

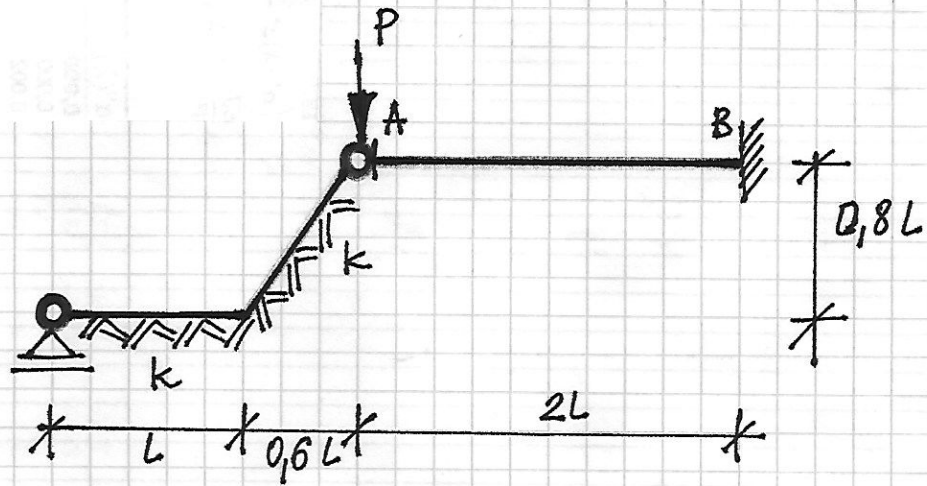
Egzamin z Mechaniki Konstrukcji (MK IPB), 11.09.2017
studia stacjonarne

NAZWISKO, Imię				
rok akademicki zaliczenia ćwiczeń		nr albumu	grupa (IPB / BZ)	tryb studiów (ST / NST)
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna

Zadanie 1.

$$EJ = const., \quad k = 0,0064 \frac{EJ}{l^4}$$

Oblicz kąt obrotu w punkcie A ramy z rys. 1 korzystając z funkcji ugięcia pręta AB .



rys. 1

Zadanie 2.

$$EJ = const., \quad k = 0,0064 \frac{EJ}{l^4}$$

Oblicz moment w utwierdzeniu A w ramie z rys. 2 korzystając z tw. Bettiego.



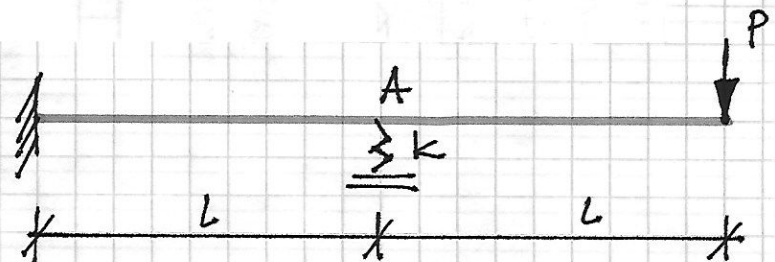
rys. 2

Zadanie 3.

$$EJ = const.$$

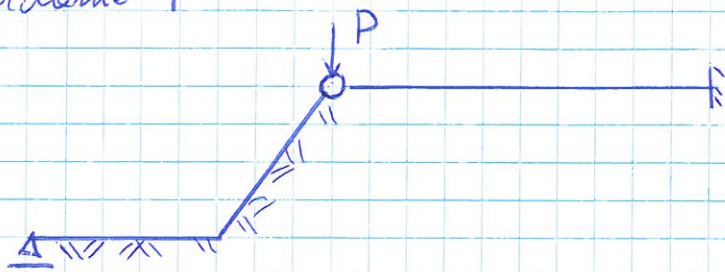
Wyznacz wartość współczynnika k , dla którego kąt obrotu w punkcie A belki z rys. 3

