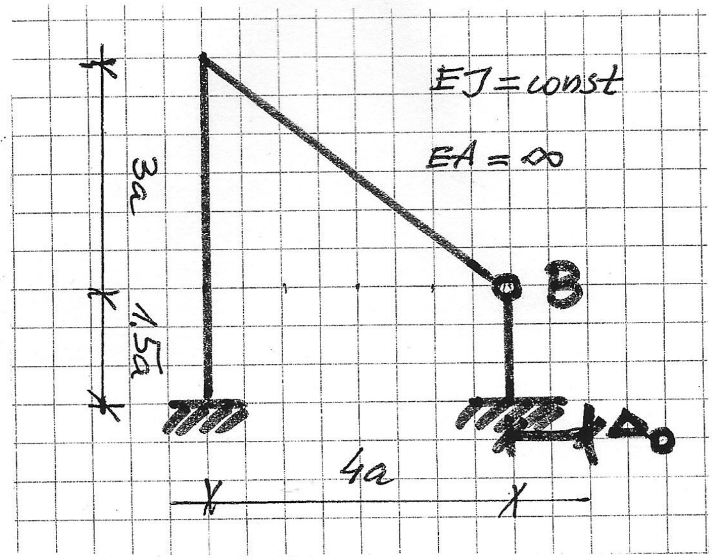
Egzamin pisemny z Mechaniki Konstrukcji I, 31 I 2018 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu	Ocena łączna Data

Zadanie 1

Dana jest rama płaska obciążona jak na rysunku; Sporządzić wykres *M* i znaleźć przemieszczenie poziome węzła B

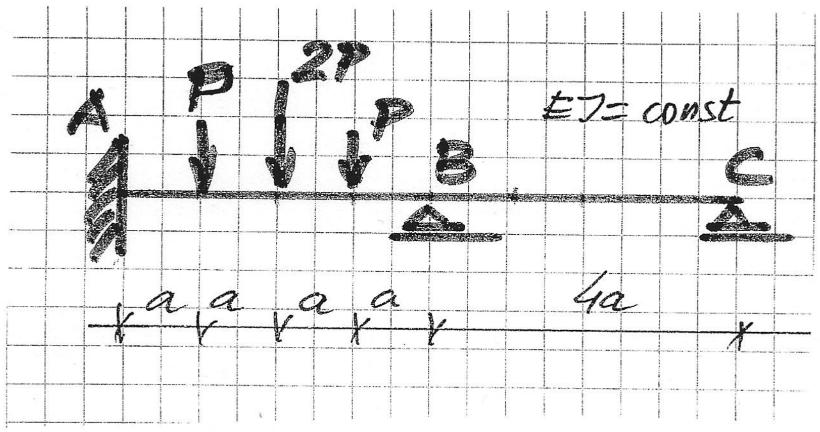
(For the given frame construct the diagram of the bending moments and find the horizontal displacement of the node B.)



Zadanie 2

Dana jest belka ciągła obciążona jak na rysunku. Znaleźć moment w utwierdzeniu A korzystając z twierdzenia Bettiego.

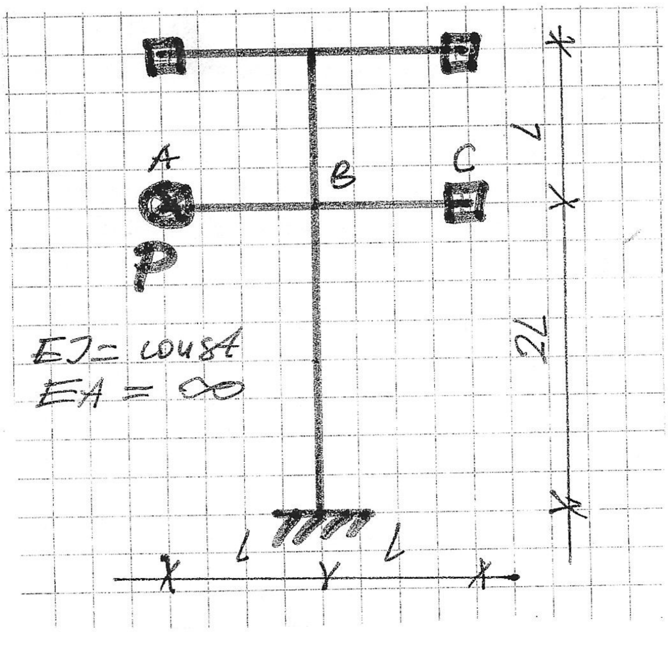
(Given is a continuous beam, loaded as shown in the figure. Compute the bending moment at the clamped end A by using Betti's theorem)



