Egzamin z Mechaniki Konstrukcji (MK3 IPB), 20.06.2018
studia stacjonarne

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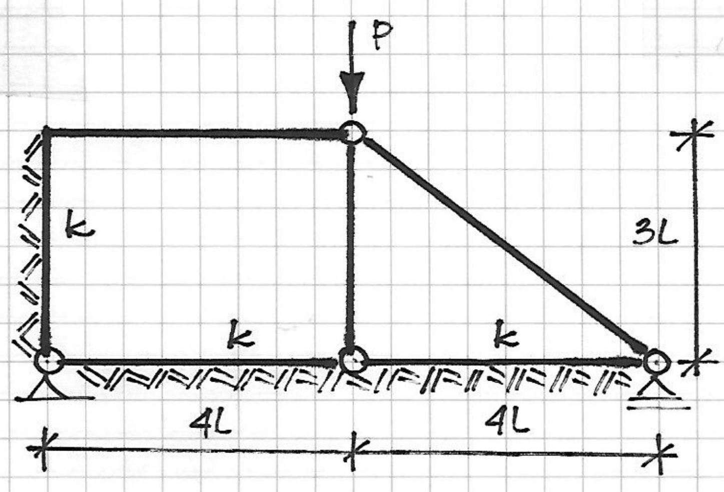
rok akademicki zaliczenia ćwiczeń		nr albumu	grupa (IPB / BZ)	tryb studiów (ST / NST)	
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna	

Zadanie 1.

$EJ = const.,\qquad k = 0{,}0064\dfrac{EJ}{l^4}$

Oblicz siły podłużne w prętach pionowych ramy z rys. 1.



rys. 1.

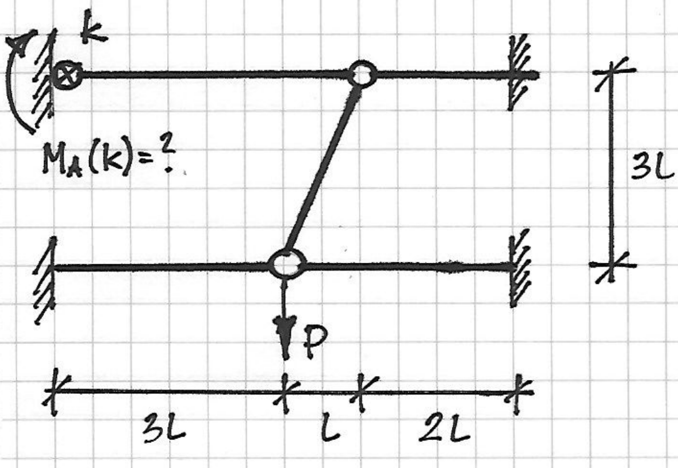
Zadanie 2.

$EJ = const.$

Oblicz wartość $M_A(k)$ w ramie z rys. 2 dla $k = \frac{EJ}{l}$, $k = 3\frac{EJ}{l}$, $k = 10\frac{EJ}{l}$, $k = +\infty$

Zadanie 3.

Wyprowadź wzór transformacyjny określający moment przywęzłowy w pręcie podpartym według schematu IIa (patrz na odwrocie) metodą kondensacji statycznej ugięcia prawego węzła w pręcie podpartym według schematu Ia.

