

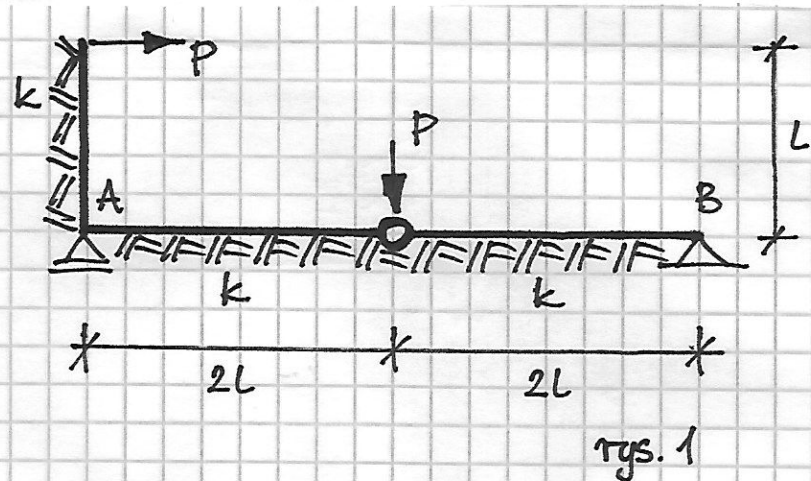
Egzamin z Mechaniki Konstrukcji (MK3 IPB), 3.09.2018
studia stacjonarne

NAZWISKO, Imię				
rok akademicki zaliczenia ćwiczeń		nr albumu	grupa (IPB / BZ)	tryb studiów (ST / NST)
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna

Zadanie 1.

$EJ = const., \quad k = 0,1024 \frac{EJ}{l^4}$

Oblicz poziome i pionowe składowe reakcji podpór A (podpora przesuwna) i B (podpora nieprzesuwna) ramy z rys. 1.

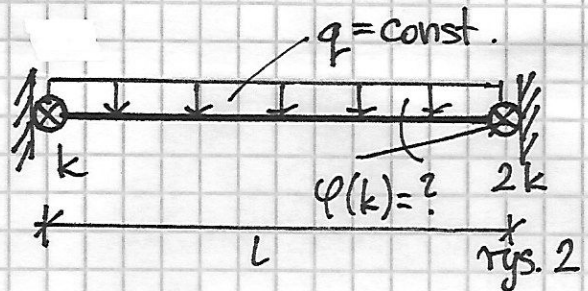


Zadanie 2.

$EJ = const.$

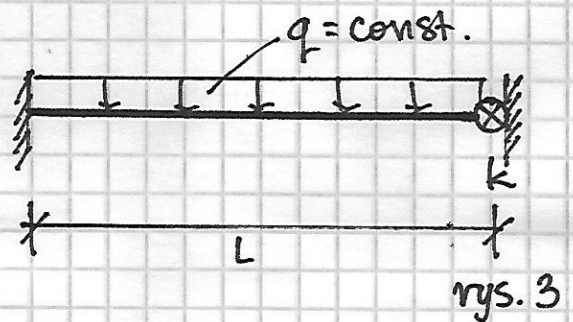
Oblicz wartość $\varphi_A(k)$ w ramie z rys. 2 dla $k = \frac{EJ}{l}$, $k = 5 \frac{EJ}{l}$, $k = 20 \frac{EJ}{l}$, $k = +\infty$

Uwaga: Podpory mają różne współczynniki sprężystości ze względu na obrót.

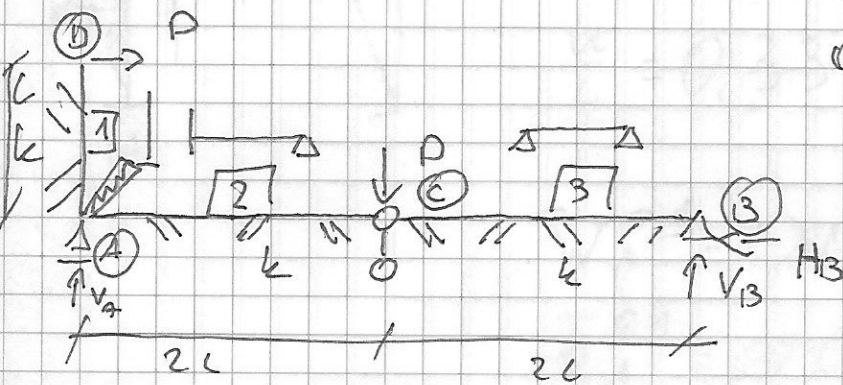


Zadanie 3.