Zadanie 3

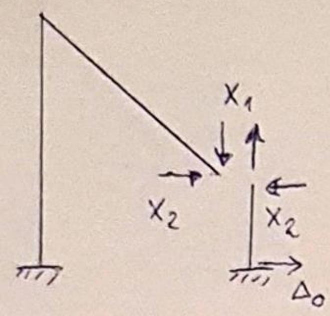
Dany jest ruszt przegubowy obciążony jak na rysunku. Znajdź funkcję opisującą ugięcie fragmentu BC belki ABC

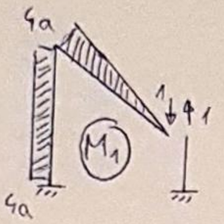
(Consider the given system of beams loaded as shown in the figure. Find the deflection function of the segment BC of the beam ABC).

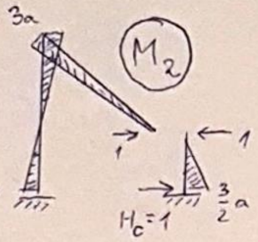


METODA SIŁ / FORCE METHOD

$n = 2$







$\delta_{11} = 98{,}667 \, \frac{a^3}{EJ}$

$\delta_{12} = -33{,}5 \, \frac{a^3}{EJ}$ $\qquad \delta_{10} = 0$

$\delta_{22} = 26{,}25 \, \frac{a^3}{EJ}$ $\qquad \delta_{20} = -1 \cdot \Delta_0$

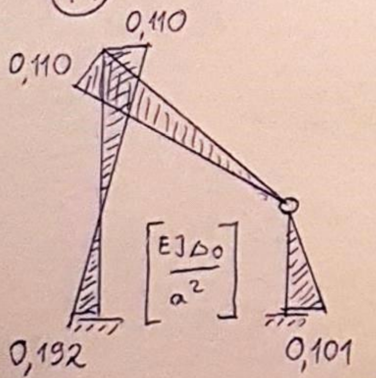
$$\mathbb{D} = \begin{bmatrix} 98{,}667 & -33{,}5 \\ -33{,}5 & 26{,}25 \end{bmatrix} \frac{a^3}{EJ} \qquad \mathbb{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\mathbb{D}_0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \Delta_0 \qquad \mathbb{D}\mathbb{X} + \mathbb{D}_0 = \mathbb{O}$$

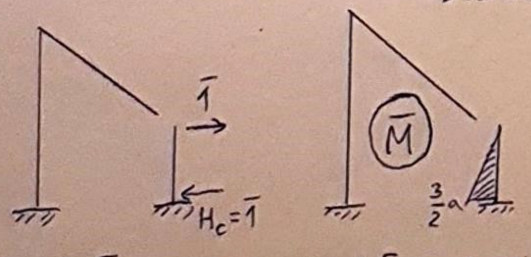
$$\Downarrow$$

$$\mathbb{X} = \begin{bmatrix} 0{,}0228 \\ 0{,}0672 \end{bmatrix} \frac{EJ\Delta_0}{a^3}$$

M

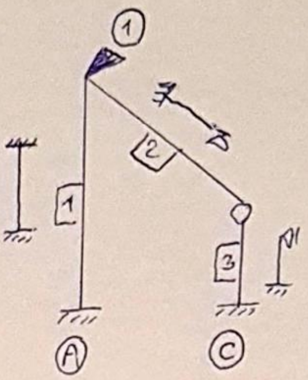


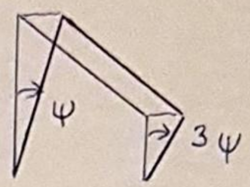
PRZEMIESZCZENIE PUNKTU B / POINT B DISPLACEMENT