$$\text{jest równy } \varphi_A = \frac{Pl^2}{2EJ}.$$



rys. 3

Zadanie 1

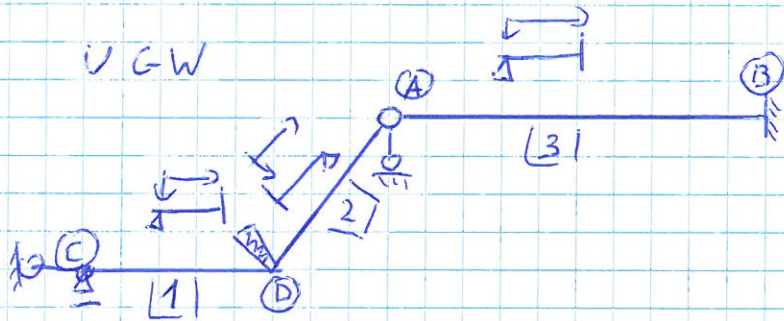


$$\lambda_1 = 0,2$$

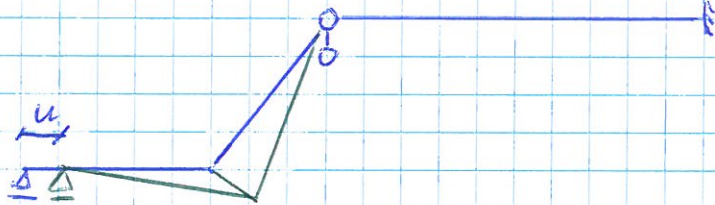
$$\lambda_2 = 0,2$$

$$\lambda_3 = 0$$

$$q_1 = \begin{bmatrix} 40 \\ u \\ c \\ v \\ c \end{bmatrix}$$

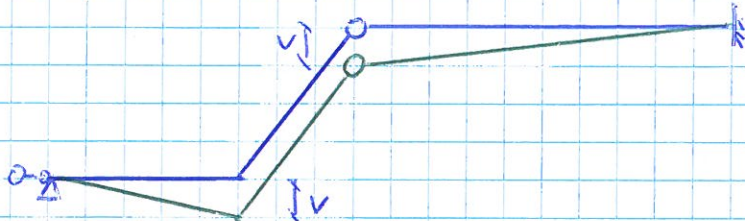


PP u:



Przet	w^*	w^*	u
1	0	$\frac{3}{4}u$	u
2	$\frac{5}{4}u$	0	0
3	0	0	0

PP v:



Przet	w^*	w^*	u
1	0	v	0
2	$\frac{3}{5}v$	$\frac{3}{5}v$	$-\frac{4}{5}v$
3	v	0	0

RR:

$$\bar{\Phi}_D^1 + \bar{\Phi}_D^2 = 0$$

$$-(\bar{W}_D^1 \cdot \frac{3}{4}u + \bar{W}_D^2 \cdot \frac{5}{4}u) = 0$$

$$-(\bar{W}_D^1 v + (\bar{W}_D^2 + \bar{W}_A^2) \frac{3}{5}v + \bar{W}_A^3 v) + P v = 0$$

$$\bar{\Phi}_D^1 = \frac{EJ}{c} (\alpha'(0,2) \varphi_D - \psi'(0,2) (\frac{3}{4}u + \frac{v}{c})) = \frac{EJ}{c} (3 \varphi_D - 2,250 \frac{u}{c} - 3,001 \frac{v}{c})$$

$$\bar{\Phi}_D^2 = \frac{EJ}{c} (\alpha'(0,2) \varphi_D + \psi'(0,2) (\frac{5}{4} \frac{u}{c} + \frac{3}{5} \frac{v}{c})) - \delta'(0,2) (\frac{3}{5} \frac{v}{c}) =$$