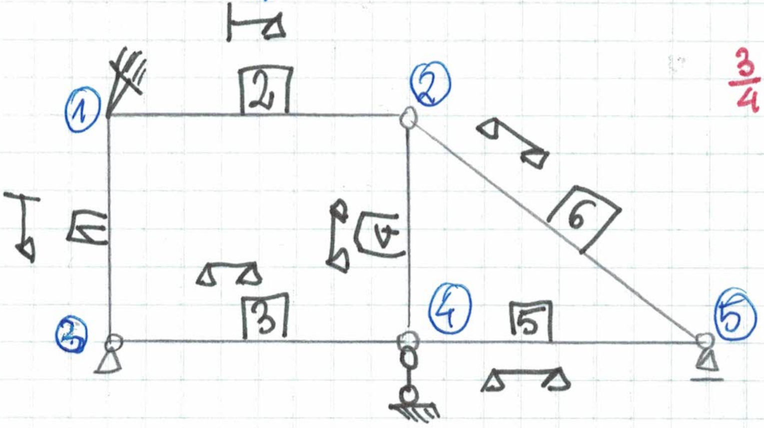


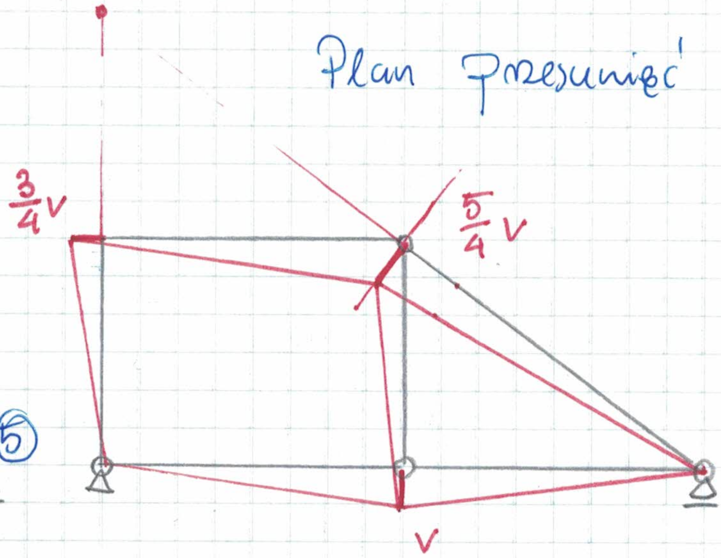
rys. 2.

ZADANIE 1 NST II IPB 20 VI 2018

UGW $q = [\varphi \ \frac{v}{l}]^T$

Plan przesunięć





Pręt	$l^{(k)}$	$\lambda^{(k)}$	$^*w$	w^*
1	$3l$	0,6	0	$-\frac{3}{4}v$
2	$4l$	0	0	v
3	$4l$	0,8	0	v
4	$3l$	0	0	$-\frac{3}{4}v$
5	$4l$	0,8	v	0
6	$5l$	0	$\frac{5}{4}v$	0

r.r. MP

$$\Phi_1^{(1)} + \Phi_1^{(2)} = 0 \qquad (1)$$

$$-\left(W_4^{(3)}\bar{v} + W_4^{(5)}\bar{v} + W_1^{(1)}\left(-\frac{3}{4}\bar{v}\right) + W_2^{(2)}\bar{v}\right) + P\bar{v} = 0 \qquad (2)$$

Wzory transformacyjne

$$\Phi_1^{(1)} = \frac{EJ}{3l}\left[\alpha'(0{,}6)\varphi - \theta'(0{,}6)\frac{\left(-\frac{3}{4}v\right)}{3l}\right] = \frac{EJ}{l}\left[1{,}003\varphi + 0{,}254\frac{v}{l}\right]$$

$$W_1^{(1)} = -\frac{EJ}{(3l)^2}\left[\theta'(0{,}6)\varphi - \gamma'(0{,}6)\left(-\frac{3}{4}v\right)\cdot\frac{1}{3l}\right] = \frac{EJ}{l^2}\left[-0{,}338\varphi - 0{,}090\frac{v}{l}\right]$$

$$\Phi_1^{(2)} = \frac{EJ}{4l}\left[\alpha'(0)\varphi - \delta'(0)\frac{v}{4l}\right] = \frac{EJ}{l}\left[0{,}75\varphi - 0{,}188\frac{v}{l}\right]$$

$$W_2^{(2)} = -\frac{EJ}{(4l)^2}\left[\delta'(0)\varphi - \lambda'(0)\frac{v}{4l}\right] = \frac{EJ}{l^2}\left[-0{,}188\varphi + 0{,}047\frac{v}{l}\right]$$

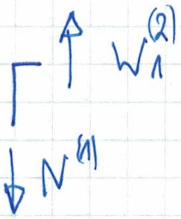
$$W_4^{(3)} = -\frac{EJ}{(4l)^2}\left[-\gamma'''(0{,}8)\frac{v}{4l}\right] = \frac{EJ}{l^2}\left[0{,}008\frac{v}{l}\right]$$

$$W_4^{(5)} = \frac{EJ}{(4l)^2}\left[\gamma'''(0{,}8)\frac{v}{4l}\right] = \frac{EJ}{l^2}\left[0{,}008\frac{v}{l}\right]$$

$$\frac{EJ}{l}\begin{bmatrix} 1,753 & 0,066 \\ 0,066 & 0,131 \end{bmatrix}\begin{bmatrix} \varphi \\ \frac{v}{l} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} Pl$$