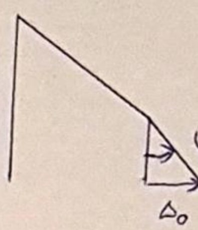


$$\delta_B = \int \frac{M\bar{M}}{EJ} ds - \Sigma \bar{R} \cdot \Delta_0 = \frac{1}{EJ}\left[\frac{1}{2} \cdot \frac{3}{2}a \cdot \frac{3}{2}a \cdot \left(-\frac{2}{3} \cdot 0{,}101 \frac{EJ\Delta_0}{a^2}\right) - (-1) \cdot \Delta_0\right] = 0{,}924 \, \Delta_0$$

METODA PRZEMIESZCZEŃ / DISPLACEMENT METHOD







$\psi_0 = -\frac{\Delta_0}{\frac{3}{2}a} = -\frac{2}{3}\frac{\Delta_0}{a}$

$$\mathbb{q} = \begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix}$$

1) $\Phi_1^1 + \Phi_1^2 = 0$

2) $(\Phi_A^1 + \Phi_1^1)\bar{\psi} + \Phi_C^3 \cdot 3\bar{\psi} = 0$

$\Phi_A^1 = \frac{2EJ}{4{,}5a}(\varphi_1 - 3\psi)$ $\qquad \Phi_1^2 = \frac{3EJ}{5a}\varphi_1$

$\Phi_1^1 = \frac{2EJ}{4{,}5a}(2\varphi_1 - 3\psi)$ $\qquad \Phi_C^3 = \frac{3EJ}{1{,}5a}(-3\psi) + \frac{3EJ}{1{,}5a} \cdot \frac{2}{3}\frac{\Delta_0}{a}$

$$\mathbb{K} = \begin{bmatrix} 1{,}489 & -1{,}333 \\ -1{,}333 & 20{,}667 \end{bmatrix} \frac{EJ}{a} \qquad \mathbb{Q} = \begin{bmatrix} 0 \\ -4 \end{bmatrix} \frac{EJ\Delta_0}{a^2}$$

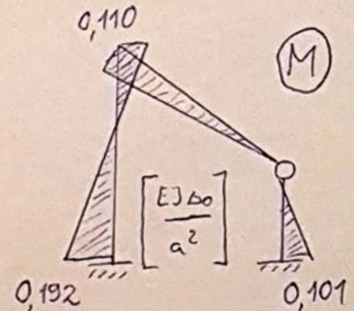
$$\mathbb{K}\mathbb{q} + \mathbb{Q} = 0 \Rightarrow \mathbb{q} = \begin{bmatrix} 0{,}1839 \\ 0{,}2054 \end{bmatrix} \frac{\Delta_0}{a}$$

$\Phi_A^1 = -0{,}192$

$\Phi_1^1 = -0{,}110$ $\qquad \left[\frac{EJ\Delta_0}{a^2}\right]$

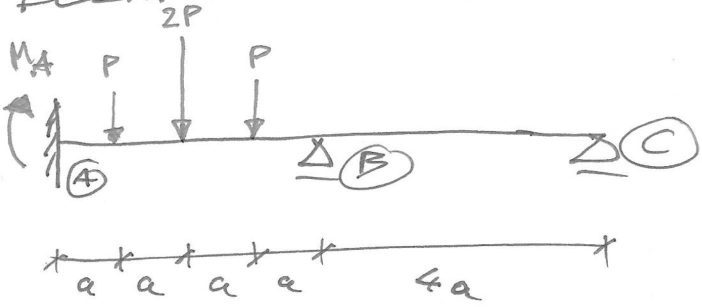
$\Phi_1^2 = 0{,}110$

$\Phi_C^3 = 0{,}101$

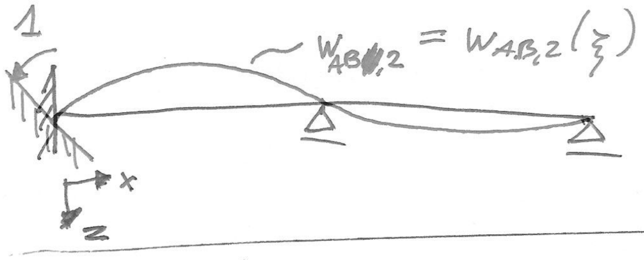


$\delta_B = 1{,}5a \cdot 3\psi = 0{,}924 \, \Delta_0$

EGZAMIN MKI 31.01.18 Zadanie 2



– układ 1



– układ 2 $\left(\xi = \frac{x}{4a}\right)$

Tw. Bettiego:

$$L_{12} = L_{21}$$

czyli:

$$P\cdot w_{AB,2}\left(\tfrac{1}{4}\right) + 2P\cdot w_{AB,2}\left(\tfrac{1}{2}\right) + P\cdot w_{AB,2}\left(\tfrac{3}{4}\right) - M_A\cdot 1 = 0$$

Wyznaczenie f. ugięcia $w_{AB,2}$ (2 oznacza układ 2):

$$\frac{EJ}{(4a)^4}\frac{d^4 w_{AB,2}}{d\xi^4} = 0 \Rightarrow w_{AB,2}(\xi) = C_0 + C_1\xi + C_2\xi^2 + C_3\xi^3$$

Warunki brzegowe:

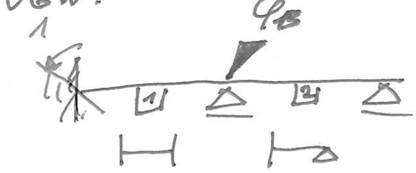
$w_{AB,2}(0) = 0$ $\quad$ $w_{AB,2}(1) = 0$

$\varphi_{AB,2}(0) = -1$ $\quad$ $\varphi_{AB,2}(1) = \varphi_B$ – należy „doliczyć"

$\frac{1}{4a}\frac{dw_{AB,2}}{d\xi}(0)$ $\quad$ $\frac{1}{4a}\frac{dw_{AB,2}}{d\xi}(1)$

Obliczenie kąta obrotu φ_B w układzie 2 – metoda przemieszczeń:

UGW:



RRMP:

1) $\Phi_B^{(1)} + \Phi_B^{(2)} = 0$

Wzory transf.:

$$\left.\begin{aligned}\Phi_B^{(1)} &= \frac{2EJ}{4a}\left((-1) + 2\cdot\varphi_B\right)\\ \Phi_B^{(2)} &= \frac{3EJ}{4a}(\varphi_B)\end{aligned}\right\} \Rightarrow \boxed{\varphi_B = \frac{2}{7}}$$

$$w_{AB,2}(\xi) = -a\left(4\xi - \frac{48}{7}\xi^2 + \frac{20}{7}\xi^3\right)$$

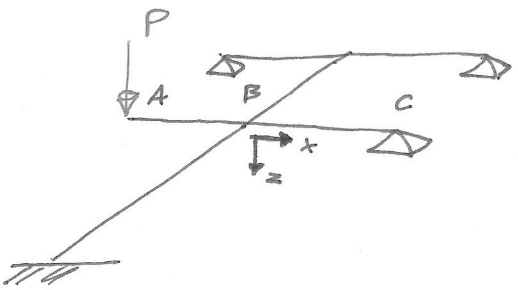
Ostatecznie na mocy tw. Bettiego:

$$M_A = P\cdot w_{AB,2}\left(\tfrac{1}{4}\right) + 2P\cdot w_{AB,2}\left(\tfrac{1}{2}\right) + P\cdot w_{AB,2}\left(\tfrac{3}{4}\right) = \boxed{-\frac{9}{4}Pl}$$ – zwrot M_A przeciwny do założonego...

Przygotował:

Karol [illegible]

EGZAMIN MKI 31.01.18 Zadanie 3



Obliczenie funkcji $w_{BC} = w_{BC}(\xi)$ $\left(\xi = \frac{x}{l}\right)$

$$\frac{1}{l^4}EJ\frac{d^4 w_{BC}}{d\xi^4}=0 \Rightarrow w_{BC}(\xi)=C_0+C_1\xi+C_2\xi^2+C_3\xi^3$$

warunki brzegowe:

$w_{BC}(0)=w_B$ — należy „doliczyć"

