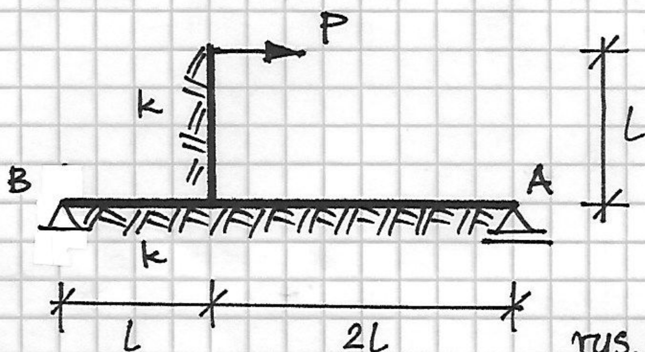
Egzamin z Mechaniki Konstrukcji (MK3 IPB), 8.09.2018
studia niestacjonarne

NAZWISKO, Imię					
rok akademicki zaliczenia ćwiczeń		nr albumu	grupa (IPB / BZ)	tryb studiów (ST / NST)	
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna	

Zadanie 1.

$EJ = const.$, $\quad k = 0{,}1024\dfrac{EJ}{l^4}$

Oblicz poziome i pionowe składowe reakcji podpór A (podpora przesuwna) i B (podpora nieprzesuwna) ramy z rys. 1.

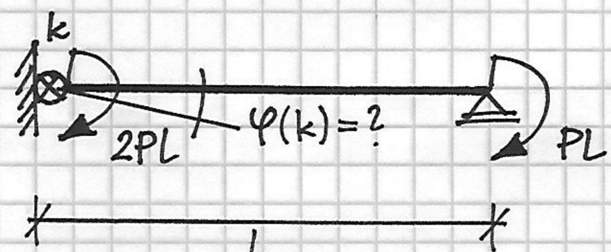


rys. 1

Zadanie 2.

$EJ = const.$

Oblicz wartość $\varphi(k)$ w ramie z rys. 2 dla $k = \frac{EJ}{l}$, $k = 5\frac{EJ}{l}$, $k = 20\frac{EJ}{l}$, $k = +\infty$.

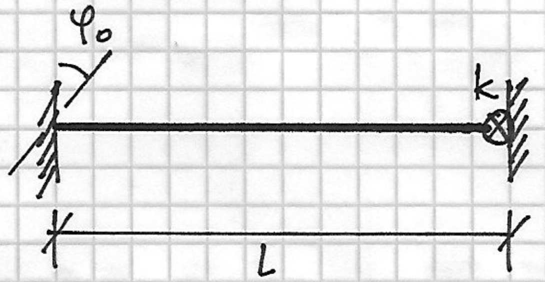


rys. 2

Zadanie 3.

Wyprowadź równanie linii ugięcia belki z rys. 3.

Belka jest obciążona obrotem lewej podpory o kąt φ_0.



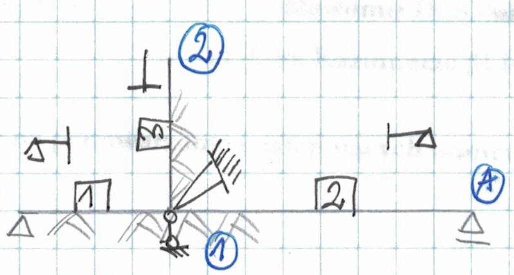
rys. 3

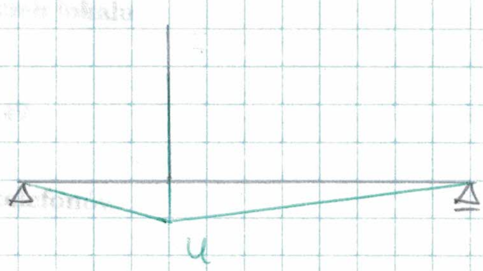
ZADANIE 1 NST II IPB 8 IX 2018

UGW

$$q = \begin{bmatrix} \varphi \\ \frac{u}{l} \end{bmatrix}$$

Plan przesunięć





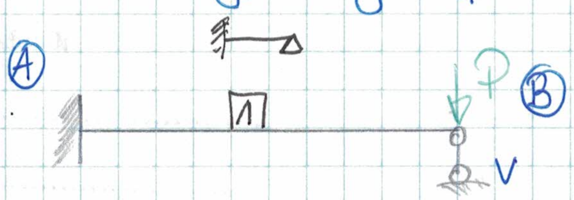
Pręt	$l^{(i)}$	$\eta^{(i)}$	$^*w$	w^*
1	l	0,4	0	u
2	$2l$	0,8	u	0
3	l	0,4	0	0

