Egzamin z Mechaniki Konstrukcji (MK3 IPB), 4.12.2018
studia niestacjonarne

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rok akademicki zaliczenia ćwiczeń		nr albumu	grupa (IPB / BZ)	tryb studiów (ST / NST)
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna

Zadanie 1.

$EJ = const.$, $\quad k = 0{,}1024\dfrac{EJ}{l^4}$

Oblicz składowe reakcji podpór ramy z rys. 1.

Zadanie 2.

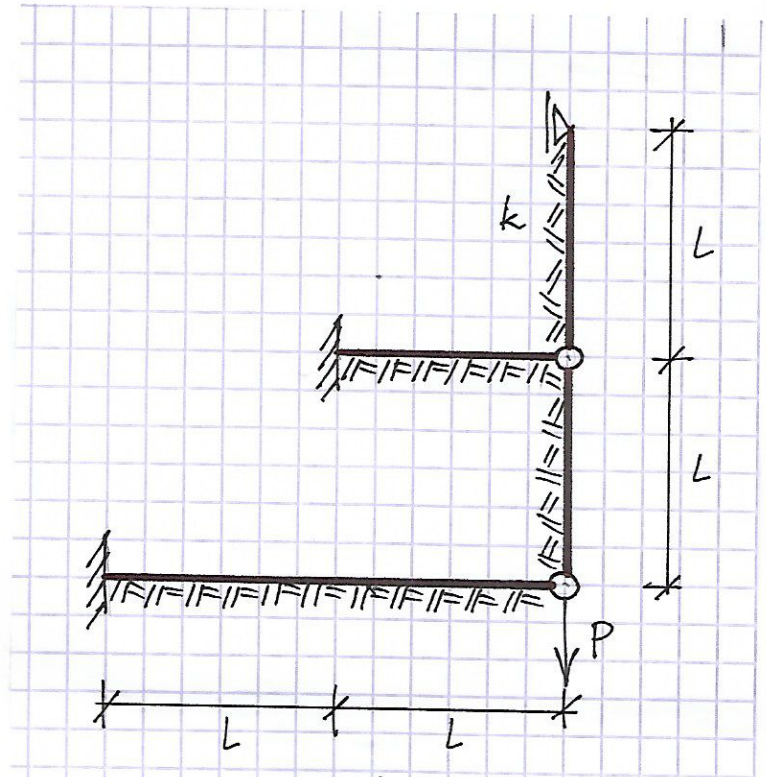
$EJ = const.$

Oblicz wartość $\varphi(k)$ w belce z rys. 2

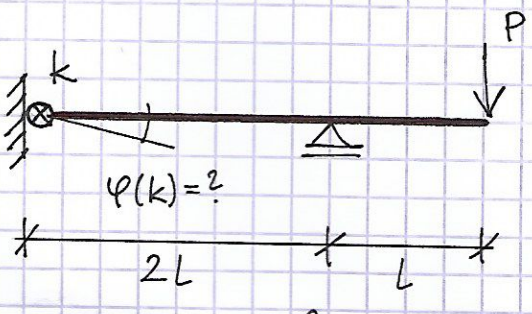
dla $k = \frac{EJ}{l}$, $k = 5\frac{EJ}{l}$, $k = 20\frac{EJ}{l}$, $k = +\infty$.

Zadanie 3.

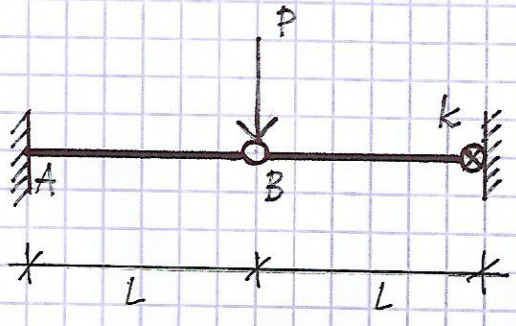
Wyprowadź równanie linii ugięcia fragmentu A-B belki z rys. 3.



