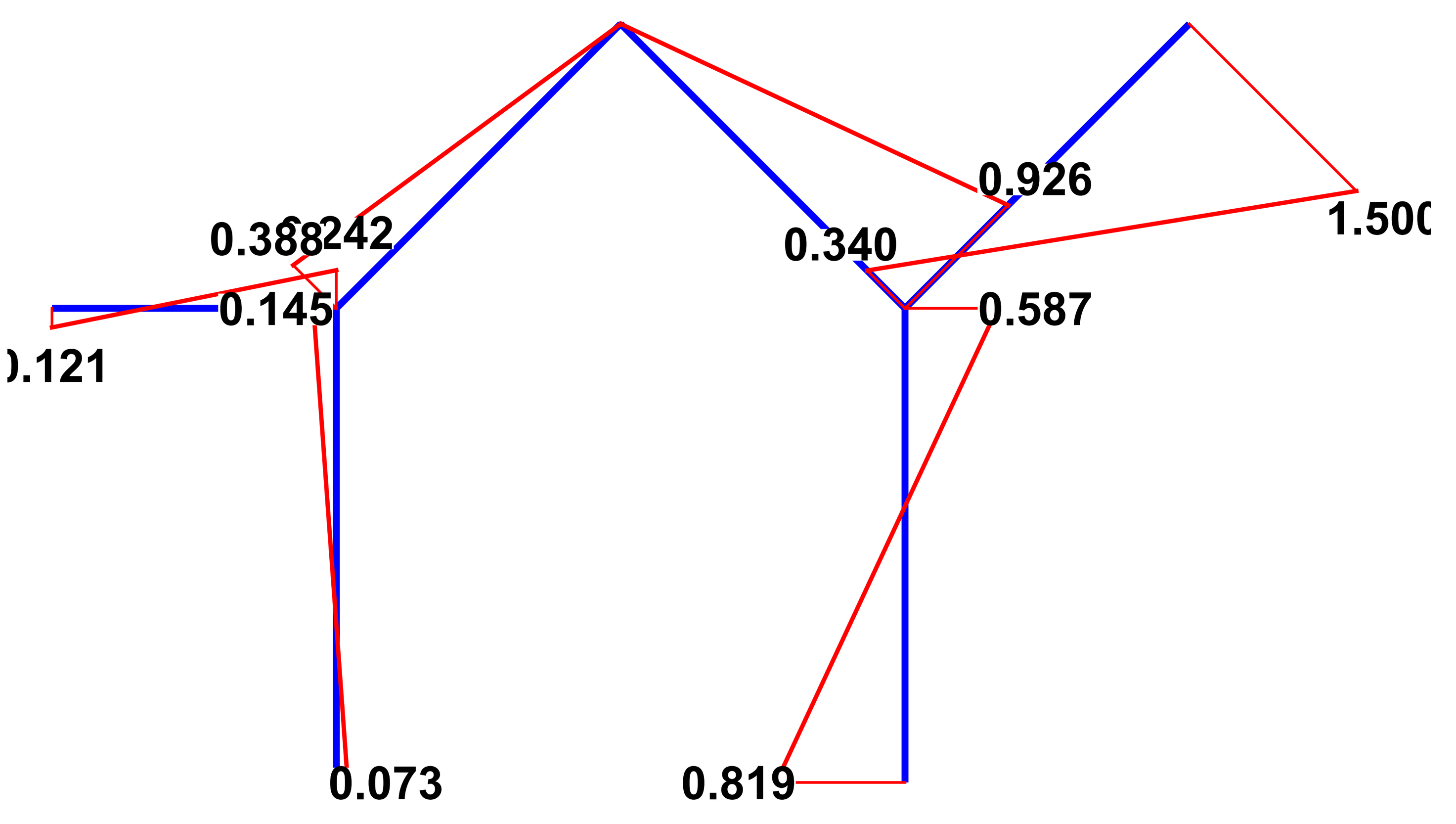
$\Phi_C^5 = -0.819\,l\,P$

$\Phi_2^5 = -0.587\,l\,P$

$\Phi_2^6 = -0.340\,l\,P$

Wykres momentów zginających:

M[l P]:

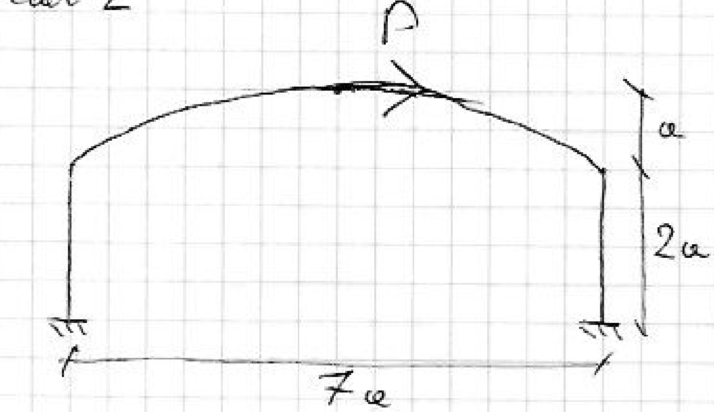


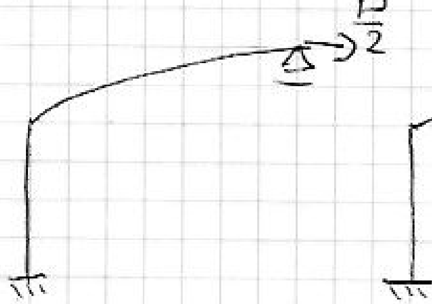
Zadanie przygotował Karol Bołbotowski.

