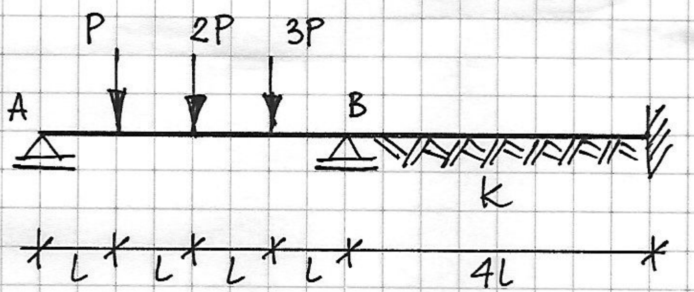
Egzamin z Mechaniki Konstrukcji (MK IPB), 31.01.2017

NAZWISKO, Imię				
rok akademicki zaliczenia ćwiczeń		nr albumu	grupa (IPB / BZ)	tryb studiów (ST / NST)
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egzaminu	ocena łączna

Zadanie 1.

$EJ = const., \qquad k = 0{,}0324\dfrac{EJ}{l^4}$

Oblicz wartości reakcji w podporach A i B w belce z rys. 1.

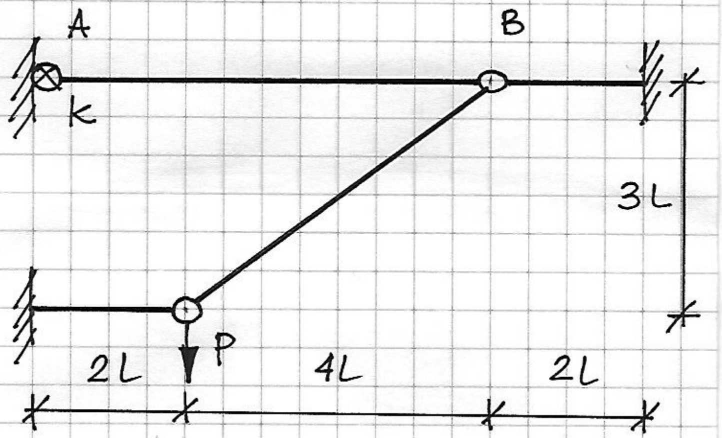


rys. 1

Zadanie 2.

$EJ = const., \qquad k = 4\dfrac{EJ}{l}$

Korzystając z Metody Przemieszczeń zapisz funkcję ugięcia pręta A-B w ramie z rys. 2



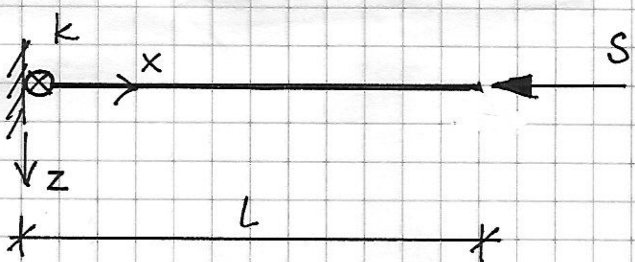
rys. 2

Zadanie 3.

$EJ = const., \qquad k = 10\dfrac{EJ}{l}$

Wyprowadź warunek określający krytyczną wartość siły S w belce z rys. 3.

Wskazówka:
Równanie osi odkształconej ma postać
$w(\xi) = A_0 + A_1\sigma\xi + A_2\cos(\sigma\xi) + A_3\sin(\sigma\xi)$
gdzie
$\xi = \dfrac{x}{l}, \sigma = l\sqrt{\dfrac{S}{EJ}}$

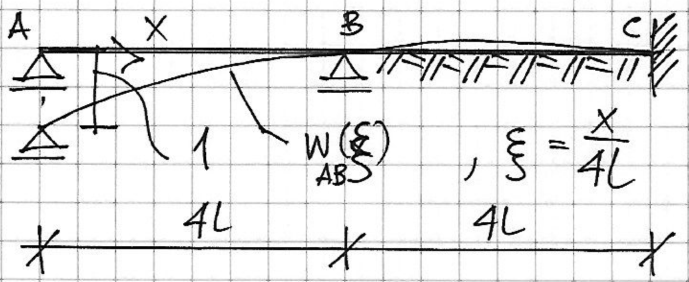


rys. 3

Zadanie 1

Obliczenie V_A z tw. Bettiego.

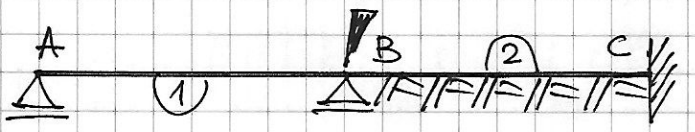
Zadanie pomocnicze – określenie $w_{AB} = w_{AB}(\xi)$.