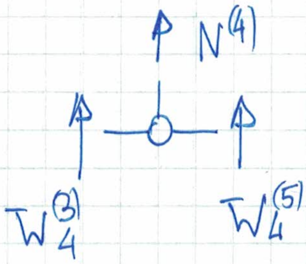
$$\varphi = -0,293\,\frac{Pl^2}{EJ} \qquad v = 7,781\,\frac{Pl^3}{EJ}$$

Węzeł ①



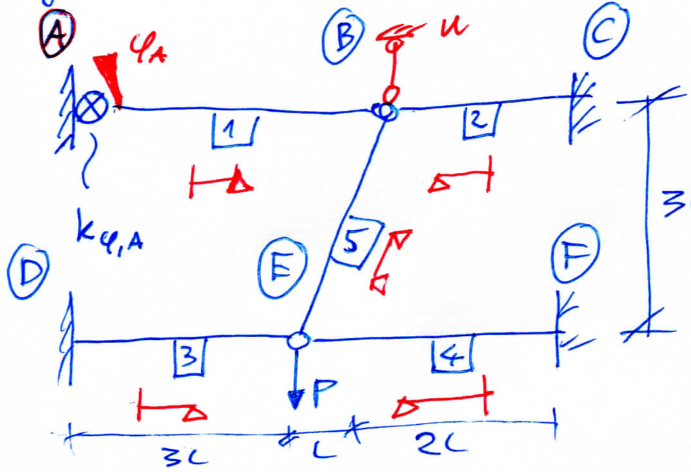
$$N^{(1)} = W_1^{(2)} = -0,420\,P$$

Węzeł ④



$$N^{(4)} = -\left(W_4^{(3)} + W_4^{(5)}\right) = -0,125\,P$$

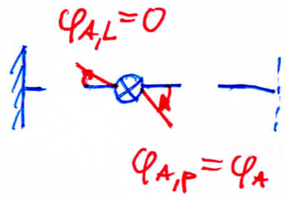
Egzamin MK IPB stac. 20.06.18r, Zadanie 2



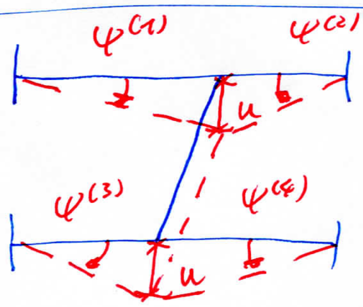
$$k_{\varphi,A} = \tau \frac{EJ}{l}$$

$$q = \begin{bmatrix} \varphi_A \\ \frac{u}{l} \end{bmatrix}$$

Kinematyka:



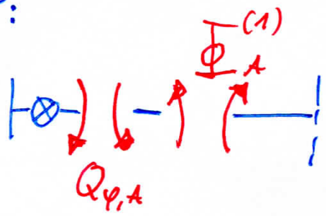
$$\Delta\varphi_A = \varphi_{A,P} - \varphi_{A,L} = \varphi_A - 0 = \varphi_A$$



$$\varphi^{(1)} = \frac{u}{4l}$$
$$\varphi^{(2)} = -\frac{u}{2l}$$
$$\varphi^{(3)} = \frac{u}{3l}$$
$$\varphi^{(4)} = -\frac{u}{3l}$$
$$\varphi^{(5)} = 0$$

RRMP:

1)



$$\Phi_A^{(1)} + Q_{\varphi,A} = 0$$

2) RPW:

$$\Phi_A^{(1)}\cdot\left(\frac{\bar{u}}{4l}\right) + \Phi_C^{(2)}\cdot\left(-\frac{\bar{u}}{2l}\right) + \Phi_D^{(3)}\cdot\left(\frac{\bar{u}}{3l}\right) + \Phi_F^{(4)}\cdot\left(-\frac{\bar{u}}{3l}\right) + P\cdot\bar{u} = 0 \;\Big|\cdot(-1)\;;\; \bar{u} = l$$

Wzory transformacyjne:

$$\Phi_A^{(1)} = \frac{EJ}{(4l)}\left[3\varphi_A - 3\left(\frac{u}{4l}\right)\right];\quad \Phi_C^{(2)} = \frac{EJ}{(2l)}\left[-3\left(-\frac{u}{2l}\right)\right];$$

$$\Phi_D^{(3)} = \frac{EJ}{(3l)}\left[-3\left(\frac{u}{3l}\right)\right];\quad \Phi_F^{(4)} = \frac{EJ}{(3l)}\left[-3\left(-\frac{u}{3l}\right)\right]$$