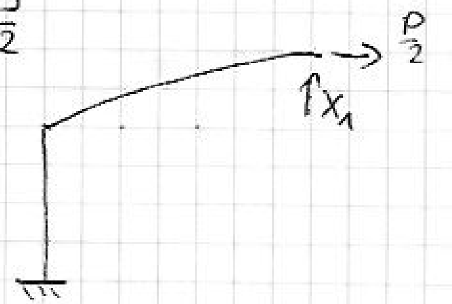
Egzamin MK I 7 II 2018

Zad 2

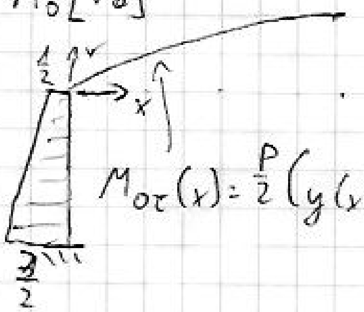
Schemat połówkowy: Schemat zastępczy:



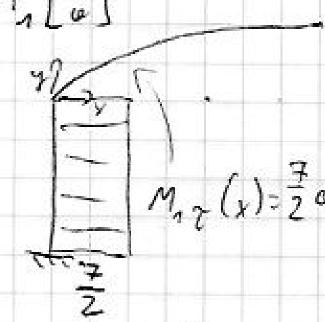




$M_0[Pa]$ $M_1[a]$ $y(x) = x(7a - x) \cdot \frac{4a}{(7a)^2}$



$M_{0\tau}(x) = \frac{P}{2}(y(x) - a)$



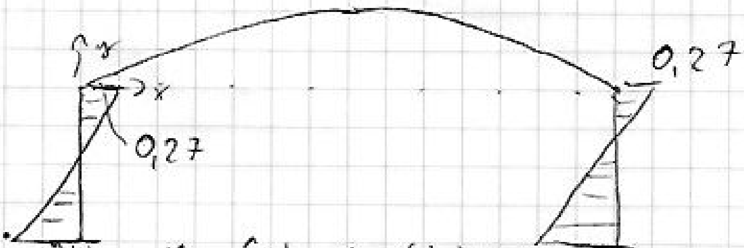
$M_{1\tau}(x) = \frac{7}{2}a - x$ (liniowe)

$$\delta_{11} = \frac{1}{EJ}\left[\left(\frac{7}{2}a\right)^2 \cdot 2a + \frac{1}{3}\left(\frac{7}{2}a\right)^3\right] = 38{,}79\,\frac{a^3}{EJ}$$

$$\delta_{10} = \frac{1}{EJ}\left[\frac{\left(\frac{1}{2}+\frac{3}{2}\right)Pa}{2} \cdot 2a \cdot \left(-\frac{7}{2}a\right) + \int_0^{\frac{7}{2}a} M_{0\tau}(x)\, M_{1\tau}(x)\, dx\right] = -8{,}53\,\frac{Pa^3}{EJ}$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = 0{,}22\,P$$

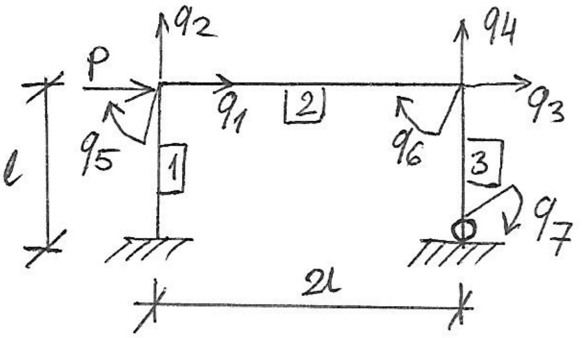
$M[Pa]$

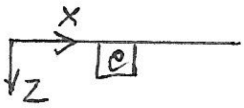


$M_\tau(x) = M_{0\tau}(x) + X_1 M_{1\tau}(x)$

$$M_\tau(x) = \frac{P}{2}(y(x) - a) + 0{,}22P\left(\frac{7}{2}a - x\right)$$

ZAD 3





$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{bmatrix}$$

$$\Delta = \mathbb{B} q \qquad \mathbb{N} = \mathbb{E}\mathbb{B} q$$

$$^*\chi = {}^*\mathbb{B} q \qquad {}^*\phi = \mathbb{D}\left(2\,{}^*\mathbb{B} q + \mathbb{B}^* q\right)$$

$$\chi^* = \mathbb{B}^* q \qquad \phi^* = \mathbb{D}\left({}^*\mathbb{B} q + 2\mathbb{B}^* q\right)$$

$$\mathbb{K} q = \mathbb{Q} \longrightarrow q$$

$$\mathbb{K} = \mathbb{B}^T \mathbb{E} \mathbb{B} + 2({}^*\mathbb{B})^T \mathbb{D}\, {}^*\mathbb{B} + ({}^*\mathbb{B})^T \mathbb{D} \mathbb{B}^* + (\mathbb{B}^*)^T \mathbb{D}\, {}^*\mathbb{B} + 2(\mathbb{B}^*)^T \mathbb{D} \mathbb{B}^*$$

$$\mathbb{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbb{E} = \frac{EA}{l}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^*\mathbb{B} = \begin{bmatrix} -\frac{1}{l} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2l} & 0 & \frac{1}{2l} & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{l} & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbb{D} = \frac{EI}{l}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\mathbb{B}^* = \begin{bmatrix} -\frac{1}{l} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2l} & 0 & \frac{1}{2l} & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{l} & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbb{Q} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} P$$