

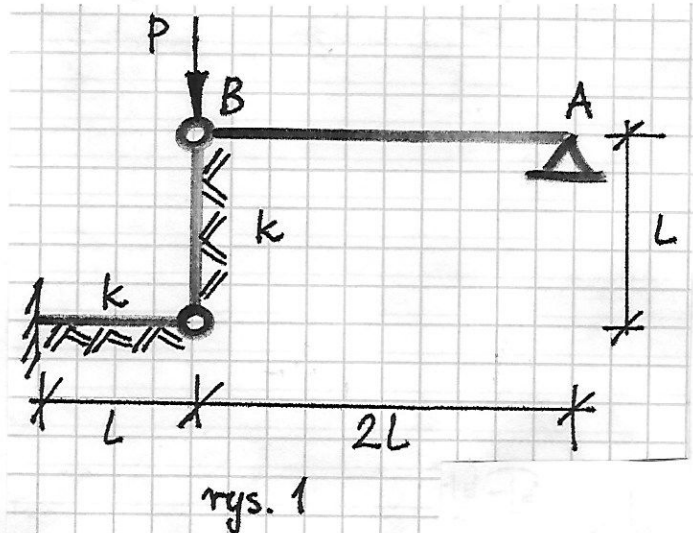
Egzamin z Mechaniki Konstrukcji (MK IPB), 8.09.2017
studia niestacjonarne

| | | | | |
|-----------------------------------|-----------------|-----------------|------------------|-------------------------|
| NAZWISKO, Imię | | | | |
| rok akademicki zaliczenia ćwiczeń | | nr albumu | grupa (IPB / BZ) | tryb studiów (ST / NST) |
| ocena zadania 1 | ocena zadania 2 | ocena zadania 3 | ocena egzaminu | ocena łączna |

Zadanie 1.

$$EJ = const., \quad k = 0,0064 \frac{EJ}{l^4}$$

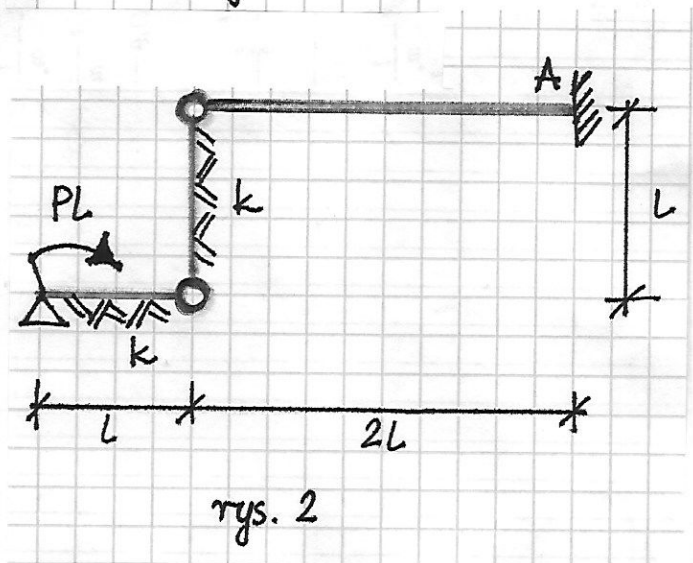
Oblicz kąt obrotu w punkcie *A* ramy z rys. 1 korzystając z funkcji ugięcia pręta *AB*.



Zadanie 2.

$$EJ = const., \quad k = 0,0064 \frac{EJ}{l^4}$$

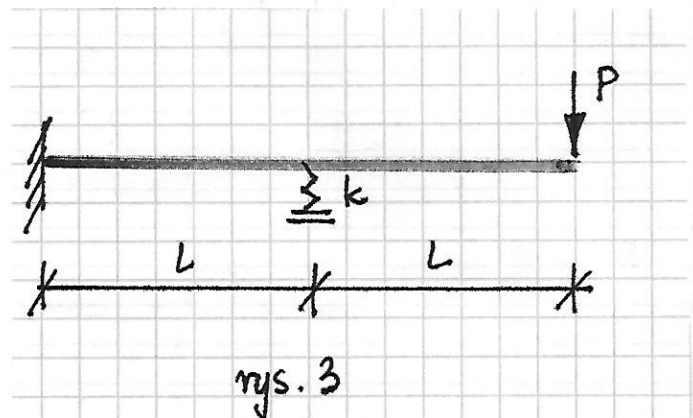
Oblicz moment w utwierdzeniu *A* w ramie z rys. 2 korzystając z tw. Bettiego.