$\lambda^4 = l^4 \frac{k}{4EJ} \rightarrow \lambda = 0{,}3$

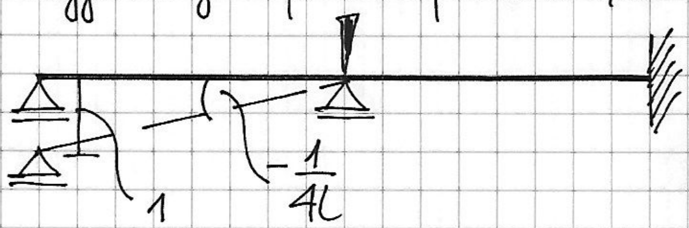
$\lambda^{(2)} = 4\lambda = 1{,}2$

Układ zastępczy



$q = [\varphi_B]$

"Wyjściowy" plan przesunięć



$$\overset{\circ}{\Phi}{}^{(1)}_B = \frac{3EJ}{4l}\left[\frac{1}{4l}\right] = \frac{3EJ}{16l^2}$$

Równanie równowagi

$$\Phi^{(1)}_B + \Phi^{(2)}_B = 0$$

$$\Phi^{(1)}_B = \frac{3EJ}{4l}\varphi_B + \frac{3EJ}{16l^2}$$

$$\Phi^{(2)}_B = \frac{EJ}{4l}\left[\alpha(1{,}2)\varphi_B\right]$$

$$\frac{EJ}{l}\left[\frac{3}{4} + \frac{1}{4}\cdot 4{,}078\right]\varphi_B + \frac{3}{16}\frac{EJ}{l^2} = 0 \rightarrow \varphi_B = -0{,}106\cdot\frac{1}{l}$$

$$w_{AB}(\xi) = A_0 + A_1\xi + A_2\xi^2 + A_3\xi^3 \quad , \quad \xi = \frac{x}{4l}$$

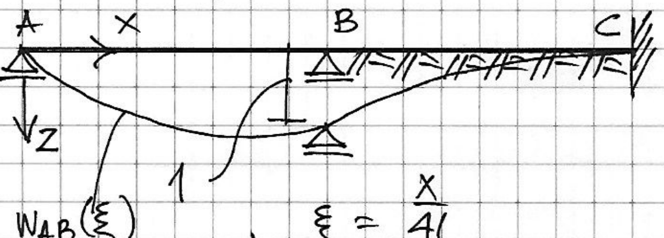
$w_{AB}(0) = 1$	$A_0 = 1$	
$M(0) = 0$	$A_1 = -1{,}288$	$\rightarrow$ $w(\xi) = 1 - 1{,}288\xi + 0{,}288\xi^3$
$w_{AB}(1) = 0$	$A_2 = 0$	
$\varphi(1) = \varphi_B$	$A_3 = 0{,}288$	

Na mocy tw. Bettiego: $V_A = P\cdot w(\frac{1}{4}) + 2P\cdot w(\frac{1}{2}) + 3P\cdot w(\frac{3}{4})$

$V_A = 1{,}933\,P$

Obliczenie V_B

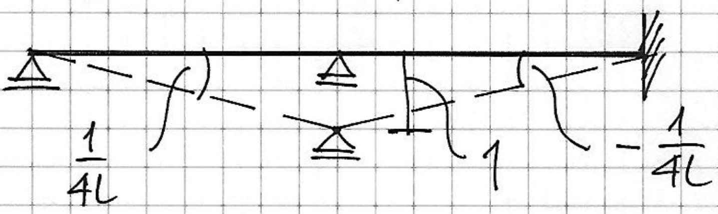
Zadanie pomocnicze



$$, \quad \xi = \frac{x}{4l}$$

Układ zastępczy – jak poprzednio

"Wyjściowy" plan przesunięć



$$\overset{\circ}{\Phi}{}^{(1)}_B = \frac{3EJ}{4l} \cdot \left(-\frac{1}{4l}\right) = -\frac{3}{16}\frac{EJ}{l^2}$$

$$\overset{\circ}{\Phi}{}^{(2)}_B = \frac{EJ}{4l}\left[\vartheta(1,2) \cdot \frac{1}{4l}\right]$$