Wyprowadź równanie linii ugięcia belki z rys. 3.

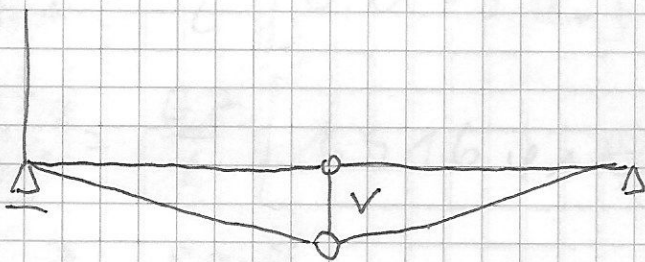


Zadanie 1



$$Q_1 = \begin{bmatrix} u_A \\ v \\ c \end{bmatrix}$$

PP



Node	w^*	w^*	u	η
1	0	0	0	0,4
2	0	v	0	0,8
3	v	0	0	0,8

RR:

$$\ominus (\bar{W}_c^2 v + \bar{W}_c^3 v) + P v = 0$$

$$\ominus \bar{\Phi}_A^1 + \ominus \bar{\Phi}_A^2 = 0$$

$$\frac{EJ}{L} \begin{bmatrix} +1,55 & \ominus 0,734 \\ \ominus 0,734 & 0,491 \end{bmatrix} \begin{bmatrix} u_A \\ v \\ c \end{bmatrix} = \begin{bmatrix} +0,99 \\ +1 \end{bmatrix} PL$$

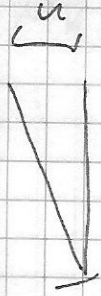
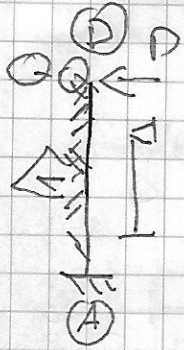
$$u_A = 5,489 \frac{PL^3}{EJ}$$

$$\frac{v}{c} = 10,242 \frac{PL^3}{EJ}$$

$$V_A = \ominus \bar{W}_A^2 = 0,757 P$$

$$V_B = \ominus \bar{W}_B^3 = 0,343 P$$

$$H_B = \ominus \bar{W}_A^1 = 0,707 P$$



$$-\left(\bar{W}_D^{01} w\right) + Pw = 0$$

$$\frac{w}{L} = 0,3331 \frac{PL^2}{EI}$$

$$\bar{W}_A^{01} = \ominus 0,987P$$

$$\bar{\Phi}_A^{01} = \ominus 0,99 PL$$

$$\bar{\Phi}_A^1 = \frac{EI}{L} \left[0,034 u_A \right] \ominus 0,99 PL$$

$$\bar{\Phi}_A^2 = \frac{EI}{L} \left[1,516 u_A \ominus 0,734 \frac{w}{L} \right]$$

$$\bar{W}_C^2 = \frac{EI}{L^2} \left[\ominus 0,734 u_A + 0,4231 \frac{w}{L} \right]$$

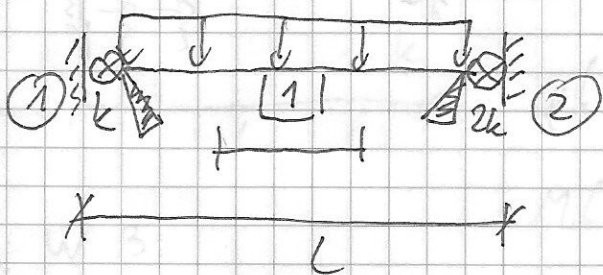
$$\bar{W}_C^3 = \frac{EI}{L^2} \left[0,0676 \frac{w}{L} \right]$$