rys. 1

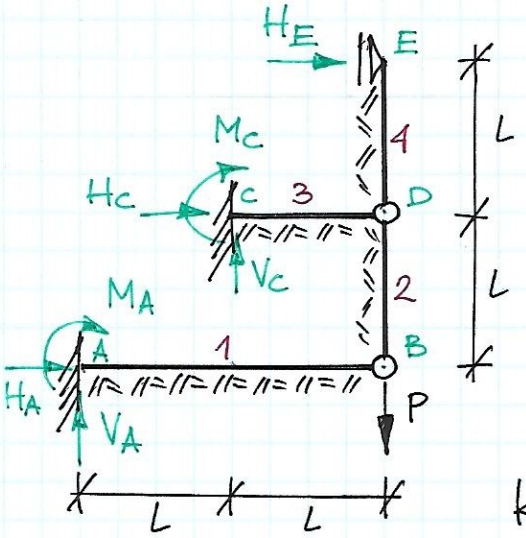


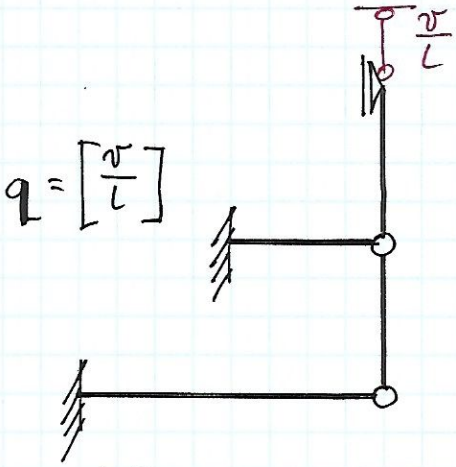
rys. 2

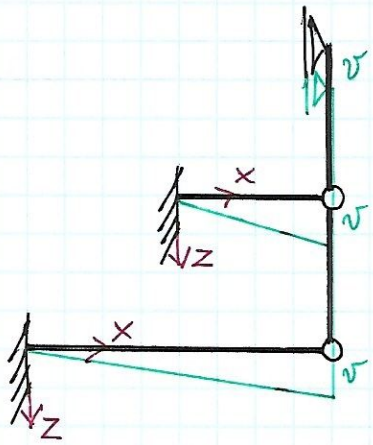


rys. 3

Zadanie 1







$$k = 0{,}1024\,\frac{EJ}{L^4} \rightarrow \lambda = 0{,}4$$

$$\lambda_1 = 2\lambda = 0{,}8 \;;\; \lambda_2 = \lambda_3 = \lambda_4 = \lambda = 0{,}4$$

Równanie równowagi:

$$W_B^1 + W_D^3 - P = 0$$

$$W_B^1 = -\frac{EJ}{4L^2}\left(-\chi'(0{,}8)\frac{v}{2L}\right)$$

$$W_D^3 = -\frac{EJ}{L^2}\left(-\chi'(0{,}4)\frac{v}{L}\right)$$

$$\frac{1}{4}\cdot 3{,}385\,\frac{v}{2L} + 3{,}024\,\frac{v}{L} = \frac{PL^2}{EJ} \rightarrow \frac{v}{L} = 0{,}29\,\frac{PL^2}{EJ}$$

$$V_A = \frac{EJ}{4L^2}\left(-\varepsilon'(0{,}8)\cdot\frac{v}{2L}\right) = -0{,}1\,P$$

$$M_A = \frac{EJ}{2L}\left(-\delta'(0{,}8)\cdot\frac{v}{2L}\right) = -0{,}213\,PL$$

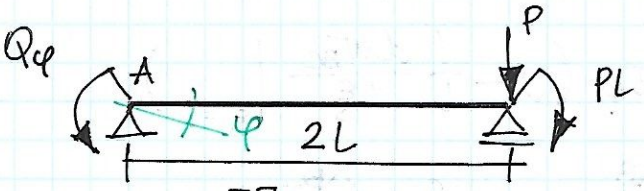
$$V_C = \frac{EJ}{L^2}\left(-\varepsilon'(0{,}4)\cdot\frac{v}{L}\right) = -0{,}1866\,P$$

$$M_C = \frac{EJ}{L}\left(-\delta'(0{,}4)\cdot\frac{v}{L}\right) = -0{,}1869\,PL$$

$$H_A = H_C = H_E = 0$$

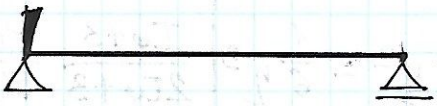
opracował: G. Dzierżanowski

Zadanie 2



$\tau_\varphi = \frac{k_\varphi L}{EJ}$ $\quad k_\varphi \equiv k$

$Q_\varphi = \frac{EJ}{L}\tau_\varphi \varphi$



$q = [\varphi]$

$$\phi_A + Q_\varphi = 0$$

$$\phi_A = \frac{3EJ}{2L}\varphi + \frac{1}{2}PL$$

$$\frac{3EJ}{2L}\varphi + \frac{1}{2}PL + \frac{EJ}{L}\tau_\varphi \varphi = 0$$

$$\frac{EJ}{L}\left(\frac{3}{2} + \tau_\varphi\right)\varphi = -\frac{1}{2}PL$$

$$\varphi = -\frac{1}{3+2\tau_\varphi}\frac{PL^2}{EJ}$$

$k_\varphi = \frac{EJ}{L} \rightarrow \tau_\varphi = 1 \rightarrow \varphi = -\frac{1}{5}\cdot\frac{PL^2}{EJ}$

$k_\varphi = 5\frac{EJ}{L} \rightarrow \tau_\varphi = 5 \rightarrow \varphi = -\frac{1}{13}\cdot\frac{PL^2}{EJ}$

$k_\varphi = 20\frac{EJ}{L} \rightarrow \tau_\varphi = 20 \rightarrow \varphi = -\frac{1}{43}\cdot\frac{PL^2}{EJ}$