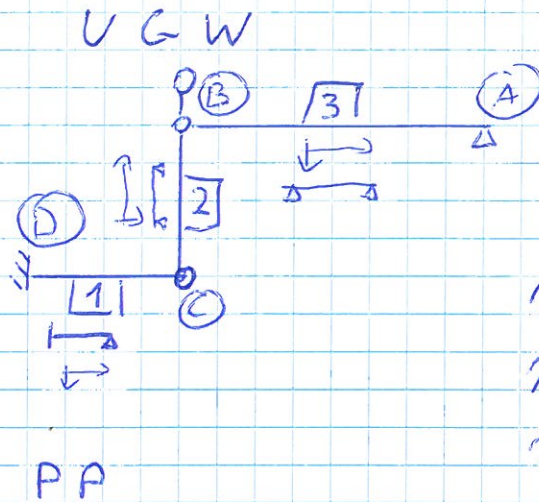
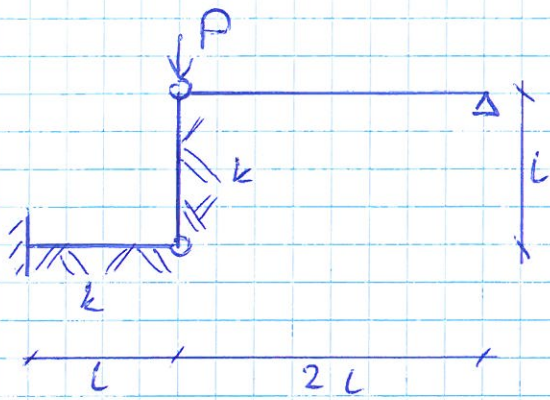
Zadanie 3.

$$EJ = const.$$

Wyznacz wartość współczynnika *k*, dla którego moment w utwierdzeniu belki z rys. 3 jest równy zero.



Zadanie 1



$$q_1 = \left\{ \frac{u}{L} \right\}$$

$$\lambda_1 = 0,2$$

$$\lambda_2 = 0,2$$

$$\lambda_3 = 0$$

RR:

$$-(\bar{W}_B^3 \bar{u} + \bar{W}_C^1 \bar{u}) + P\bar{u} = 0$$

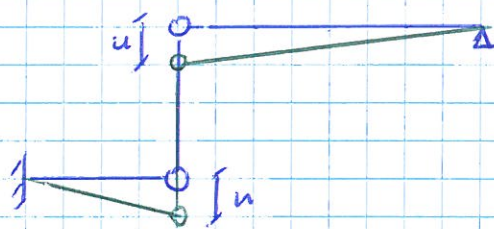
$$\bar{W}_B^3 + \bar{W}_C^1 = P$$

$$\bar{W}_B^3 = 0$$

$$\bar{W}_C^1 = -\frac{EJ}{L^2} \left(-\chi'(0,2) \frac{u}{L} \right) = \frac{EJ}{L^2} \left(3,002 \frac{u}{L} \right)$$

$$\frac{EJ}{L^2} \left(3,002 \frac{u}{L} \right) = P$$

$$\frac{u}{L} = \frac{PL^2}{EJ} \cdot \frac{1}{3,002}$$



| P_{ret} | w^* | w^* | u |
|-----------|-------|-------|------|
| 1 | 0 | u | 0 |
| 2 | 0 | 0 | $-u$ |
| 3 | u | 0 | 0 |

WB:

$$w(\xi) = A_0 + A_1 \xi + A_2 \xi^2 + A_3 \xi^3$$

$$\varphi(\xi) = \frac{1}{2L} w'(\xi)$$

$$M(\xi) = -\frac{EJ}{(2L)^2} w''(\xi)$$

$$w(0) = u \quad M(0) = 0$$

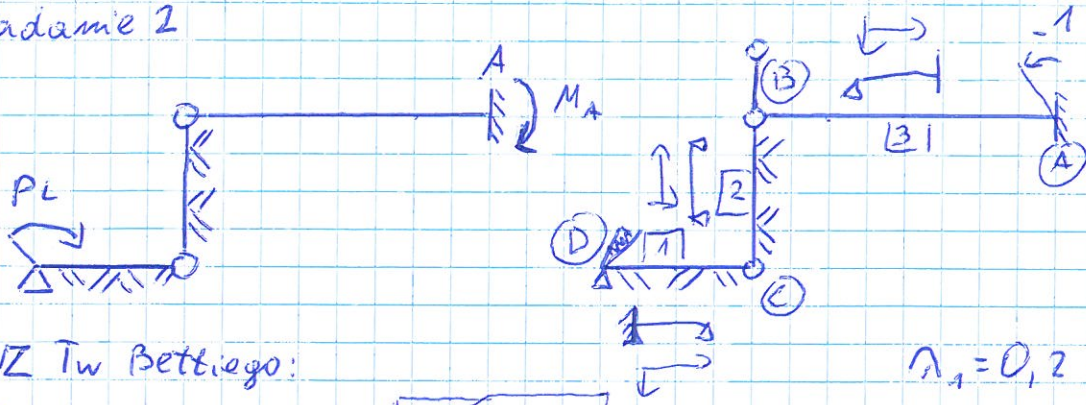
$$w(1) = 0 \quad M(1) = 0$$

$$w(\xi) = u - u \xi$$

$$\varphi(\xi) = -\frac{u}{2L}$$

$$\varphi_B = \varphi(0) = -\frac{u}{2L} = -\frac{PL^2}{EJ} \frac{1}{6,004}$$

Zadanie 2



$$q_1 = \begin{bmatrix} \varphi_D \\ \frac{u}{L} \end{bmatrix}$$

Z Tw Bettiego:

$$(-1) M_A + \varphi_D \cdot PL \Rightarrow M_A = PL \varphi_D$$

$$\lambda_1 = 0,2$$

$$\lambda_2 = 0,2$$

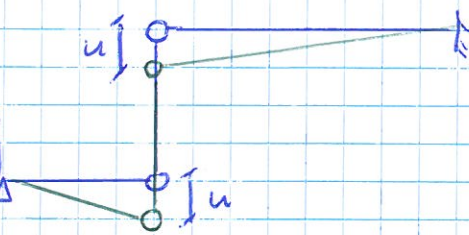
$$\lambda_3 = 0$$

RR:

$$\Phi_D^1 = 0$$

$$-W_c^1 \bar{u} - W_B^3 \bar{u} = 0$$

PP



$$\Phi_D^1 = \frac{E\alpha}{L} \left(\delta'(0,2) \varphi_D - \gamma'(0,2) \frac{u}{L} \right)$$

$$= \frac{E\alpha}{L} \left(3 \varphi_D - 3 \frac{u}{L} \right)$$

$$W_c^1 = - \frac{E\alpha}{L^2} \left(\delta'(0,2) \varphi_D - \gamma'(0,2) \frac{u}{L} \right) =$$

| P_{int} | w | w^* | u |
|-----------|-----|-------|------|
| 1 | 0 | u | 0 |
| 2 | 0 | 0 | $-u$ |

$$= + \frac{E\alpha}{L^2} \left(-3 \varphi_D + 3,002 \frac{u}{L} \right)$$

$$W_B^3 = \frac{E\alpha}{(2L)^2} \left(\delta'(0) (-1) + \gamma'(0) \frac{u}{2L} \right) =$$

| P_{int} | w | w^* | u |
|-----------|-----|-------|-----|
| 3 | u | 0 | 0 |

$$= \frac{E\alpha}{L^2} \left(-\frac{3}{4} + \frac{3u}{8L} \right)$$

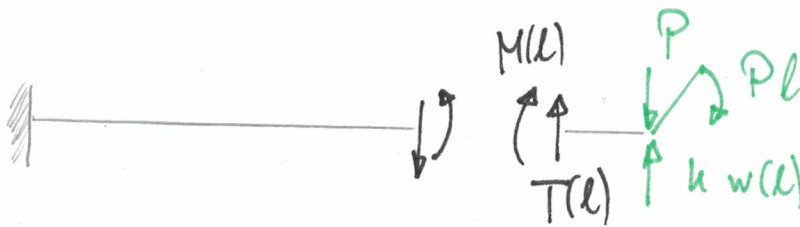
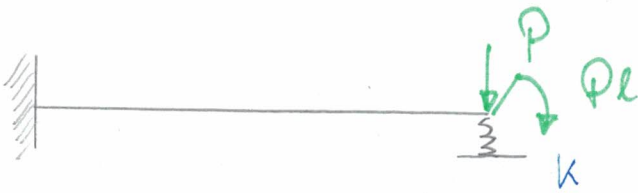
$$\Phi_D^1 = 0 \Rightarrow \varphi_D = \frac{u}{L}$$

$$W_c^1 + W_B^3 = 0 \Rightarrow \frac{E\alpha}{L^2} \left(-3\varphi_D + 3,002\varphi_D - \frac{3}{4} + \frac{3}{8} \varphi_D \right) = 0$$

$$\varphi_D = \frac{6}{3,016} \approx 1,989$$

$$M_A = PL \varphi_D = 1,989 PL$$

Układ zredukowany



$$w(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$$

w.b

$$w(0) = 0$$

$$M(l) = -E\gamma w''(l) = -Pl$$

$$\varphi'(0) = w'(0) = 0$$

$$T(l) = -E\gamma w'''(l) = P - kw(l)$$

$$C_0 = C_1 = 0$$

$$C_2 = \frac{Pl(12E\gamma - kl^3)}{4E\gamma(3E\gamma + kl^3)}$$

$$C_3 = \frac{P(kl^3 - E\gamma)}{4E\gamma(3E\gamma + kl^3)}$$

Moment w utwierdzeniu

$$M(0) = -E\gamma w''(0) = 0 \iff k = \frac{12E\gamma}{l^3}$$