r. r. MP

$$\Phi_1^{(1)} + \Phi_1^{(2)} + \Phi_1^{(3)} = 0 \qquad (1)$$

$$-\left(W_1^{(1)}\bar{u} + W_1^{(2)}\bar{u}\right) = 0 \qquad (2)$$

Moment wyjściowy $\Phi_1^{(03)}$



$$-W_B^{(1)}\bar{v} + P\bar{v} = 0$$

$$W_B^{(1)} = P$$

$$W_B^{(1)} = -\frac{EJ}{l^2}\left(-\alpha'(0,4)\frac{v}{l}\right) = 3,024\frac{EJ}{l^3}v = P$$

$$v = 0,331\frac{Pl^3}{EJ} \qquad \Phi_A^{(1)} = \frac{EJ}{l}\left(-\delta'(0,4)\frac{v}{l}\right) = -0,992\,Pl = \Phi_1^{(03)}$$

$$W_A^{(1)} = W_1^{(03)} = \frac{EJ}{l^2}\left(-\varepsilon'(0,4)\frac{v}{l}\right) = -0,988\,P$$

Wzory transformacyjne

$$\Phi_1^{(1)} = \frac{EJ}{l}\left[\alpha'(0,4)\varphi - \theta'(0,4)\frac{u}{l}\right] = \frac{EJ}{l}\left[3,002\varphi - 3,009\frac{u}{l}\right]$$

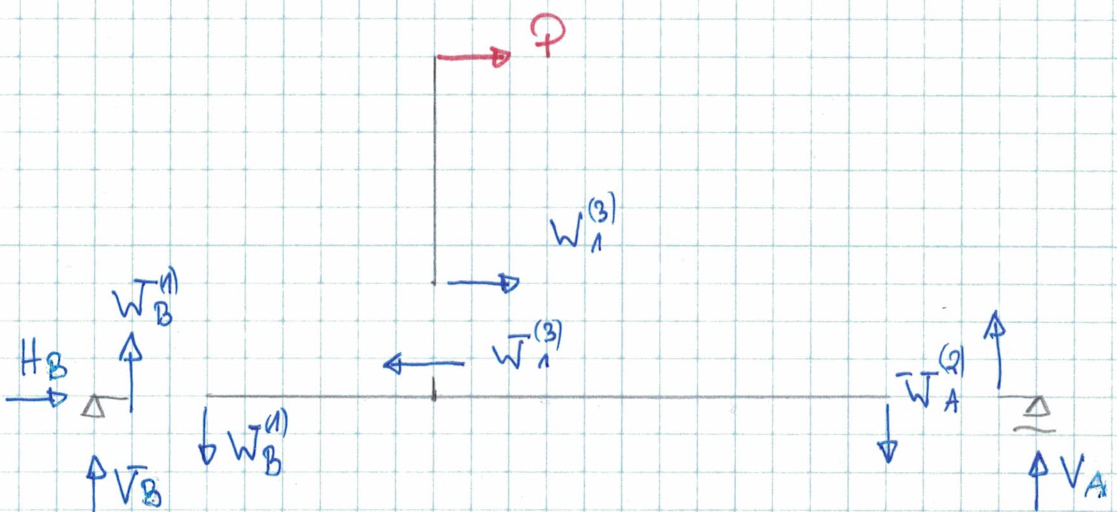
$$\Phi_1^{(2)} = \frac{EJ}{2l}\left[\alpha'(0,8)\varphi + \theta'(0,8)\frac{u}{2l}\right] = \frac{EJ}{l}\left[1,516\varphi + 0,785\frac{u}{l}\right]$$

$$\bar{\Phi}_1^{(3)} = \frac{EJ}{l}\left[\alpha''(0,4)\varphi\right] - 0,992Pl = \frac{EJ}{l}\left[0,034\varphi\right] - 0,992Pl$$

$$\bar{W}_1^{(2)} = \frac{EJ}{4l^2}\left[\Theta'(0,8)\varphi + \gamma'(0,8)\frac{u}{2l}\right] = \frac{EJ}{l^2}\left[0,785\varphi + 0,474\frac{u}{l}\right]$$

$$\bar{W}_1^{(1)} = -\frac{EJ}{l^2}\left[\Theta'(0,4)\varphi - \gamma'(0,4)\frac{u}{l}\right] = \frac{EJ}{l^2}\left[-3,009\varphi + 3,050\frac{u}{l}\right]$$

$$\frac{EJ}{l}\left[\begin{array}{c|c} 4,552 & -2,224 \\ \hline -2,224 & 3,524 \end{array}\right]\begin{bmatrix}\varphi \\ \frac{u}{l}\end{bmatrix} = \begin{bmatrix} 0,992 \\ 0 \end{bmatrix} Pl \Rightarrow \begin{array}{l} \varphi = 0,315\frac{Pl^2}{EJ} \\ u = 0,199\frac{Pl^3}{EJ} \end{array}$$