$$= \frac{1}{16} \cdot 6{,}429 \cdot \frac{EJ}{l^2}$$

Równanie równowagi

$$\Phi^{(1)}_B + \Phi^{(2)}_B = 0$$

$$\Phi^{(1)}_B = \frac{3EJ}{4l}\varphi_B - \frac{3}{16}\frac{EJ}{l^2}$$

$$\Phi^{(2)}_B = \frac{EJ}{4l}\left[\alpha(1,2)\varphi_B\right] + \frac{6{,}429}{16}\frac{EJ}{l^2}$$

$$\varphi_B = -0{,}121 \cdot \frac{1}{l}$$

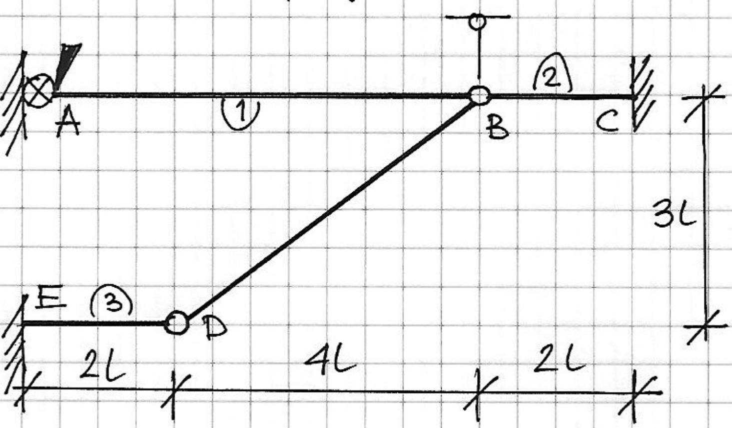
$$w_{AB}(\xi) = A_0 + A_1\xi + A_2\xi^2 + A_3\xi^3$$

$$\left.\begin{array}{l} w_{AB}(0) = 0 \\ M(0) = 0 \\ w_{AB}(1) = 1 \\ \varphi(1) = \varphi_B \end{array}\right| \rightarrow \left.\begin{array}{l} A_0 = 0 \\ A_1 = 1{,}742 \\ A_2 = 0 \\ A_3 = 0{,}742 \end{array}\right| \rightarrow w(\xi) = 1{,}742\xi - 0{,}742\xi^3$$

$$V_B = 4{,}96P$$

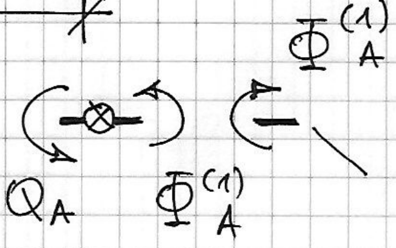
Zadanie 2

Układ zastępczy



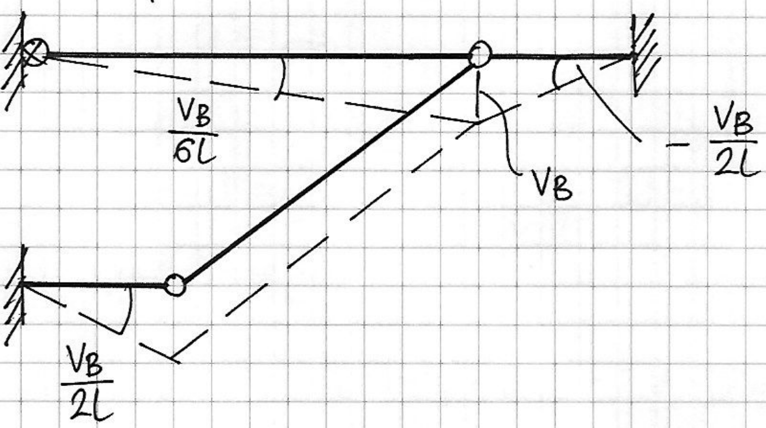
$$q = \begin{bmatrix} \varphi_A \\ \frac{v_B}{L} \end{bmatrix}$$

Węzeł podatny :



$$Q_A = \frac{4EJ}{L}\varphi_A$$

Plan przesunięć :