$$= \frac{EJ}{c} (3 \varphi_D + 3,751 \frac{u}{c})$$

$$\bar{W}_D^1 = -\frac{E\alpha}{l^2} (v'(0,2) \varphi_0 - \gamma'(0,2) (\frac{3}{4} \frac{u}{l} + \frac{v}{l})) = \frac{E\alpha}{l^2} (-3,001 \varphi_0 + 2,252 \frac{u}{l} +$$

$$\bar{W}_D^2 = \frac{E\alpha}{l^2} (v'(0,2) \varphi_0 + \gamma'(0,2) (\frac{5}{4} \frac{u}{l} + \frac{3}{5} \frac{v}{l}) - \varepsilon'(0,2) \frac{3}{5} \frac{v}{l}) =$$

$$= \frac{E\alpha}{l^2} (3,001 \varphi_0 + 3,754 \frac{u}{l} + 0,02 \frac{v}{l})$$

$$\bar{W}_A^2 = -\frac{E\alpha}{l^2} (\delta'(0,2) \varphi_0 + \varepsilon'(0,2) (\frac{5}{4} \frac{u}{l} + \frac{3}{5} \frac{v}{l}) - \chi'(0,2) \frac{3}{5} \frac{v}{l}) =$$

$$\frac{E\alpha}{l^2} (-3 \varphi_0 - 3,749 \frac{u}{l} + 0,001 \frac{v}{l})$$

$$\bar{W}_A^3 = \frac{3E\alpha}{4l^2} (-\frac{v}{2l}) = \frac{E\alpha}{l^2} (+\frac{3}{8} \frac{v}{l})$$

$$\frac{E\alpha}{l} \begin{bmatrix} 6 & 1,5 & -3 \\ 1,5 & 6,382 & 2,255 \\ -3 & 2,255 & 3,38 \end{bmatrix} \begin{bmatrix} \varphi_0 \\ \frac{u}{l} \\ \frac{v}{l} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ PL} \Rightarrow \begin{matrix} \varphi_0 = 1,66 \frac{\text{Pl}^2}{E\alpha} \\ \frac{u}{l} = -1,329 \frac{\text{Pl}^2}{E\alpha} \\ \frac{v}{l} = 2,656 \frac{\text{Pl}^2}{E\alpha} \end{matrix}$$

$$w(\xi) = A_0 + A_1 \xi + A_2 \xi^2 + A_3 \xi^3$$

$$\varphi(\xi) = \frac{1}{2l} w'(\xi)$$

$$M(\xi) = -\frac{E\alpha}{(2l)^2} w''(\xi)$$

W.B.:

$$w(0) = v$$

$$w(1) = 0$$

$$M(0) = 0$$

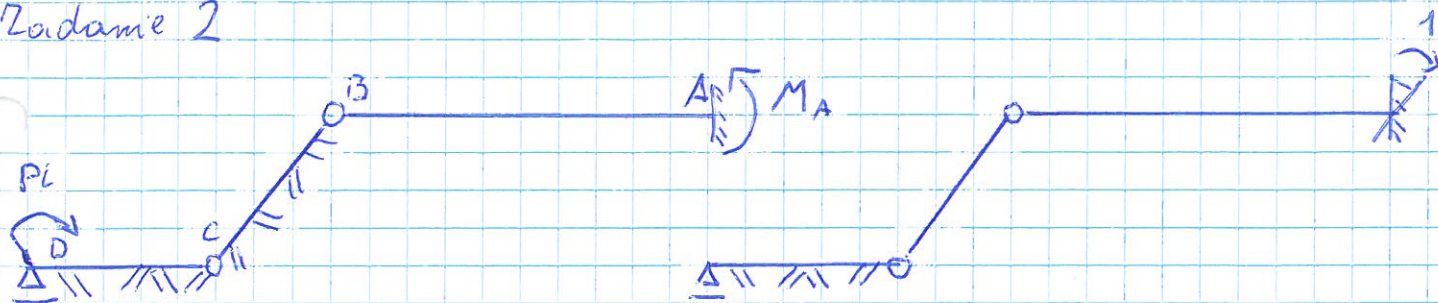
$$\varphi(1) = 0$$

$$w(\xi) = (2,656 + 3,984\xi + 1,328\xi^3) \frac{\text{Pl}^3}{E\alpha}$$

$$\varphi(\xi) = 1,992(\xi^2 - 1) \frac{\text{Pl}^2}{E\alpha}$$

$$\varphi(0) = -1,992 \frac{\text{Pl}^2}{E\alpha}$$

Zadanie 2



Z tw Bettiego:

$$-M_A \cdot 1 + PL \varphi_D = 0 \Rightarrow M_A = PL \varphi_D$$

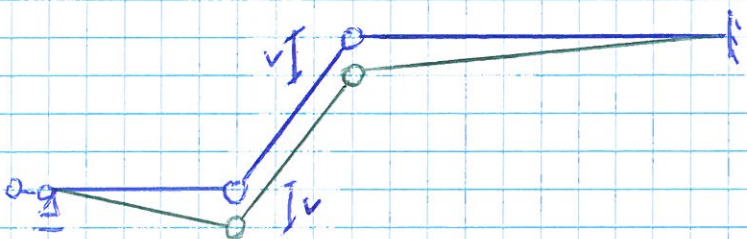
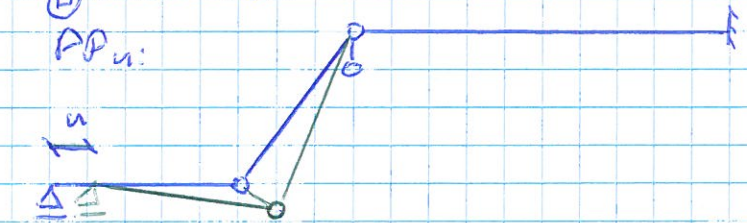
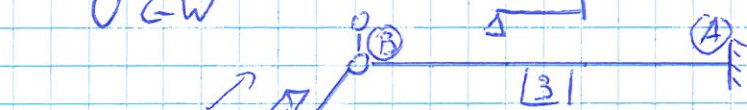
UGW

$$\lambda_1 = 0,2$$

$$\lambda_2 = 0,2$$

$$\lambda_3 = 0$$

$$q_1 = \begin{bmatrix} u \\ v \\ u \end{bmatrix}$$



Przet	w	w^*	u
1	0	$\frac{3}{4}u$	u
2	$\frac{5}{4}u$	0	0
3	0	0	0

Przet	w	w^*	u
1	0	v	0
2	$\frac{3}{5}v$	$\frac{2}{5}v$	$-\frac{4}{5}v$
3	v	0	0

RR:

$$\int -(\bar{W}_C^{-1} \cdot \frac{3}{4}u + \bar{W}_C^{-2} \cdot \frac{5}{4}u) = 0$$

$$\int -(\bar{W}_C^{-1} v + (\bar{W}_C^{-2} + \bar{W}_B^{-2}) \frac{3}{5}v + \bar{W}_B^{-3} v) = 0$$

$$\bar{W}_C^{-1} = \frac{EJ}{l^2} (0,002 \frac{u}{l} + 0,002 \frac{v}{l}) \quad \bar{W}_C^{-2} = \frac{EJ}{l^2} (0,003 \frac{u}{l} + 0,002 \frac{v}{l})$$

$$\bar{W}_B^{-2} = \frac{EJ}{l^2} (0,001 \frac{u}{l} + 0,002 \frac{v}{l}) \quad \bar{W}_B^{-3} = \frac{EJ}{l^2} (\frac{3}{4} + \frac{3}{8} \frac{v}{l})$$

$$\frac{EJ}{l} \begin{bmatrix} 0,005 & 0,004 \\ 0,004 & 0,379 \end{bmatrix} \begin{bmatrix} \frac{u}{l} \\ \frac{v}{l} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{3}{4} \end{bmatrix} \frac{EJ}{l}$$