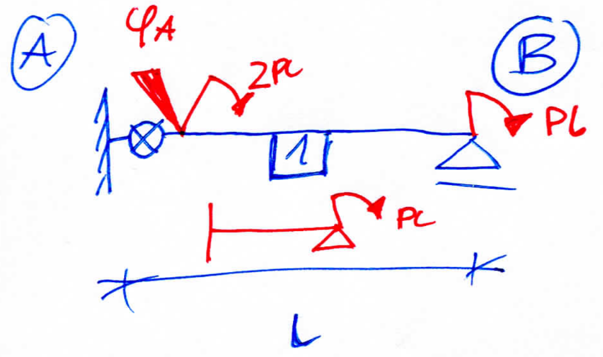


$$H_B = W_1^{(3)} = \frac{EJ}{l^2}\left[\Theta''(0,4)\varphi\right] - 0,988P = -0,972P$$

$$V_B = -W_B^{(1)} = -\frac{EJ}{l^2}\left[\delta'(0,4)\varphi - \varepsilon'(0,4)\frac{u}{l}\right] = -0,350P$$

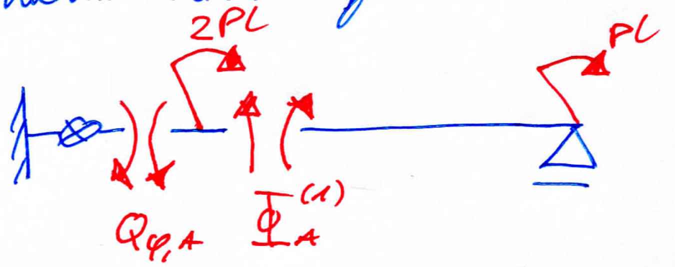
$$V_A = -W_A^{(2)} = \frac{EJ}{4l^2}\left[\delta'(0,8)\varphi + \varepsilon'(0,8)\frac{u}{2l}\right] = 0,300P$$

Egzamin MK IPB nstac. 8.09.2018 Zadanie 2



$$q = [\varphi_A]$$

Równania równowagi MP:



$$1)\ \Phi_A^{(1)} + Q_{\varphi,A} - 2PL = 0$$

Wzór transformacyjny pręta 1:

$$\Phi_A^{(1)} = \left\{ \varphi_A \ \ PL \right\} = \frac{EJ}{L}\left[\alpha'(0)\cdot\varphi_A\right] + \Phi_A^{(01)} = \frac{EJ}{L}\left[3\cdot\varphi_A\right] + \frac{PL}{2}$$

Związek konstytutywny węzła podatnego:

$$Q_{\varphi,A} = k\cdot\Delta\varphi_A = \left\{\Delta\varphi_A = \varphi_A - 0\right\} = \frac{EJ}{L}\left[\tau\cdot\varphi_A\right], \text{ gdzie } \boxed{\tau = \frac{k\cdot L}{EJ}}$$

$$1)\ \frac{EJ}{L}\left[3\cdot\varphi_A\right] + \frac{PL}{2} + \frac{EJ}{L}\left[\tau\cdot\varphi_A\right] - 2PL = 0$$

$$\frac{EJ}{L}\left[(3+\tau)\cdot\varphi_A\right] = \frac{3}{2}PL$$

$$\boxed{\varphi_A = \frac{3}{2(3+\tau)}\frac{PL^2}{EJ}}$$

$k = \frac{EJ}{L} \Rightarrow \tau = 1 \Rightarrow \varphi_A = \frac{3}{8}\frac{PL^2}{EJ}$

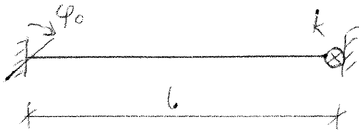
$k = 5\frac{EJ}{L} \Rightarrow \tau = 5 \Rightarrow \varphi_A = \frac{3}{16}\frac{PL^2}{EJ}$