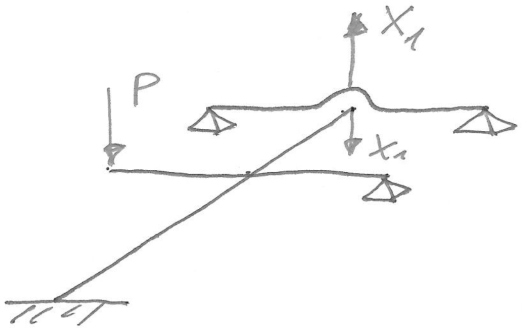
$M_{BC}(0)=-Pl \Rightarrow -\frac{EJ}{l^2}\frac{d^2 w_{BC}}{d\xi^2}(0)=-Pl$

$w_{BC}(1)=0$

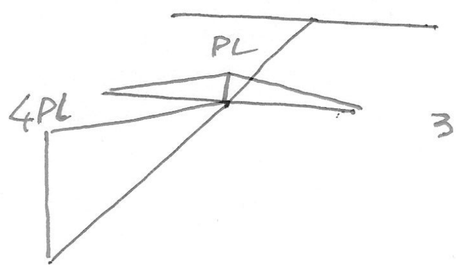
$M_{BC}(1)=0 \Rightarrow -\frac{EJ}{l^2}\frac{d^2 w_{BC}}{d\xi^2}(1)=0$

Obliczanie przemieszczenia w_B — metoda sił:

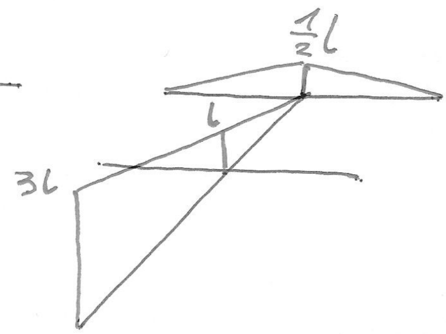
ukł. zast.:



M_0:

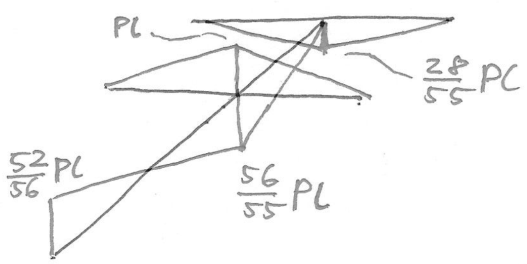


M_1:

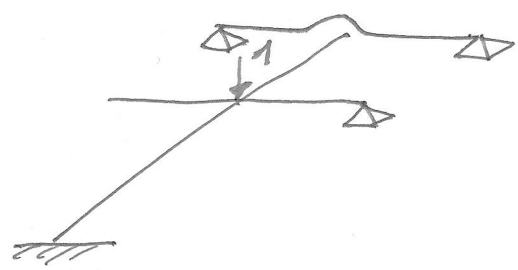


$$\delta_{11}=\frac{55}{6}\frac{l^3}{EJ},\quad \delta_{10}=\frac{28}{3}\frac{Pl^3}{EJ};\quad \delta_{11}X_1+\delta_{10}=0 \Rightarrow \boxed{X_1=-\frac{56}{55}P}$$

Ostateczny wykres momentów M:

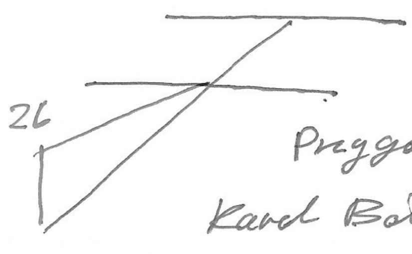


Obliczenie w_B ze wzoru M-M, przykładając obciążenie wirtualne w układzie zast. na mocy tw. redukcyjnego:



$$\Downarrow$$

$\bar{M}$:



$$w_B=\int_{RUSZT}\frac{\bar{M}M}{EJ}dx=\boxed{\frac{32}{55}\frac{Pl^3}{EJ}}$$

Ostatecznie:

$$w_{BC}(\xi)=\frac{Pl^3}{EJ}\left(\frac{32}{55}-\frac{151}{165}\xi+\frac{1}{2}\xi^2-\frac{1}{6}\xi^3\right)$$

Przygotował:
Karol Bołbotowski