$k_\varphi = +\infty \rightarrow \tau_\varphi = +\infty \rightarrow \varphi = 0$

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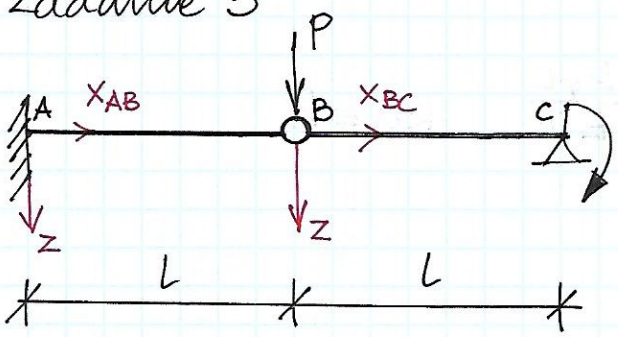
Zadanie 3



$$Q_\varphi = \frac{EJ}{L} r_\varphi (0 - \varphi_C)$$

$$Q_\varphi = \frac{EJ}{L} r_\varphi \left(0 - \frac{1}{L}\frac{dw_{BC}}{d\xi}\Big|_{\xi=1}\right)$$

$$\eta = \frac{X_{AB}}{L} \qquad \xi = \frac{X_{BC}}{L}$$

$$w_{AB}(\eta) = C_0 + C_1\eta + C_2\eta^2 + C_3\eta^3 = \left(\frac{1}{2}\eta^2 - \frac{1}{6}\eta^3\right)\frac{r_\varphi+3}{2r_\varphi+3}\frac{PL^3}{EJ}$$

$w_{AB}(0) = 0 \leftarrow w_A = 0 \rightarrow C_0 = 0$

$\frac{1}{L}\frac{dw_{AB}}{d\eta}\Big|_{\eta=0} = 0 \leftarrow \varphi_A = 0 \rightarrow C_1 = 0$

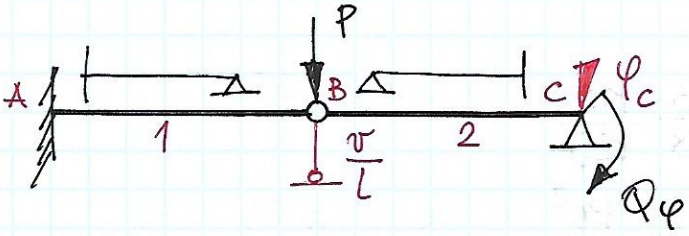
$C_2 = \frac{r_\varphi+3}{2(2r_\varphi+3)}\frac{PL^3}{EJ}$

$C_3 = -\frac{r_\varphi+3}{6(2r_\varphi+3)}\frac{PL^3}{EJ}$

$-\frac{EJ}{L^2}\frac{d^2w_{AB}}{d\eta^2}\Big|_{\eta=1} = 0 \leftarrow M_B = 0 \rightarrow -\frac{EJ}{L^2}(2C_2 + 6C_3) = 0$

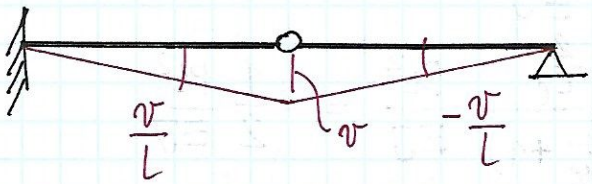
$w_{AB}(1) = v \leftarrow w_B = v \rightarrow C_0 + C_1 + C_2 + C_3 = \frac{r_\varphi+3}{3(2r_\varphi+3)}\frac{PL^3}{EJ}$

Obliczamy v metodą przemieszczeń



$$q = \begin{bmatrix} \varphi_C \\ \frac{v}{L} \end{bmatrix}$$

1. $\Phi_C^2 - Q_\varphi = 0$



2. $\Phi_A^1 \cdot \frac{v}{L} + \Phi_C^2 \cdot \left(-\frac{v}{L}\right) + Pv = 0$

$$\Phi_A^1 = \frac{3EJ}{L}\left(-\frac{v}{L}\right)$$

$$\Phi_C^2 = \frac{3EJ}{L}\left(\varphi_C + \frac{v}{L}\right)$$

1. $\frac{3EJ}{L}\left(\varphi_C + \frac{v}{L}\right) + \frac{EJ}{L} r_\varphi \varphi_C = 0$

2. $\frac{3EJ}{L}\left(\frac{v}{L}\right) + \frac{3EJ}{L}\left(\varphi_C + \frac{v}{L}\right) = PL$

$$\varphi_C = -\frac{1}{2r_\varphi+3}\frac{PL^2}{EJ}$$

$$v = \frac{r_\varphi+3}{3(2r_\varphi+3)}\frac{PL^3}{EJ}$$

opracował: G. Dzierżanowski