$k = 20\frac{EJ}{L} \Rightarrow \tau = 20 \Rightarrow \varphi_A = \frac{3}{46}\frac{PL^2}{EJ}$

$k = \infty \Rightarrow$ z definicji węzeł staje się sztywny $\Rightarrow \varphi_A = 0$

Przygotował: Karol Bołbotowski

ZADANIE 3



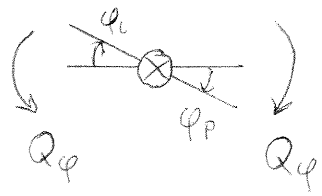
RÓWNANIE RÓŻNICZKOWE: $\dfrac{d^4 w(x)}{dx^4} = 0$

STĄD: $w(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$

$$\varphi(x) = \frac{dw(x)}{dx} = C_1 + 2C_2 x + 3C_3 x^2$$

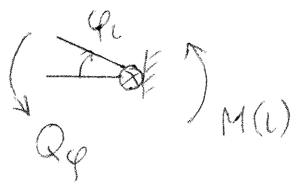
$$M(x) = -EJ\frac{d^2 w(x)}{dx^2} = (2C_2 + 6C_3 x)(-EJ)$$

WĘZEŁ PODATNY:



$\Rightarrow$

W TYM PRZYPADKU:



$\Delta\varphi = \varphi_P - \varphi_L$ $\quad\Rightarrow\quad$ $\Delta\varphi = 0 - \varphi_L$

$Q_\varphi = k\cdot\Delta\varphi$ $\quad\Rightarrow\quad$ $Q_\varphi = -k\cdot\varphi_L$

SPOSÓB 1:

WARUNKI BRZEGOWE:

(1) $w(0) = 0 \Rightarrow C_0 = 0$

(2) $\varphi(0) = \varphi_0 \Rightarrow C_1 = \varphi_0$

(3) $w(l) = 0$

(4) $M(l) = -Q_\varphi = k\cdot\varphi(l)$

$$\begin{cases} C_2 l^2 + C_3 l^3 = 0 \\ -EJ(2C_2 + 6C_3 l) = k(\varphi_0 + 2C_2 l + 3C_3 l^2) \end{cases}$$

$$(3)+(4) \Rightarrow \begin{cases} C_2 = -\dfrac{2\varphi_0(3EJ + kl)}{l(4EJ + kl)} \\ C_3 = \dfrac{\varphi_0(2EJ + kl)}{l^2(4EJ + kl)} \end{cases}$$

$$\boxed{w(x) = \varphi_0 x - \frac{2\varphi_0(3EJ + kl)}{l(4EJ + kl)}x^2 + \frac{\varphi_0(2EJ + kl)}{l^2(4EJ + kl)}x^3}$$

SPOSÓB 2:

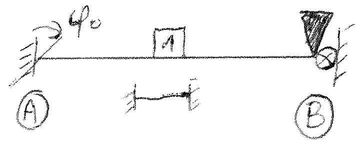
(1) $w(0) = 0 \Rightarrow C_0 = 0$

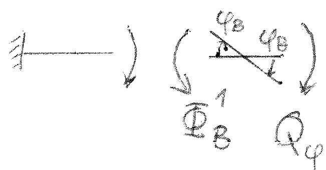
(2) $\varphi(0) = \varphi_0 \Rightarrow C_1 = \varphi_0$

(3) $w(l) = 0$

(4) $\varphi(l) = \varphi_B$

Z METODY PRZEMIESZCZEŃ





$q = [\varphi_B]$

$$\Phi_B^1 - Q_\varphi = 0$$

$$\Phi_B^1 = \frac{2EJ}{l}(2\varphi_B + \varphi_0)$$

$$Q_\varphi = -k\cdot\varphi_B$$

$$\boxed{\varphi_B = -\frac{2EJ\varphi_0}{4EJ + kl}}$$

(4) $\varphi_0 + 2C_2 l + 3C_3 l^2 = -\dfrac{2\varphi_0 EJ}{4EJ + kl}$

$$(3)+(4) \Rightarrow \begin{cases} C_2 = -\dfrac{2\varphi_0(3EJ + kl)}{l(4EJ + kl)} \\ C_3 = \dfrac{\varphi_0(2EJ + kl)}{l^2(4EJ + kl)} \end{cases}$$

$$\boxed{w(x) = \varphi_0 x - \frac{2\varphi_0(3EJ + kl)}{l(4EJ + kl)}x^2 + \frac{\varphi_0(2EJ + kl)}{l^2(4EJ + kl)}x^3}$$