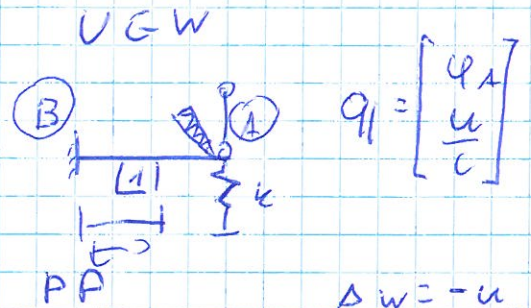
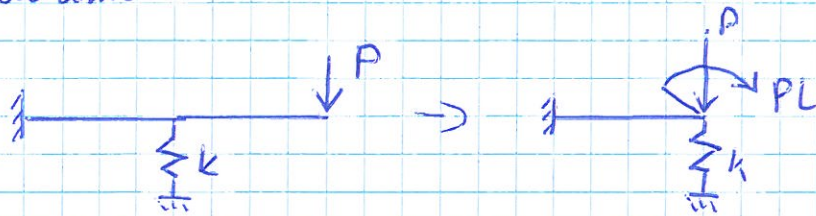
$$\frac{u}{l} = -1,760$$

$$\frac{v}{l} = 1,995$$

Poniemaj $\Phi_D^1 = 0, \varphi_D = \frac{EJ}{l} (\lambda_1(0,2) \varphi_D - \delta_1(0,2) (\frac{3}{4}u + v)) = 0 \Rightarrow \varphi_D = \frac{3}{4}u + v$

$$\varphi_D = 0,675 \Rightarrow M_A = 0,675 q PL$$

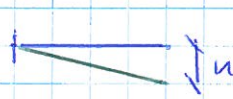
Zadanie 3



$$q_1 = \begin{bmatrix} \varphi_A \\ \frac{u}{l} \end{bmatrix}$$

$$\Delta w = -u$$

$$Q_w = -ku$$



RR:

$$\begin{cases} -\bar{\Phi}_B^1 + Pl = 0 \\ -\bar{W}_A^1 \bar{u} + Q_w \bar{u} + P\bar{u} = 0 \end{cases} \Rightarrow \begin{cases} \bar{\Phi}_B^1 = Pl \\ \bar{W}_A^1 + k\bar{u} = P \end{cases}$$

$$\bar{\Phi}_B^1 = \frac{EJ}{l} \left(4\varphi_B - 6\frac{u}{l} \right) \quad k = \tilde{l} \frac{EJ}{l^3}$$

$$\bar{W}_B^1 = \frac{EJ}{l^2} \left(-6\varphi_B + 12\frac{u}{l} \right)$$

$$\frac{EJ}{l} \begin{bmatrix} 4 & -6 \\ -6 & 12 + \tilde{l} \end{bmatrix} \begin{bmatrix} \varphi_B \\ \frac{u}{l} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} Pl$$

$$\varphi_B = \frac{1}{4} \frac{Pl^2}{EJ} + \frac{3}{2} \frac{u}{l}$$

$$-6\varphi_B + (12 + \tilde{l}) \frac{u}{l} = \frac{Pl^2}{EJ} \Rightarrow -\frac{3}{2} \frac{Pl^2}{EJ} - \frac{9}{2} \frac{u}{l} + (12 + \tilde{l}) \frac{u}{l} = \frac{Pl^2}{EJ}$$

$$(3 + \tilde{l}) \frac{u}{l} = \frac{5}{2} \frac{Pl^2}{EJ}$$

$$\varphi_B = \frac{Pl^2}{EJ} \Rightarrow \frac{u}{l} = \frac{Pl^2}{6EJ}$$

$$(3 + \tilde{l}) \frac{Pl^2}{6EJ} = \frac{5}{2} \frac{Pl^2}{EJ}$$

$$\tilde{l} = 12 \Rightarrow k = 12 \frac{EJ}{l^3}$$