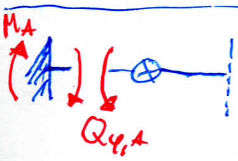
Związek konstytutywny węzła podatnego:

$$Q_{\varphi,A} = k_{\varphi,A}\cdot\Delta\varphi_A$$

$$Q_{\varphi,A} = \frac{EJ}{l}\left[\tau\cdot\varphi_A\right]$$

$$\Rightarrow \frac{EJ}{l}\begin{bmatrix} \frac{3}{4}+\tau & -\frac{3}{16} \\ -\frac{3}{16} & \frac{341}{576} \end{bmatrix}\begin{bmatrix} \varphi_A \\ \frac{u}{l} \end{bmatrix} = \begin{bmatrix} 0 \\ Pl \end{bmatrix} \Rightarrow \varphi_A = \varphi_A(\tau) = \frac{108}{258+341\tau}\frac{Pl^2}{EJ}$$

$$u = u(\tau) = \ldots$$



$$\Rightarrow \boxed{M_A = -Q_{\varphi,A} = -\frac{EJ}{l}[\tau\cdot\varphi_A] = -\frac{108\tau}{258+341\tau}Pl =: M_A(\tau)}$$

$$M_A(\infty) = \Phi_A^{(1)} = -0.291\,Pl$$

$$\Uparrow$$

$$-0.291\,Pl$$

$$M_A(1) = -0.172\,Pl$$
$$M_A(3) = -0.236\,Pl$$
$$M_A(10) = -0.242\,Pl$$

$$\boxed{M_A(\infty) = \lim_{\tau\to\infty} M_A(\tau) = -\frac{108}{341}Pl = -0.291\,Pl}$$

Albo: $k_{\varphi,A} = \infty \xRightarrow{\text{z definicji}} \Delta\varphi_A = 0 \Rightarrow \varphi_A = 0 \Rightarrow$

$$\Rightarrow -\frac{3}{16}\cdot 0 + \frac{341}{576}\frac{u}{l} = Pl \Rightarrow u = \frac{576}{341}\frac{Pl^3}{EJ} \Rightarrow \Phi_A^{(1)} = \frac{EJ}{(4l)}\left[0 - 3\left(\frac{u}{4l}\right)\right] =$$

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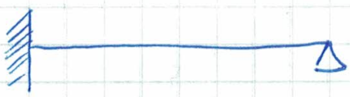
Schemat I a



Dobieramy w^* tak, aby $W^* = 0$

$$W^* = -\frac{EJ}{\ell^2}\left[\delta'(\nu)\,{}^*\varphi + \varepsilon'(\nu)\frac{{}^*w}{\ell} - \chi'(\nu)\frac{w^*}{\ell}\right] = 0$$

$\Rightarrow$

$$w^* = \frac{\ell}{\chi'(\nu)}\left[\delta'(\nu)\,{}^*\varphi + \varepsilon'(\nu)\frac{{}^*w}{\ell}\right]$$

Podstawiamy do wzoru na ${}^*\Phi$

$${}^*\Phi = \frac{EJ}{\ell}\left[\alpha'(\nu)\,{}^*\varphi + \Theta'(\nu)\frac{{}^*w}{\ell} - \frac{\delta'(\nu)}{\chi'(\nu)}\left(\delta'(\nu)\,{}^*\varphi + \varepsilon'(\nu)\frac{{}^*w}{\ell}\right)\right] =$$

$$= \frac{EJ}{\ell}\left[\left(\alpha'(\nu) - \frac{(\delta'(\nu))^2}{\chi'(\nu)}\right){}^*\varphi + \left(\Theta'(\nu) - \frac{\delta'(\nu)\,\varepsilon'(\nu)}{\chi'(\nu)}\right)\frac{{}^*w}{\ell}\right] =$$

$$= \frac{EJ}{\ell}\left[\alpha''(\nu)\,{}^*\varphi + \Theta''(\nu)\frac{{}^*w}{\ell}\right]$$

Otrzymujemy wzór na $\overset{*}{\Phi}$ pręta podpartego wg schematu II a.