$$\bar{W}_A^1 = \frac{EI}{L^2} \left[0,051 u_A \right] + \bar{W}_A^{01}$$

$$\bar{W}_A^2 = \frac{EI}{L^2} \left[0,785 u_A \ominus 0,347 \frac{w}{L} \right]$$

$$\bar{W}_B^3 = \frac{EI}{L^2} \left[0,0335 \frac{w}{L} \right]$$

Zadanie 2



$$q_1 = \begin{cases} q_1 \\ q_2 \end{cases}$$

$$Q_1 = q_1 k$$

$$Q_2 = -q_2 \cdot 2k$$

RR:

$$\Phi_1' + Q_1 = 0$$

$$\Phi_1' = \frac{EJ}{L} [4\varphi_1 + 2\varphi_2] - \frac{1}{12} qL^2$$

$$\Phi_2' - Q_2 = 0$$

$$\Phi_2' = \frac{EJ}{c} [2\varphi_1 + 4\varphi_2] + \frac{1}{12} qL^2$$

$$\frac{EJ}{L} \begin{bmatrix} 4+k & 2 \\ 2 & 4+2k \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{12} \\ -\frac{1}{12} \end{bmatrix} qL^2$$

$$\varphi_A(k) = \varphi_2 = - \frac{6+k}{24(6+6k+k^2)} \frac{qL^3}{EJ}$$

$$\varphi_A\left(\frac{EJ}{c}\right) = - \frac{7}{312} \frac{qL^3}{EJ}$$

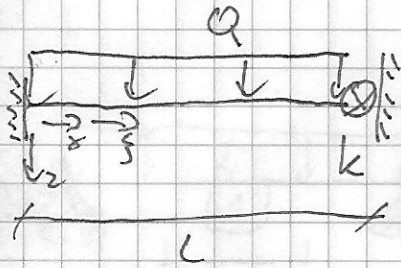
$$\varphi_A\left(5\frac{EJ}{c}\right) = - \frac{11}{1464} \frac{qL^3}{EJ}$$

$$\varphi_A\left(20\frac{EJ}{c}\right) = - \frac{13}{6312} \frac{qL^3}{EJ}$$

$$\varphi_A(\infty) = 0$$

Zadanie 3

$$\xi = \frac{x}{L}$$



$$w(\xi) = C_0 + C_1 \xi + C_2 \xi^2 + C_3 \xi^3 + \frac{qL^4}{24EI} \xi^4$$

$$\varphi(\xi) = \frac{1}{L} \frac{dw(\xi)}{d\xi} = \frac{1}{L} \left(C_1 + 2C_2 \xi + 3C_3 \xi^2 + \frac{qL^4}{6EI} \xi^3 \right)$$

$$M(\xi) = \ominus \frac{EI}{L^2} \frac{d^2w(\xi)}{d\xi^2} = \ominus \frac{EI}{L^2} \left(2C_2 + 6C_3 \xi + \frac{qL^4}{2EI} \xi^2 \right)$$

WB:

$$w(0) = 0$$

$$C_0 = 0$$

$$\varphi(0) = 0$$

$$C_1 = 0$$

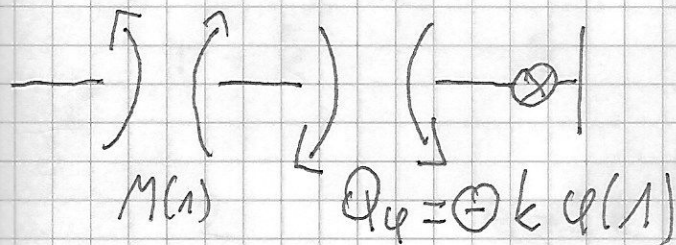
$$w(1) = 0$$

\Rightarrow

$$C_2 = \frac{qL^4}{24EI} \left(\frac{6EI + kL}{4EI + kL} \right)$$

$$M(1) = \varphi(1)k$$

$$C_3 = \ominus \frac{qL^4}{12EI} \left(\frac{5EI + kL}{4EI + kL} \right)$$



$$w(\xi) = \frac{qL^4}{24EI} \left(\frac{6EI + kL}{4EI + kL} \xi^2 \ominus 2 \frac{5EI + kL}{4EI + kL} \xi^3 + \xi^4 \right)$$