Równania równowagi:

$$\begin{cases} \Phi_A^{(1)} + Q_A = 0 \\ \Phi_A^{(1)} \cdot \frac{\overline{v_B}}{6L} + \Phi_C^{(2)} \cdot \left(-\frac{\overline{v_B}}{2L}\right) + \Phi_E^{(3)} \cdot \frac{\overline{v_B}}{2L} + P\overline{v_B} = 0 \end{cases}$$

$$\begin{cases} \Phi_A^{(1)} + \frac{4EJ}{L}\varphi_A = 0 \\ -\frac{1}{6}\Phi_A^{(1)} + \frac{1}{2}\Phi_C^{(2)} - \frac{1}{2}\Phi_E^{(3)} - PL = 0 \end{cases}$$

$$\Phi_A^{(1)} = \frac{3EJ}{6L}\left[\varphi_A - \frac{1}{6}\frac{v_B}{L}\right]$$

$$\Phi_C^{(2)} = \frac{3EJ}{2L}\left[\frac{v_B}{2L}\right]$$

$$\Phi_E^{(3)} = \frac{3EJ}{2L}\left[-\frac{v_B}{2L}\right]$$

$$\frac{EJ}{l}\left\{\left[\frac{1}{2}+4\right]\varphi_A-\frac{1}{12}\frac{v_B}{l}\right\}=0$$

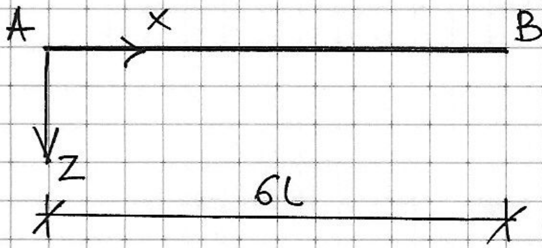
$$\frac{EJ}{l}\left\{\left[-\frac{1}{12}\right]\varphi_A+\left[\frac{1}{72}+\frac{3}{8}+\frac{3}{8}\right]\frac{v_B}{l}\right\}-Pl=0$$

$$\varphi_A=0{,}024\,\frac{Pl^2}{EJ}$$

$$\frac{v_B}{l}=1{,}312\,\frac{Pl^2}{EJ}$$

Linia ugięcia w_{AB}

$$w_{AB}=w_{AB}(\xi)\,,\qquad \xi=\frac{x}{6l}$$



$$w_{AB}(\xi)=A_0+A_1\xi+A_2\xi^2+A_3\xi^3$$

Warunki brzegowe:

$$\left.\begin{aligned} w_A&=0\\ \varphi_A&=0{,}024\,\frac{Pl^2}{EJ}\\ w_B&=1{,}312\,\frac{Pl^3}{EJ}\\ M_B&=0\end{aligned}\right| \longrightarrow \begin{aligned} A_0&=0\\ A_1&=0{,}144\,\frac{Pl^3}{EJ}\\ A_2&=1{,}752\,\frac{Pl^3}{EJ}\\ A_3&=-0{,}584\,\frac{Pl^3}{EJ}\end{aligned}$$

$$w_{AB}(\xi)=\frac{Pl^3}{EJ}\left(0{,}144\,\xi+1{,}752\,\xi^2-0{,}584\,\xi^3\right)$$

Zadanie 3

Warunki brzegowe:

$$w(0)=0 \qquad\qquad w(0)=0$$

$$M(0)=-k\varphi(0) \qquad\qquad -\frac{EJ}{l^2}w''(0) = -k\cdot\frac{1}{l}\cdot w'(0)$$

$$M(1)=0 \qquad\qquad -\frac{EJ}{l^2}w''(1)=0 \qquad\qquad \sigma^2=\frac{Sl^2}{EJ}$$

$$T(1)=0 \qquad\qquad -\frac{EJ}{l^3}\left[w'''(1)+\sigma^2 w'(1)\right]=0$$

$$A_0+A_2=0$$

$$-\frac{EJ}{l^2}\left[-A_2\sigma^2\right]+10\frac{EJ}{l^2}\left[A_1\sigma+A_3\sigma\right]=0$$

$$-\frac{EJ}{l^2}\left[-A_2\sigma^2\cos\sigma-A_3\sigma^2\sin\sigma\right]=0$$

$$-\frac{EJ}{l^3}\left[A_2\sigma^3\sin\sigma-A_3\sigma^3\cos\sigma+\sigma^3A_1 -A_2\sigma^3\sin\sigma+A_3\sigma^3\cos\sigma\right]=0$$

$$\begin{bmatrix} 1 & & 1 & \\ & -10\sigma & -\sigma^2 & -10\sigma \\ & & \sigma^2\cos\sigma & \sigma^2\sin\sigma \\ & -\sigma^3 & & \end{bmatrix}\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\mathbb{C}(\sigma)\qquad \mathbb{A} = \mathbb{O}$$

$$\det\mathbb{C}(\sigma)=\sigma^6\left[\sigma\sin\sigma-10\cos\sigma\right]$$

$$\det\mathbb{C}(\sigma)=0 \iff \sigma=0 \quad\vee\quad \sigma\,\mathrm{tg}\,\sigma=10$$