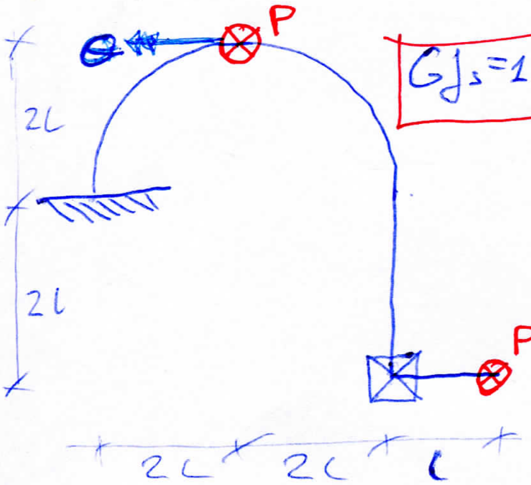
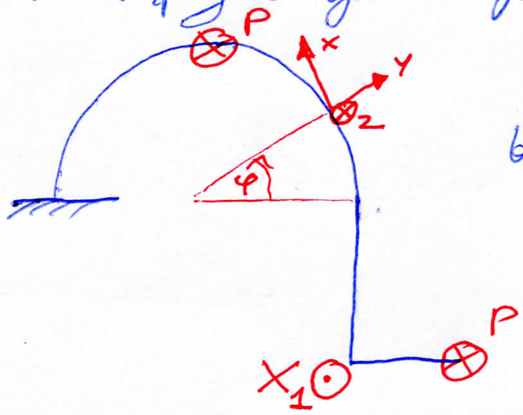
KOLOKWIUM POPRAWKOWE IPB RA. 2018/19.

OBLICZYĆ KĄT SKRĘCENIA Θ:



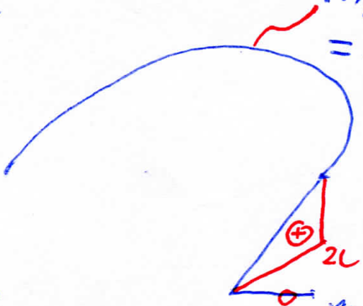
$\boxed{GJ_s = 1 \cdot EJ}$

Układ zastępczy statycznie wyznaczalny:



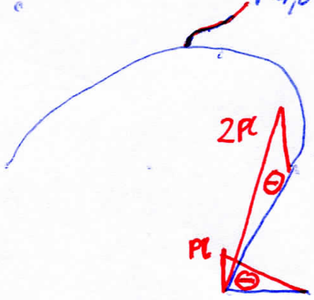
Układ biegunowy: $\varphi \in [0, \pi]$

M_1:



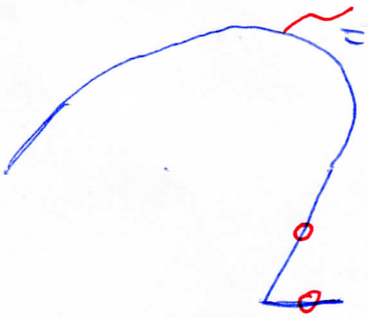
$M_{\alpha,1}(\varphi) = 2L \cdot \sin\varphi + 2L\cos\varphi$

M_0:



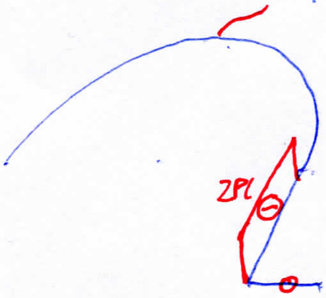
$M_{\alpha,0}(\varphi) = \begin{cases} -3PL\sin\varphi - 2PL\cos\varphi & \text{dla } \varphi \in [0, \frac{\pi}{2}] \\ -3PL\sin\varphi & \text{dla } \varphi \in [\frac{\pi}{2}, \pi] \end{cases}$

$\mathfrak{M}_1$:



$\mathfrak{M}_{\alpha,1}(\varphi) = -2L(1-\cos\varphi) - 2L\sin\varphi$

$\mathfrak{M}_0$:



$\mathfrak{M}_{\alpha,0}(\varphi) = \begin{cases} 2PL - 3PL\cos\varphi + 2PL\sin\varphi & \text{dla } \varphi \in [0, \frac{\pi}{2}] \\ 4PL - 3PL\sin\varphi & \text{dla } \varphi \in [\frac{\pi}{2}, \pi] \end{cases}$

$$\delta_{11} = \frac{1}{EJ}\left(\frac{1}{2} \cdot 2L \cdot 2L\right)\left(+\frac{2}{3} \cdot 2L\right) + \frac{1}{EJ}\int_0^{\pi}\left[2L \cdot \sin\varphi + 2L \cdot \cos\varphi\right]^2 \cdot \overset{2L}{R}\, d\varphi + \frac{1}{\underset{EJ}{GJ_s}}\int_0^{\pi}\left[-2L(1-\cos\varphi) - 2L\sin\varphi\right]^2 \cdot \overset{2L}{R}\, d\varphi = 110.065\frac{L^3}{EJ}$$

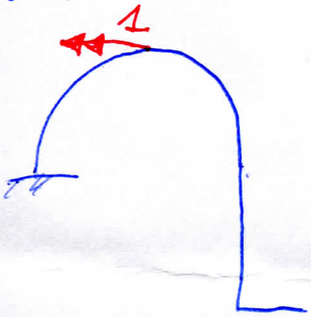
$$\delta_{10} = \frac{1}{EJ}\left(\frac{1}{2} \cdot 2PL \cdot 2L\right)\left(-\frac{2}{3} \cdot 2L\right) + \frac{1}{EJ}\left(\int_0^{\frac{\pi}{2}}\left[2L\sin\varphi + 2L\cos\varphi\right]\left[-3PL\sin\varphi - 2PL\cos\varphi\right] \cdot R\, d\varphi + \int_{\frac{\pi}{2}}^{\pi}\left[2L\sin\varphi + 2L\cos\varphi\right]\left[-3PL\sin\varphi\right] \cdot R\, d\varphi\right) +$$

$$+ \frac{1}{GJ_s}\left(\int_0^{\frac{\pi}{2}}\left[-2L(1-\cos\varphi) - 2L\sin\varphi\right] \cdot \left[2PL - 3PL\cos\varphi + 2PL\sin\varphi\right] \cdot R\, d\varphi + \int_{\frac{\pi}{2}}^{\pi}\left[-2L(1-\cos\varphi) - 2L\sin\varphi\right]\left[4PL - 3PL\sin\varphi\right] \cdot R \cdot d\varphi\right)$$

$$= -130.631\frac{PL^3}{EJ}$$

$\boxed{\delta_{11}X_1 + \delta_{10} = 0 \Rightarrow X_1 = 1.187P}$

Obciążenie wirtualne w układzie zastępczym na mocy Tw. Redukcyjnego:



$\overline{M}_\alpha(\varphi) = \begin{cases} 0 & \text{dla } \varphi \in [0, \frac{\pi}{2}] \\ -\sin(\varphi - \frac{\pi}{2}) & \text{dla } \varphi \in [\frac{\pi}{2}, \pi] \end{cases}$

$\overline{\mathfrak{M}}_\alpha(\varphi) = \begin{cases} 0 & \text{dla } \varphi \in [0, \frac{\pi}{2}] \\ -\cos(\varphi - \frac{\pi}{2}) & \text{dla } \varphi \in [\frac{\pi}{2}, \pi] \end{cases}$

$$\Theta = \frac{1}{EJ} \cdot \int_{\frac{\pi}{2}}^{\pi}\left[\left(2L \cdot \sin\varphi + 2L \cdot \cos\varphi\right) \cdot 1.187P - 3PL\sin\varphi\right] \cdot \left[-\sin\left(\varphi - \frac{\pi}{2}\right)\right] \cdot \overset{2L}{R}\, d\varphi + \frac{1}{\underset{EJ}{GJ_s}}\int_{\frac{\pi}{2}}^{\pi}\left[\left(-2L(1-\cos\varphi) - 2L\sin\varphi\right) \cdot 1.187P + 4PL - 3PL\sin\varphi\right] \cdot \left[-\cos\left(\varphi - \frac{\pi}{2}\right)\right] \cdot \overset{2L}{R}\, d\varphi = \boxed{4.205\frac{PL^2}{EJ}}$$

Przygotował Karol Bołbotowski